Math-3A

12-1

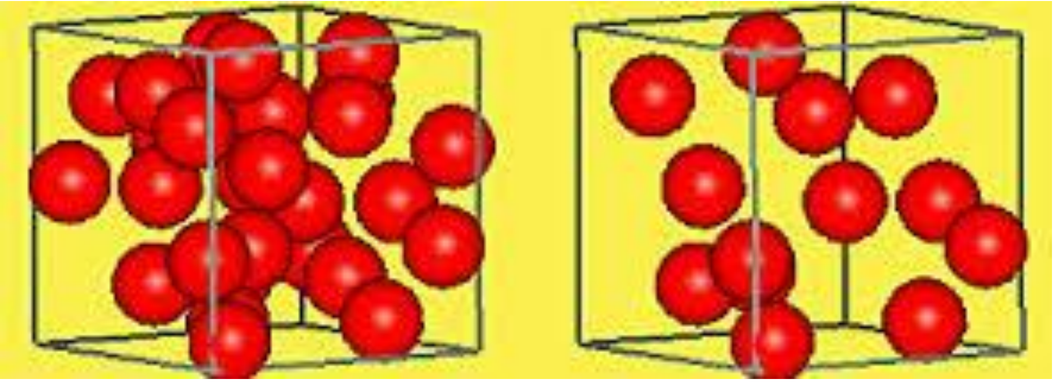
Modeling Density

and

Rewriting formulas for the variable of interest

How would you compare the two collections?

Devise a "rate" type quantity so that we can compare the two amounts.



The boxes above have a side length of 2 inches.

$$\frac{27\text{ spheres}}{8\text{ in}^3} = \frac{3.375\text{ spheres}}{\text{in}^3} \qquad \frac{12\text{ spheres}}{8\text{ in}^3} = \frac{1.5\text{ spheres}}{\text{in}^3}$$

number/unit volume

mass/unit volume = density

The total mass of steel used in the construction of a car is 1800 lbm.

The density of steel is 490 pounds (mass) per cubic foot.

What is the volume of steel in a car?

$$density = \frac{mass}{vol}$$

$$\frac{490\ \mathrm{lbm}}{\mathrm{ft}^3} = \frac{1800\ \mathrm{lbm}}{vol}$$

$$\frac{\mathrm{ft}^3}{490\ \mathrm{lbm}} = \frac{vol}{1800\ \mathrm{lbm}}$$

$$\frac{1800\ \cancel{\mathrm{lbm}} * \mathrm{ft}^3}{490\ \cancel{\mathrm{lbm}}} = vol$$

$$vol = 3.67\ \mathrm{ft}^3$$

What does “surface area” mean?

<u>Surface area</u>: The area of the surface of the shape.

Why would this information be important?

Helps you to know how much material you need to <u>build, paint, or cover</u> the item.

<u>Formula</u>: an equation that shows the relationship between two or more quantities.

Examples of formulas you've seen are:

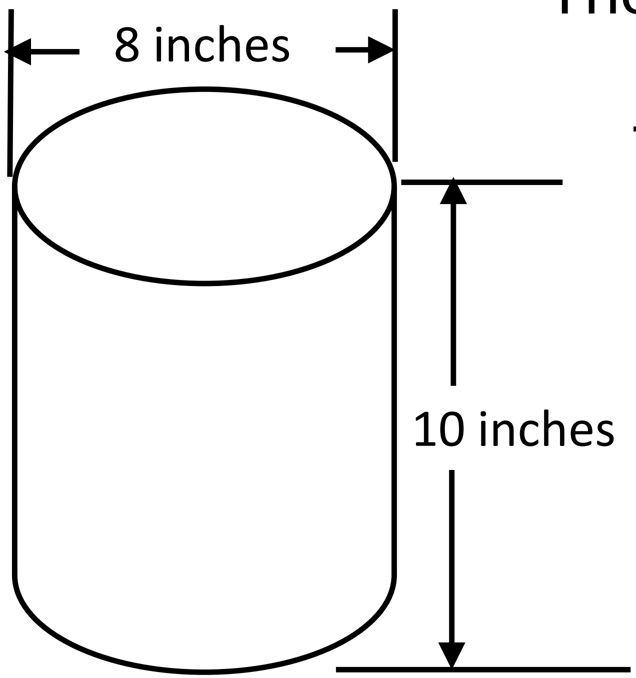
$$A_{cylinder} = 2(\pi\ \mathrm{r}^2) + 2\pi rh$$

