Math-3A

Lesson 12-6

Add, Subtract, Multiply, and Divide Rational Expressions

Adding Fractions

We can add "like fractions"

$$\frac{2}{3} + \frac{1}{3} + \frac{4}{3}$$

Combine the numerator over a <u>common</u> denominator.

$$= \frac{2+1+4}{3} \qquad = \frac{7}{3}$$

Vocabulary

A <u>rational number</u> can be written as a ratio of integers. $\dfrac{2}{5}, \dfrac{3}{1}$

A <u>rational expression</u> can be written as a ratio of expressions.

$$\frac{x}{(x+1)}$$

We will be looking at <u>ratios of polynomial expressions</u>.

<u>Excluded Value</u>: the value for 'x' that results in division by zero

$$\frac{x}{(x+1)}, \quad x \neq -1$$

Simplifying Fractions

You must FACTOR the fractions.

$$\frac{32}{44} = \frac{4*8}{4*11} \qquad \frac{x^2-4}{x^2-3x+2} = \frac{(x-2)(x+2)}{(x-2)(x-1)}$$

Break them apart into the product of fractions.

$$= \frac{4}{4} * \frac{8}{11} \qquad = \frac{(x-2)}{(x-2)} * \frac{(x+2)}{(x-1)}$$

Notice the fractions that equal '1'

$$= 1*\frac{8}{11} \quad = \frac{8}{11} \qquad = 1*\frac{(x+2)}{(x-1)} = \frac{(x+2)}{(x-1)}$$

Adding/Subtraction Rational Expressions

The <u>easy</u> problem: $\dfrac{2}{7} + \dfrac{3}{7} = \dfrac{2+3}{7} = \dfrac{5}{7}$

Combine the numerator over a <u>common</u> denominator.

The <u>easy</u> problem: $\dfrac{4}{(x-5)} + \dfrac{3x}{(x-5)} = \dfrac{4+3x}{(x-5)}$

Combine the numerator over a <u>common</u> denominator.

Can you combine 4 and 3x ?

Why not?

Your turn: add/subtract

$$\frac{x+2}{2x^2} + \frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{\cancel{2}(x-1)}{\cancel{2}*x^2} = \frac{(x-1)}{x^2}$$

What property are we using?

Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2} + \frac{x-4}{2x^2} = \frac{\cancel{x}+\cancel{2}}{\cancel{2}*\cancel{x}*x} + \frac{\cancel{x}-(\cancel{2}*2)}{\cancel{2}*\cancel{x}*x} = \frac{1}{x} + \frac{-2}{x}$$

Why not?   You CANNOT use the Inverse Property of Multiplication on <u>addends.</u>

Your turn: add/subtract

$$\frac{x+2}{2x^2} + \frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{2(x-1)}{2*x^2}$$

I will not allow you to simplify using the Inverse Property of Multiplication until you have factored it into two fractions.

$$= \frac{2}{2} * \frac{(x-1)}{x^2} = \frac{(x-1)}{x^2}$$

Only then will you be able to see how the Inverse Property of Multiplication changes the rational expression into multiplication by one.

Your turn: add/subtract

Which one is correct?

$$\frac{(2x-7)}{x^2+2} - \frac{(x-4)}{x^2+2}$$

$$= \frac{2x-7-x-4}{x^2+2}$$

$$= \frac{2x-7-x+4}{x^2+2}$$

$$\frac{(2x-7)-(x-4)}{x^2+2} = \frac{2x-7-x-(-4)}{x^2+2} = \frac{x-3}{x^2+2}$$

Subtract <u>every term </u>in the right side numerator! (Half of you will make this mistake on the HW and on the Test).

Can you factor this into two fractions multiplied together?

<u>One third of you will miss this on the test.</u>

Multiplying Rational Expressions

Simplify before you multiply.

$$\frac{(x-1)}{2(x+3)} * \frac{x^2-9}{(x+1)} = \frac{(x-1)}{2(x+3)} * \frac{(x+3)(x-3)}{(x+1)}$$

$$= \frac{\cancel{(x+3)}}{\cancel{(x+3)}} * \frac{(x-1)(x-3)}{2(x+1)} = \frac{(x-1)(x-3)}{2(x+1)}$$

DON'T multiply the simplified version of the product, just leave it in factored form.

Your turn: Multiply the expressions

$$\frac{3(x-4)}{(x-3)} * \frac{(x-2)}{(x-4)} * \frac{(x-3)}{6(x-2)} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{(x+3)}{(x-5)} * \frac{(x^2-16)}{(x+4)} = \frac{(x+3)(x-4)(x+4)}{(x-5)(x+4)}$$

$$= \frac{(x+3)(x-4)}{(x-5)}$$

Your turn: Multiply the expressions   (Solutions on this slide)

$$\frac{x^2 + x - 12}{x^2 - 9} * \frac{x^2 - 2x - 15}{x^2 - 16}$$

$$\frac{(x-3)(x+4)}{(x-3)(x+3)} * \frac{(x-5)(x+3)}{(x+4)(x-4)} = \frac{(x-5)}{(x-4)}$$

Your turn:

Multiply the expressions

$$\frac{2x^2 - 8x - 24}{x^2 + 2x - 3} * \frac{x^2 + 7x + 12}{x^2 - 2x - 24}$$

$$\frac{2(x^2 - 4x - 12)}{(x+3)(x-1)} * \frac{(x+3)(x+4)}{(x-6)(x+4)} = \frac{2(x^2 - 4x - 12)}{(x-1)(x-6)}$$

$$= \frac{2(x-6)(x+2)}{(x-1)(x-6)} \qquad = \frac{2(x+2)}{(x-1)}$$

Divide Rational Expressions

$$\frac{2}{3} \div \frac{5}{7}$$ What do we do?   <u>Multiply by the reciprocal</u>

$$\frac{2}{3} * \frac{7}{5} \qquad = \frac{14}{15}$$

Dividing Rational Expressions

$$\frac{x+3}{x^2+x-6} \div \frac{x-8}{x-2} = ? \qquad = \frac{x+3}{x^2+x-6} * \frac{x-2}{x-8}$$

simplify then multiply!

$$= \frac{x+3}{(x+3)(x-2)} * \frac{x-2}{x-8} \qquad = \frac{(x+3)(x-2)}{(x+3)(x-2)(x-8)}$$

$$\frac{1}{(x-8)} \quad \text{OR} \quad (x-8) \qquad ?$$

Your turn:
$$\frac{x+3}{x^2+x-6} \div \frac{x-8}{x^2+x-6}$$

$$\frac{x+3}{x^2+x-6} * \frac{x^2+x-6}{x-8} = \frac{(x+3)}{(x^2+x-6)} * \frac{(x^2+x-6)}{(x-8)} = \frac{(x+3)}{(x-8)}$$

Your turn:

$$\frac{x^2 + 2x - 35}{x^2 - 4x - 12} \div \frac{x^2 - 2x - 15}{x^2 + 9x + 14}$$

$$\frac{x^2 + 2x - 35}{x^2 - 4x - 12} * \frac{x^2 + 9x + 14}{x^2 - 2x - 15} = \frac{(x+7)(x-5)}{(x-6)(x+2)} * \frac{(x+2)(x+7)}{(x-5)(x+3)}$$

$$= \frac{(x+7)(x+7)}{(x-6)(x+3)}$$