# Math-3

## Lesson 2-1

Factoring

"Expression" (a math "phrase") A name or a symbol for a number

$4$ $x + 3$ $3x + 4y - 2$

Do you see an equal sign in an expression?

"Statement" (a math sentence)

A meaningful assertion that is either true or false.

The most common "statement" is an equation.

$$x + 3 = 5$$

Another "statement" could be an inequality.

$$x + 3 \leq 5$$

Equivalence?

Consult with your neighbor to define "equivalence" as it applies to mathematics.

Fill in the blank: $7 - 4 = \underline{\quad 5 - 2 \quad}$

Are there any other possible "equivalences"?

$$"3" = \{3, \frac{6}{2}, \frac{3x}{x}, (5-2), \ldots\}$$

Solution: the number (or numbers) that when substituted in for the "letter" (x, y, m, etc.) make the statement true.

Equivalent Equations — Equations that look different by have the same solutions.

$x = 2$ and $2x = 4$ are equivalent equations.

Can an expression have a solution?

Are expressions math statements (that are either true or false)?

"Variable" vs. "Unknown Value"

variable: A letter or symbol can have many values as the solution.

$x + 3 = 5$

'x' is an unknown value

$3x + 4y = 12$

'x' and 'y' are the variables

What is it?

a. Statement
b. Equation
c. expression

1. $3 + 4 – 1 = 6$
2. $x + 2y$
3. $ax + by > c$

<u>Terms</u> The individual numbers in an expression or an expression or equation that are <u>separated by either a "+" or "-" symbol</u>.

$\boxed{4x}$ — 1 term — "<u>Monomial</u>"

$\boxed{x + 3}$ — 2 terms — "<u>Binomial</u>"

$\boxed{2x^2 + 3x - 4}$ — 3 terms — "<u>Trinomial</u>"

$\boxed{x^3 - 5x^2 + x - 1}$ — More than 3 terms? — "<u>Polynomial</u>"

Coefficient The number in front of a variable in an expression or an equation.

$$3x + 4y - 2$$

3 is the coefficient of 'x'

4 is the coefficient of 'y'

Constant A term in an expression or an equation that does not contain a variable

$3x + 4y - 2$ -2 is a constant (it's "constantly" -2 regardless of the values of 'x' or 'y')

$2x + 3 = 5$ Both 3 and 5 are constants

Factor (noun) a number (or expression) that is being multiplied by another number (or expression).

$2x$ Factors: 2, x.

$2(x+3)$ Factors: 2, (x + 3).

Why is (x + 3) a factor? (it looks like a sum)

Because it is an expression that is being multiplied by ‘2’.

$$2 * (x + 3)$$

To Factor (verb) to break a number or an expression into two (or more) parts (factors) that are multiplied together.

10 → 2*5

Common Factor (noun) a number that is a factor of more than one term in an expression.

The expression 2x + 6 has the common factor '2' in both terms

We can see this if we factor each term individually:

2x + 6 → (2*x) + (2*3)

"Factoring out" a common Factor from an expression means to rewrite the expression as the *common factor multiplied by* the expression.

2x + 6 → 2(x + 3)

"Factoring out the common factor" is actually the reverse of the distributive property!

distributive property: an expression of terms being added that is multiplied by another number or expression.

2(x + 3) = 2x + 6

Factoring out the common factor: the "reverse" of the distributive property.

Identify the factors in each expression.

$$5x(3x+1)(2x-5) \rightarrow x^2, (x-2), (x+3)$$

$$x^2(x-2)(x+3) \rightarrow 5, x, (3x+1), (2x-5)$$

Factors can be an expression made up of terms being added.

## Sometimes the common factor is an integer

$$3x - 12$$
$$(3 * x) - (3 * 4)$$
$$3(x - 4)$$

$$-4x^2 + 8x + 12$$
$$(-4 * x * x) + (-4 * -2 * x) + (-4 * -3)$$
$$-4(x^2 - 2x - 3)$$

