Math-3A
Lesson 3-2
Graphing Polynomial Equations

Conclusion: To determine the end behavior (and shape of the graph) we need to identify the (1) degree, and (2) if the function has been reflected across the x-axis t

$$y = (2x+8)(3x+6)(x-1)(x-3)$$

Lead term: "left * left * left * left": $2x * 3x * x * x = 6x^4$

$y = 6x^4 + \cdots$ Positive (lead coefficient), even degree

End behavior: Up on left, up on right.

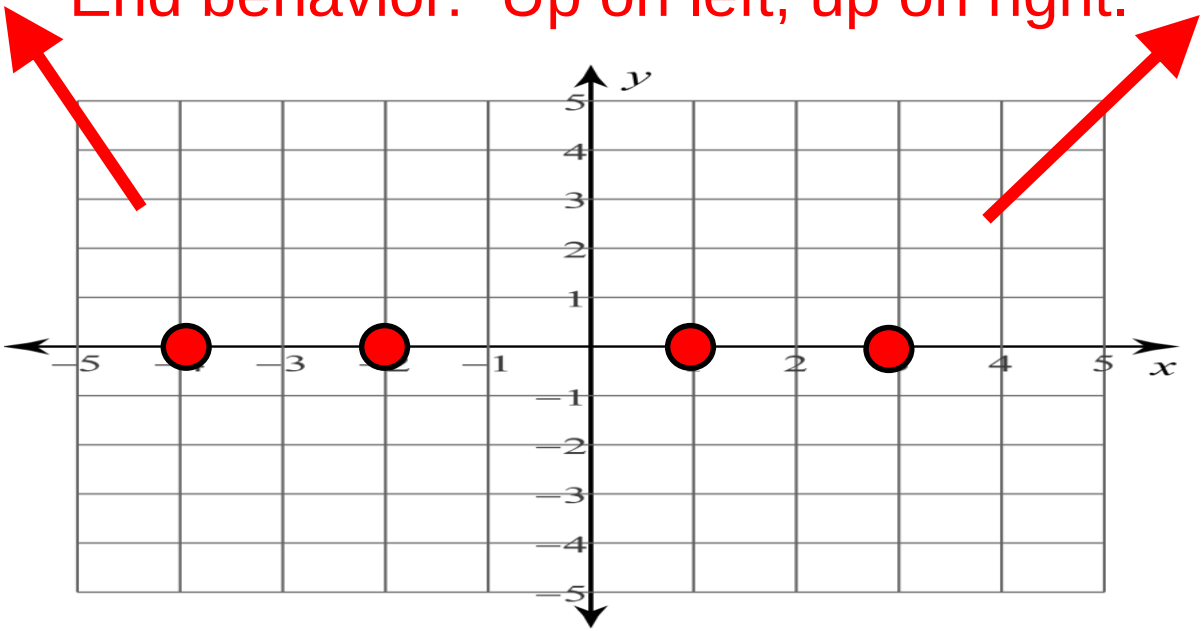
Zeroes:

$x = -4$

$x = -2$

$x = 1$

$x = 3$



We need to fill in the graph (without using a graphing calculator)!