$$V = L * w * h$$

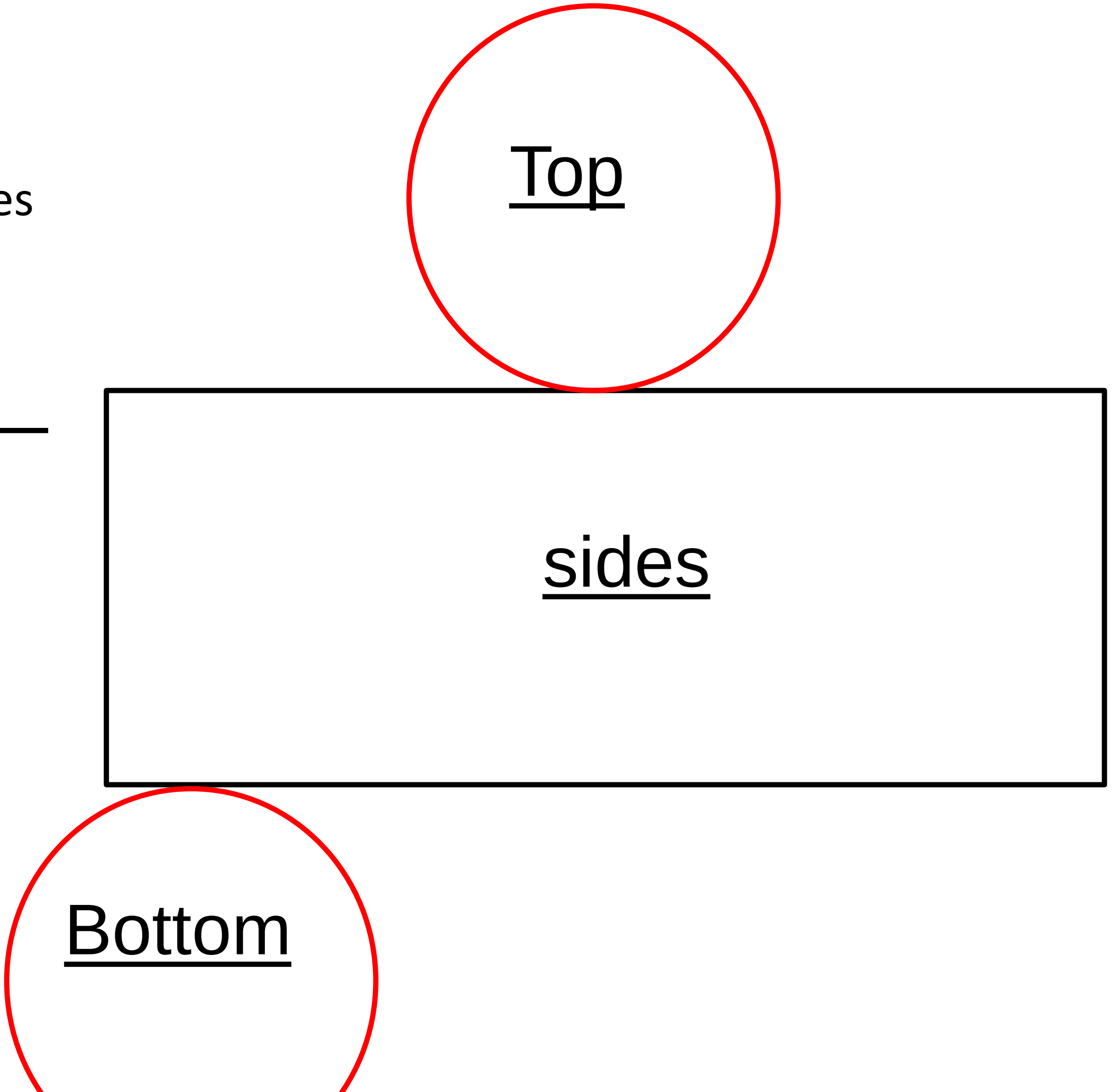
$$A = \pi r^2$$

$$C = 2\pi r$$

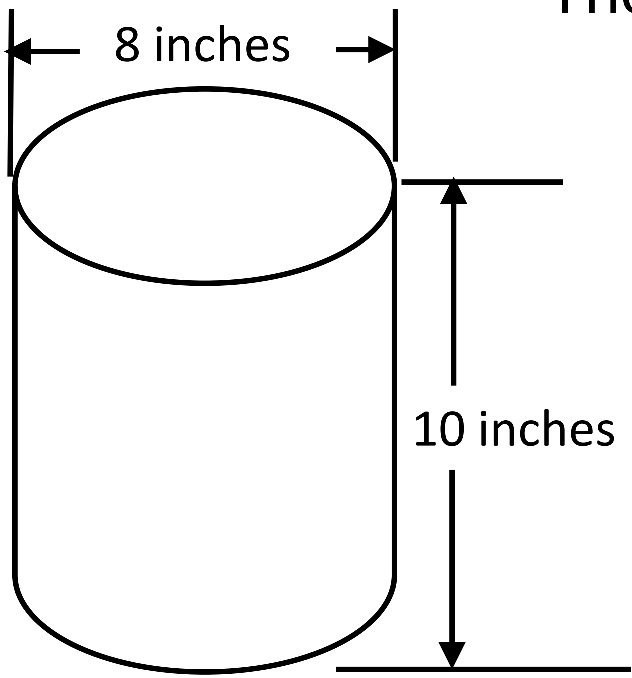
The surface area of a cylinder is….?