## Sometimes the common factor is a variable

$$x^2 + x$$
$$(x * x) + (1 * x)$$

$$x^3 + x^2 + x$$
$$(x * x^2) + (x * x) + x * 1$$

"x" is a common factor both terms

$$x(x + 1)$$

$$x(x^2 + x + 1)$$

## Sometimes the common factors are both an **integer** and a **variable**.

$$4x^2 - 16x$$

$$(4 * x * x) - (4 * 4 * x)$$

$$4x(x - 4)$$

$$5x^3 + 15x^2 + 10x$$

$$(5 * x * x * x) + (3 * 5 * x * x) + (2 * 5 * x)$$

$$5x(x^2 + 3x + 2)$$

Factor the following expressions

$$-50b + 90$$

$$-10 + 20n^3$$

$$-60x^5 - 100x^4 - 30x^2$$

$$-81r - 63r^3 - 63r^4$$

$$-24x^4 + 40x^3 - 80x^2 + 16x$$

$$-40x^6 + 20x^2 + 4x + 8$$

# Multiplying Binomials

The "Box Method"

$(x-3)(x+4)$

$\boxed{x^2+x-12}$

| | x | 4 |
|---|---|---|
| x | $x^2$ | 4x |
| -3 | -3x | -12 |

Standard Form Quadratic Expression

$(x-1)(x+5)$

| | x | 5 |
|---|---|---|
| x | $x^2$ | 5x |
| -1 | -x | -5 |

$\boxed{x^2+4x-5}$

$(x+2)(x+6)$

| | x | 6 |
|---|---|---|
| x | $x^2$ | 6x |
| 2 | 2x | 12 |

$\boxed{x^2+8x+12}$

$(x-4)(x+4)$

| | x | 4 |
|---|---|---|
| x | $x^2$ | 4x |
| -4 | -4x | -16 |

$x^2+0x-16$

$\boxed{x^2-16}$

# Your turn:

$$(x+2)(x+3)$$

Multiply the two binomials

What method did you use?

Arrows

Distributive Property (twice)

FOIL

Box method

$$(x+2)(x+3)$$ <u>multiply</u>

$$x^2$$ "<u>left</u> times <u>left</u> is the <u>left</u> term"

$$(x+2)(x+3)$$ "<u>right</u> times <u>right</u> is the <u>right</u> term"

$$x^2 \qquad\qquad +6$$

$$(x+2)(x+3)$$ "<u>inner</u>"

$$x^2 \quad +2x \qquad\qquad +6$$

$$(x+2)(x+3)$$ "<u>outer</u>"

$$x^2+2x+3x+6 = x^2+(2+3)x+(2*3)$$

$$(x+2)(x+3)$$

$$= x^2 + (2+3)x + (2*3)$$

$$= x^2 + 5x + 6$$

Left times left is left

Right plus right is middle

Right times right is right

$$(x+4)(x+5)$$

$$= x^2 + (4+5)x + (4*5)$$

Left times left is left

Right plus right is middle

Right times right is right

$$= x^2 + 9x + 20$$

$$(x-6)(x+1)$$

$$= x^2 + (-6+1)x + (-6*1)$$

Left times left is left

Right plus right is middle

Right times right is right

$$= x^2 - 5x - 6$$

$$x^2 + 5x + 6$$

Left times left is left

$$(\_\_\_ + \_\_\_)(\_\_\_ + \_\_\_)$$

$$(x + \_\_\_)(x + \_\_\_)$$

Right times right is right

$$(x + \_\_\_)(x + \_\_\_)$$

Right plus right is middle

$$(x+2)(x+3)$$

**What are the factors of 6 that add up to 5?**

# Try the following:

$$x^2 - 3x - 4 \qquad = (x-4)(x+1)$$

$$(x + \_\_\_)(x + \_\_\_)$$

<u>Right</u> times <u>right</u> is <u>right</u>

$$(x + \_\_\_)(x + \_\_\_)$$

<u>Right</u> plus <u>right</u> is <u>middle</u>

$$(-4)(1) = -4$$

$$(-4) + (1) = -3$$

**What are the factors of -4 that add up to -3?**

# Try the following:

$$x^2+8x+15 \quad = (x+3)(x+5)$$

$$(x+\underline{\quad\quad})(x+\underline{\quad\quad})$$

<u>Right</u> times <u>right</u> is <u>right</u>

$$(x+\underline{\quad\quad})(x+\underline{\quad\quad})$$

<u>Right</u> plus <u>right</u> is <u>middle</u>

$$(3)(5)=15$$

**What are the factors of 15 that add up to 8?**

$$3+5=8$$

# Try the following:

$$x^2+10x+21 \quad =(x+3)(x+7)$$

$$x^2-6x-16 \quad =(x-8)(x+2)$$

$$x^2-9x+18 \quad =(x-6)(x-3)$$

$$2x^2+4x+2$$

Always factor out the common factor first.