The General Shape of the Graph of a Polynomial

$$f(x) = (x-2)(x-3)(x+4)$$

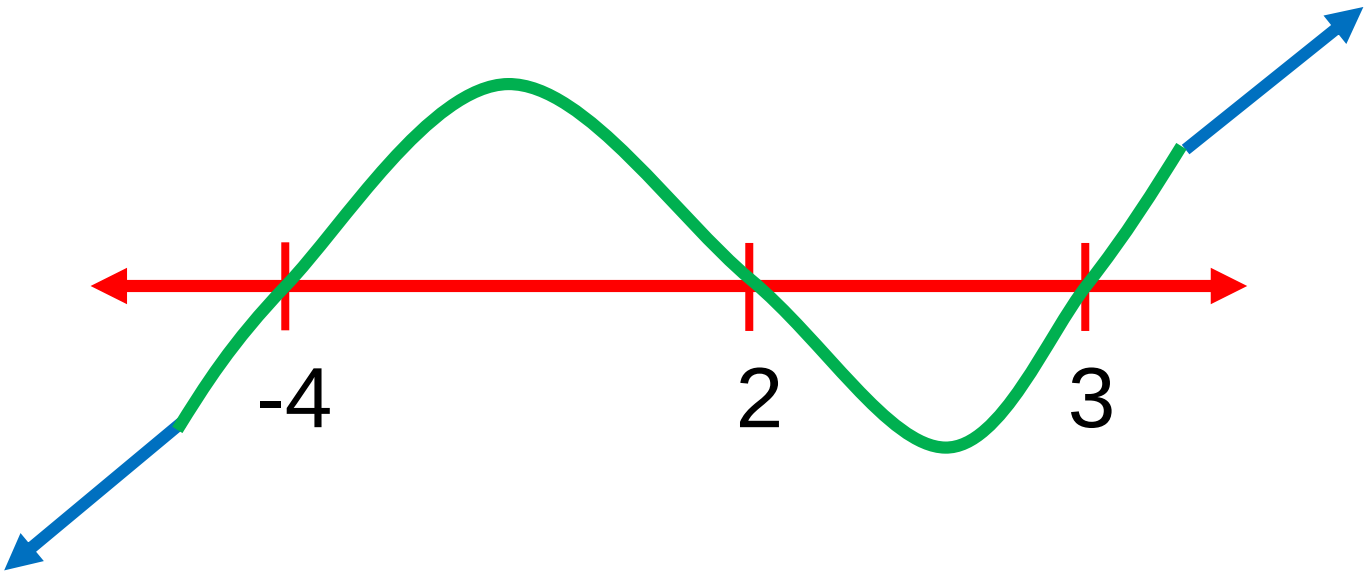
zeroes: x = 2, 3, and -4.

All are real numbers.

All are x-intercepts.

positive lead coefficient and an odd degree.

The end behavior is **Up on right, down on left**.



The General Shape of the Graph of a Polynomial

$$f(x) = x(x+1)(x-1)(x-2)$$

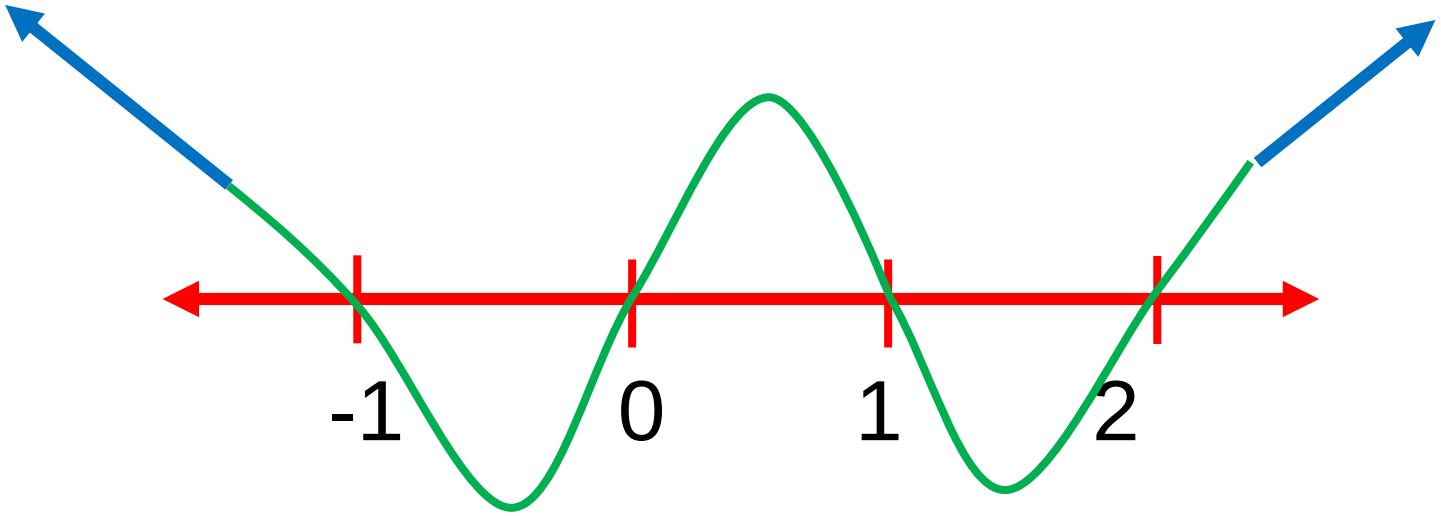
zeroes: x = 0, -1, 1, and 2.