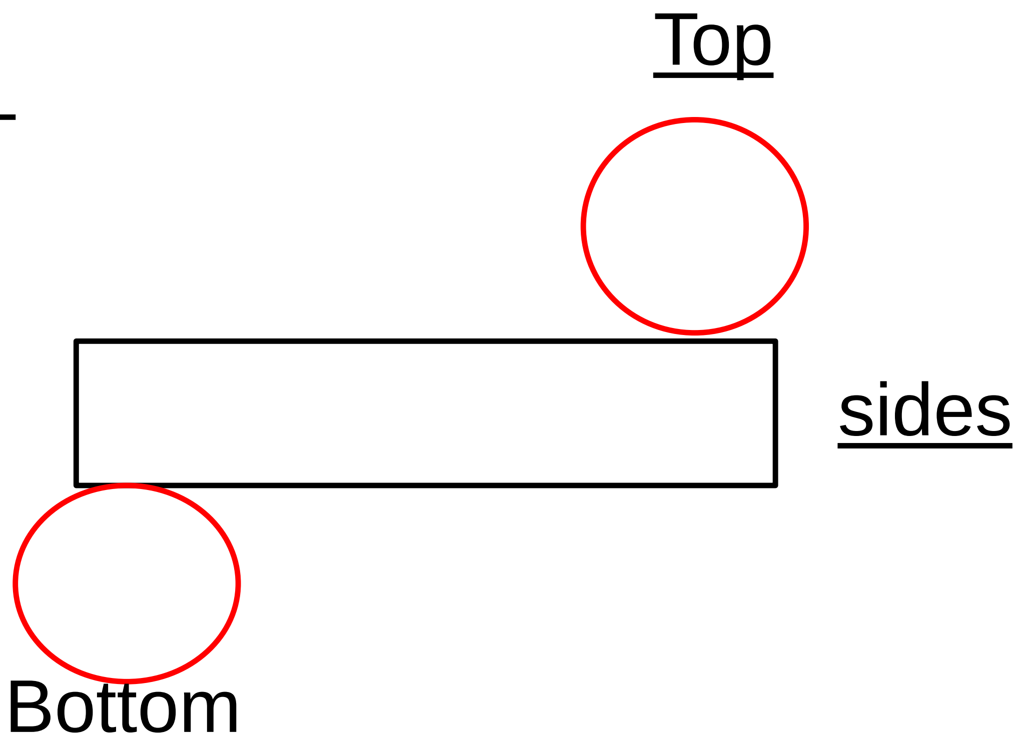
Net a the “flattened” version of a 3-dimensional shape.

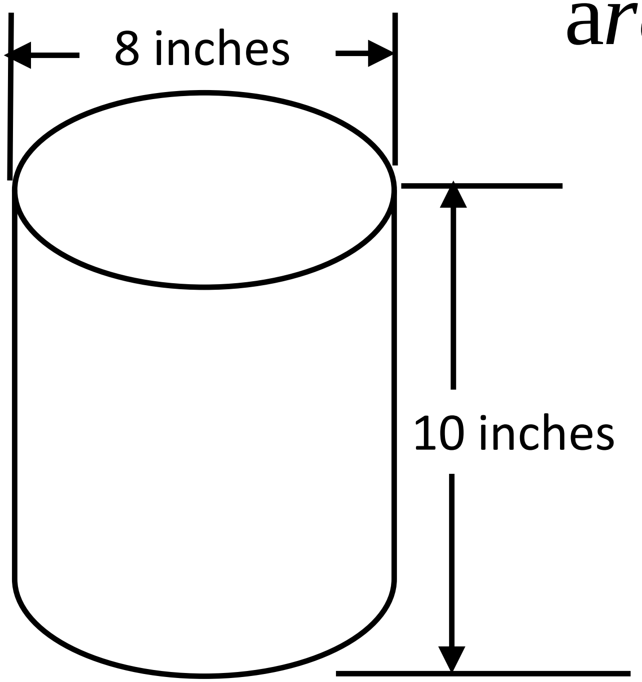


The surface area of a cylinder is….?



