Math-3
Lesson 12-4
Quadratic Formula

Your turn: Solve by factoring.

1. $y = x^2 - 5x + 4$

2. $y = x^2 - 6x - 27$

Vocabulary

Quadratic Formula: gives the solutions (x-intercepts) to ANY quadratic equation in standard form.

$$y = ax^2 + bx + c \qquad y = 2x^2 + 3x + 4$$

a = 2 b = 3 c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How did we get the quadratic formula?

We covert the following standard form quadratic equation to vertex form. $y = ax^2 + bx + c$

Which of the letters are variables? x and y

Set y = 0 so that we are finding "zeroes" of the equation.

$0 = ax^2 + bx + c$ x-coordinate of vertex: x = -b/2a

The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right) = a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c$$

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c$$

The x-coordinate of the vertex is: x = -b/2a
The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c \quad = \frac{b^2}{4a} - \frac{b^2}{2a} + c$$

Obtain common denominators then Add fractions

$$f\left(-\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a} \qquad = \frac{-b^2 + 4ac}{4a}$$

Write in Vertex form:

$$0 = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{-b^2+4ac}{4a}\right)$$

Vertex form: $0 = a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a}\right)$

Isolate the square, undo the square

$$\frac{b^2 - 4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$\pm\frac{\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Your turn: $y = ax^2 + bx + c$

Identify ‘a’ ‘b’ and ‘c’ for each of these standard form quadratic equations.

$$y = x^2 - 4x + 3$$

$$y = -2x^2 + 3x - 7$$

$$y = 3x^2 - 10$$

$$y = -x^2 + 3x$$

What if it's not in standard form?

$$2x = 3x^2 - 5$$

Use math properties to get it into standard form
(same thing left/right, combine like terms, etc.)

$$0 = 3x^2 - 2x - 5$$ a = 3 b = -2 c = -5

a = ? b = ? c = ?

Determine the following values: a = ?, b = ?, c = ?

$$y = 3 - 12x^2 - 4x$$

$$5x = 3x^2 - 5x + 1$$

Solve using the Quadratic formula.

$$y = ax^2 + bx + c$$

$$y = x^2 - 6x + 4$$

a = 1 b = -6 c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$x = \frac{6 \pm \sqrt{4}\sqrt{5}}{2}$$

$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

If the quadratic CANNOT be factored, the solutions are “ugly.”

Can you "plug" 'a', 'b', and 'c' into the Quadratic formula?

$$y = ax^2 + bx + c$$

Identify 'a', 'b', and 'c' in the standard form equation.

$$y = x^2 + 3x + 2$$

a = 1 b = 3 c = 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Replace the 'a', 'b', and 'c' with parentheses!!!

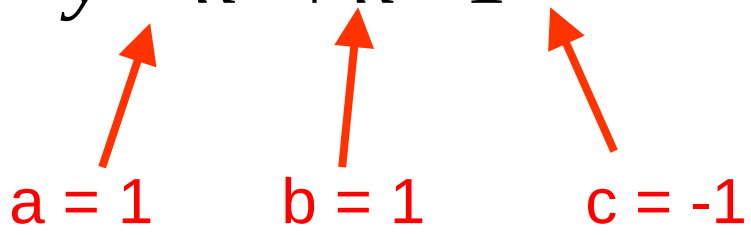
$$x = \frac{-(\ \ ) \pm \sqrt{(\ \ )^2 - 4(\ \ )(\ \ )}}{2(\ \ )}$$

Put the numerical values of 'a', 'b', 'c' into the parentheses.

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(2)}}{2(1)}$$

$$y = ax^2 + bx + c$$

$$y = x^2 + x - 1$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

"Gotcha" parts of the formula.

$$y = ax^2 + bx + c$$

$$y = x^2 - 15x - 1$$

a = 1

b = -15

c = -1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$$

If you don't use the parentheses, it would be easy to write this as -15

With the parentheses, -(-15) simplifies to +15!!!

If you don't use the parentheses, it would be easy to write this as -15^2 instead of $(-15)^2$

$$-15^2 = -225$$

$$(-15)^2 = +225$$

“Gotcha” parts of the formula.

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$$

It is easy to make mistakes with the negatives.

If you use parentheses, then you can type the expression under the radical into your calculator and it will be correct. $(-15)^2 - 4(1)(-1) = 229$

$$x = \frac{15 \pm \sqrt{229}}{2}$$

Plug in and simplify

$$y = ax^2 + bx + c$$

$$y = x^2 + x - 1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{-1}{2} + \frac{\sqrt{5}}{2}$$

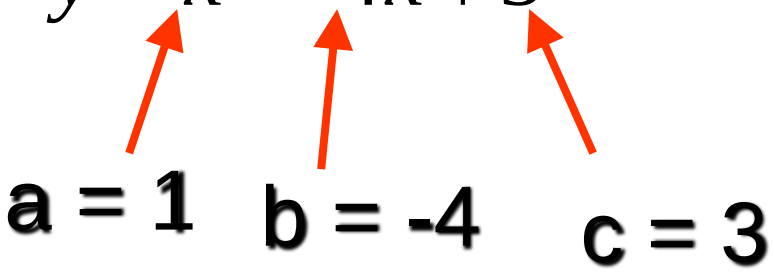
$$x = \frac{-1}{2} - \frac{\sqrt{5}}{2}$$

Another one (be careful of the “gotcha’s”)

$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = x^2 - 4x + 3$$



$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

Can you simplify the result?

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16-12}}{2}$$

$$x = \frac{4 \pm \sqrt{4}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4}{2} \pm \frac{2}{2}$$

$$x = 2 \pm 1$$

$$x = 1, 3$$