All are real numbers.

All are x-intercepts.

positive lead coefficient and an even degree.

The end behavior is **Up on right, up on left**?



$$f(x) = (x+1)^2(x+3)(x-4)$$

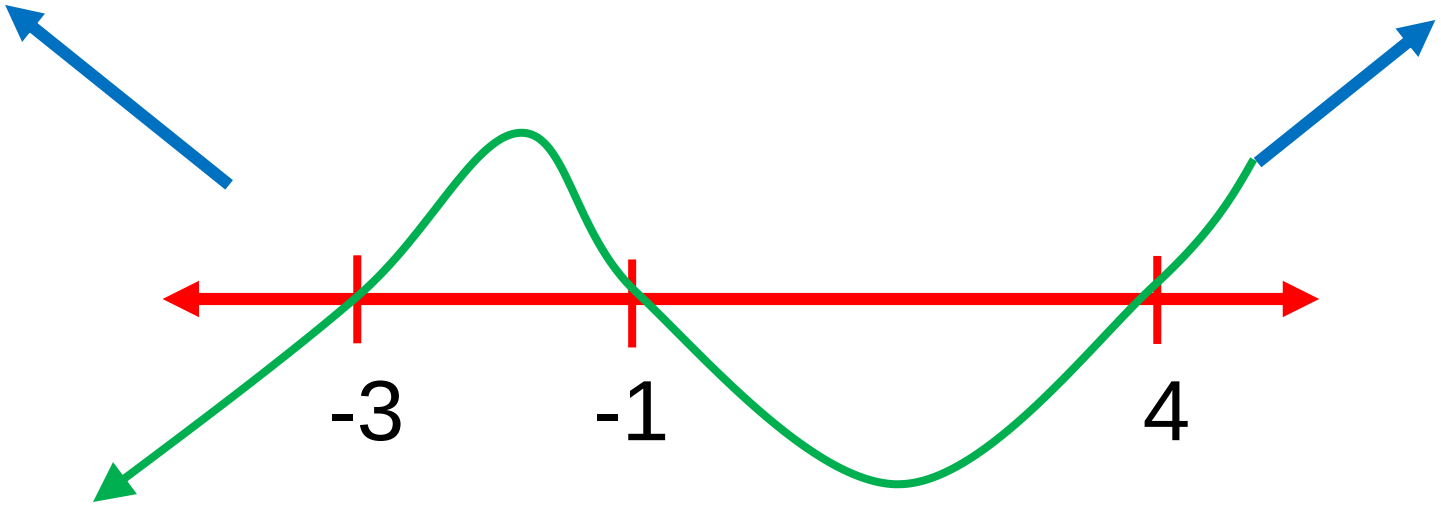
zeroes: x = -1, -1, -3, and 4.

All are real numbers.

All are x-intercepts.

positive lead coefficient and an even degree.

The end behavior is Up on right, up on left?