$$2(x^2+2x+1)$$

Now factor the trinomial.

$$2(x+1)(x+1)$$

Your turn:

$$6x^2 + 24x + 18$$

Always factor out the common factor 1st.

$$6(x^2 + 4x + 3)$$

Now factor the trinomial.

$$6(x+1)(x+3)$$

# Skills we need

- How to “factor out the common factor” from an expression
- How to factor a trinomial into two binomials
- **How to factor some special binomials into two binomials.**

$$x^2 - 1$$

"the difference of two squares"

$$x^2 + 0x - 1$$

Two numbers multiplied = (-1) and added = 0

$$(-1)(+1)$$

$$(x-1)(x+1)$$

# Vocabulary

<u>Conjugate pair </u>(of binomials)

two binomials whose terms are exactly the same <u>except</u> +/- for one pair of terms

$$(x-1)(x+1)$$

$$(-x+1)(x+1)$$

Which of the following are NOT conjugate pairs?

$$(x-4)(x+4)$$

Yes, they are.

$$(5-x)(5+x)$$

Yes, they are.

$$(3x+2)(3x+2)$$

NO, they are NOT.

$$(x-\sqrt{5})(x+\sqrt{5})$$

Yes, they are.

$$x^2 - 2$$

We can call this the "difference of two squares"

$$x^2 + 0x - 2$$

Two numbers multiplied = (-2) and added = 0

$$(-\sqrt{2})(+\sqrt{2})$$

$$(x - \sqrt{2})(x + \sqrt{2})$$

Your turn: Multiply the conjugate pairs.

$$(x-1)(x+1) = x^2 - 1$$

$$(x-\sqrt{2})(x+\sqrt{2}) = x^2 - 2$$

Can we use this as a pattern in order to <u>factor</u> the difference of two squares?

$$(x-\sqrt{3})(x+\sqrt{3}) = x^2 - 3$$

$$(x-\sqrt{4})(x+\sqrt{4}) = x^2 - 4 = (x-2)(x+2)$$

A “nice” one.

$$(x-\sqrt{5})(x+\sqrt{5}) = x^2 - 5$$

$$(x-\sqrt{a})(x+\sqrt{a}) = x^2 - a$$

$$(x-\sqrt{17})(x+\sqrt{17}) = x^2 - 17$$

Your turn: factor the following binomials

$$x^2 - 6 \qquad = (x - \sqrt{6})(x + \sqrt{6})$$

$$x^2 - 9 \qquad = (x - \sqrt{9})(x + \sqrt{9})$$

$$= (x - 3)(x + 3)$$

Multiply this out: $(x+i)(x-i)$

$x^2 \quad -xi \qquad +xi \qquad -i^2$

“i” terms “cancel” *i* squared = -1

$x^2 \quad -(-1)$

$x^2 \quad +1$

$x^2+1$

Multiply this out: $(x+i\sqrt{2})(x-i\sqrt{2})$

$$x(x-i\sqrt{2})+i\sqrt{2}(x-i\sqrt{2})$$

$$x^2 \quad \cancel{-xi\sqrt{2}} \quad \cancel{+xi\sqrt{2}} \quad -i^2\sqrt{2}*\sqrt{2})$$

"i" terms cancel.

i –squared = -1

$$x^2 \quad -(-1)\sqrt{2}*\sqrt{2})$$

$$x^2 \quad +\sqrt{2}*\sqrt{2})$$

$$x^2+2$$

What about the <u>sum of two squares</u>?

$$x^2+1=(x-i)(x+i)$$

$$x^2+2=(x-i\sqrt{2})(x+i\sqrt{2})$$ Can you see the pattern?

$$x^2+3=?=(x-i\sqrt{3})(x+i\sqrt{3})$$

$$x^2+4=?=(x-i\sqrt{4})(x+i\sqrt{4})=(x-2i)(x+2i)$$

$$x^2+a=(x-i\sqrt{a})(x+i\sqrt{a})$$ General form.

$$x^2+7=?=(x-i\sqrt{7})(x+i\sqrt{7})$$