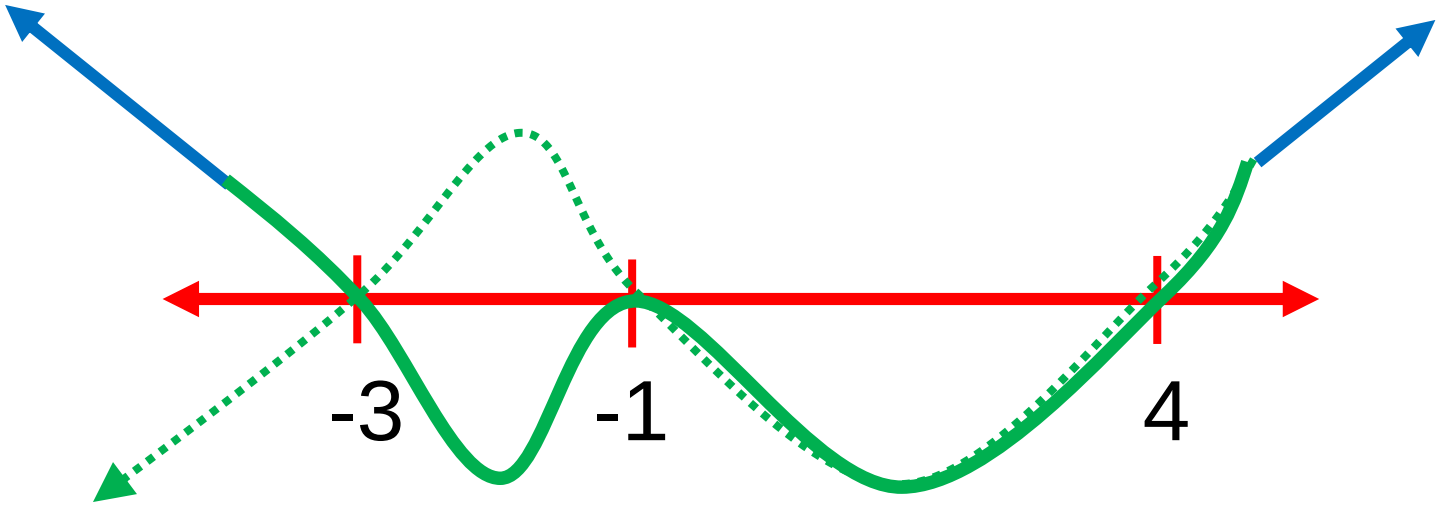


Why doesn't the "end behavior" line up?

$$f(x) = (x+1)^2(x+3)(x-4)$$

It has the following zeroes: x = -1, -1, -3, and 4.

The zero with an <u>EVEN</u> "multiplicity will just "kiss" the x-axis. Remember $y = x^2$?



Draw the Graph of a Polynomial

$$f(x) = (x+2i)(x-2i)(x-4)^2(x+2)$$

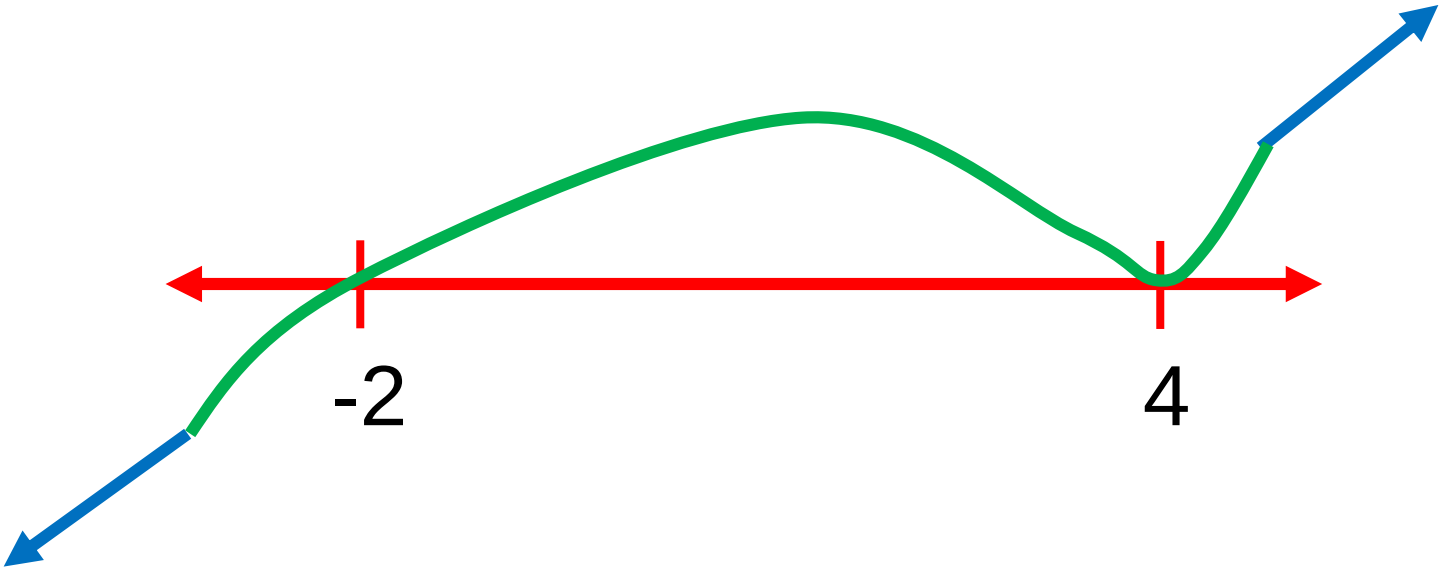
It has the following zeroes: x = 2i, -2i, 4, 4, and -2.

Only 4 and -2 are real numbers. These are x-intercepts.

positive lead coefficient and an odd degree.

The end behavior is **Up on right, down on left**

The graph "kisses" at x = 4



Summary

Even degree polynomials: up on right, up on left

odd degree polynomials: up on right, down on left

Negative coefficient switches the end behavior.

Polynomial degree tells the number of zeroes and the maximum number of x-intercepts.

Constant term is the y-intercept.

What are the x-intercepts?

$$y = (x+2)(x-2)(x-3)$$

-2 +2 +3

Write the equation in standard form.

$$y = (x^2 - 4)(x-3)$$

$$y = x^2(x-3) - 4(x-3)$$

$$y = x^3 - 3x^2 - 4x + 12$$

Product of Left side terms

Product of right side terms

$$y = -2(x-3)(x+3)(x+1)$$

a) What is the degree of the polynomial?

3rd degree

b) What is the lead coefficient? -2

c) What is the end-behavior? Up left, down right

d) What are the x-intercepts? 3, -3, -1

e) What is standard form of the polynomial?

$$y = -2x^3 - 2x^2 + 18x + 18$$

f) What is y-intercept? $(0,\ 18)$

$$y = (x - \sqrt{2})(x + \sqrt{2})(x + 6)$$

a) What is the degree of the polynomial?

3rd degree

b) What is the lead coefficient?

1

c) What is the end-behavior?

down left, up right

c) What are the x-intercepts?

$$\sqrt{2}, -\sqrt{2}, -6$$

d) What is standard form of the polynomial?

$$y = x^3 + 6x^2 + 2x + 12$$

$$y = 2(x-1)(-2x+2)(x-3)(x+4)$$

a) What is the degree of the polynomial?

4^{th} degree

b) What is the lead coefficient? -4

c) What is the end-behavior? Up left, down right

d) What are the x-intercepts? 1 (mult.=2), 3, -4

e) What is standard form of the polynomial?

$$y = -4x^4 - 4x^3 - 52x^2 + 100x + 48$$

e) What is the constant? 48

How to get the standard form:

$$y = 2(x-1)(x+2)(x-3)(x+4)$$

$$y = 2(x^2+x-2)(x^2+x-12)$$

	x^2	x	-12
x^2	x^4	x^3	$-12x^2$
x	x^3	x^2	$-12x$
-2	$-2x^2$	$-2x$	24

$$y = x^4 + 2x^3 - 13x^2 - 14x + 24$$

Diagonals have “like” terms

$$y = 5(x+5)(x-6)(x+7)$$

a) What is the degree of the polynomial?

3rd degree

b) What is the lead coefficient?

5

c) What is the end-behavior?

down left, up right

c) What are the x-intercepts?

-5, 6, -7

d) What is standard form of the polynomial?

$$y = 5x^3 + 30x^2 - 50x - 1050$$

Your turn: What are the x-intercepts?

$$y = 5(x+5)(x-6)(x+7)$$

$$y = (x-\sqrt{2})(x+\sqrt{2})(x+6)$$

$$y = (x-3i)(x+3i)(x-2)$$

Given the following solutions, write the standard from polynomial.

$$x = -4, 4, \sqrt{2}, -\sqrt{2}$$

$$y = (x+4)(x-4)(x-\sqrt{2})(x+\sqrt{2})$$

Multiply out:

$$y = (x^2 - 16)(x^2 - 2)$$

$$y = x^2(x^2 - 2) - 16(x^2 - 2)$$

$$y = x^4 - 2x^2 - 16x^2 + 32$$

$$y = x^4 - 18x^2 + 32$$

Quadratic form (comes from 2 pairs of conjugate pairs).

Quadratic “Form” $y = x^4 + 10x^2 + 9$

Factor the polynomial $(x^2 - 9)(x^2 - 1) = 0$

$$(x+3)(x-3)(x+1)(x-1) = 0$$

x-intercepts = 3, -3, 1, -1

Convert to “intercept form” (by factoring) then find the zeros of the polynomial.

$$y = x^4 - 5x^2 + 4$$

$$y = x^4 - 8x^2 + 16$$

Solve by factoring.

$$0 = 5x^2 - 10x + 5$$

$$0 = 2x^5 - 16x^4 + 32x^3$$

$$0 = 4x^2 - 16x + 16$$