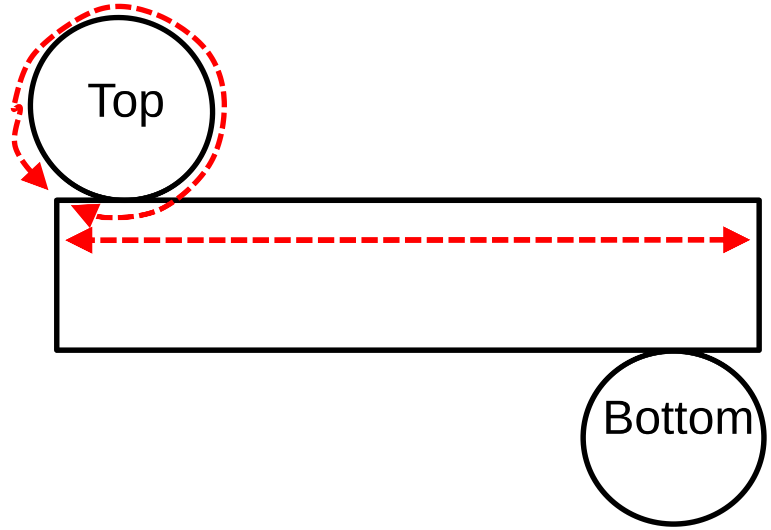
The surface area of a cylinder is made up of 2 circles and 1 rectangle.





$$area_{\text{circle}} = \pi\, \mathrm{r}^2 = \pi\,(4\text{in})^2$$

$$area_{\text{circle}} = 16\pi\ \text{in}^2$$



$$area_{\text{rectangle}} = l * w$$

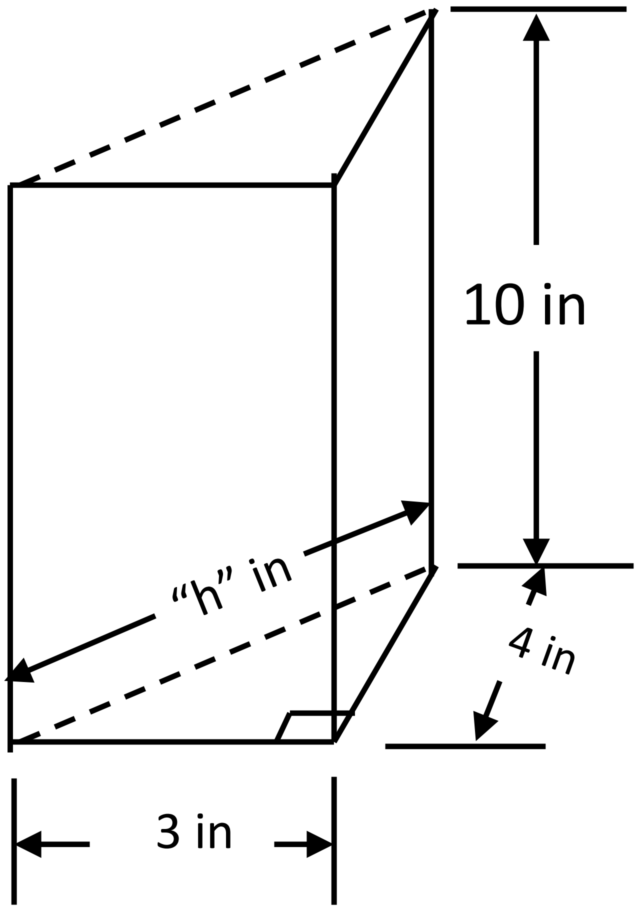
$$\text{area}_{\text{rectangle}} = \text{height} * \text{circumference of top}$$

$$area_{\text{rectangle}} = h*(2\pi r) = 10in*(2\pi*4in) = 80\pi\ \text{in}^2$$

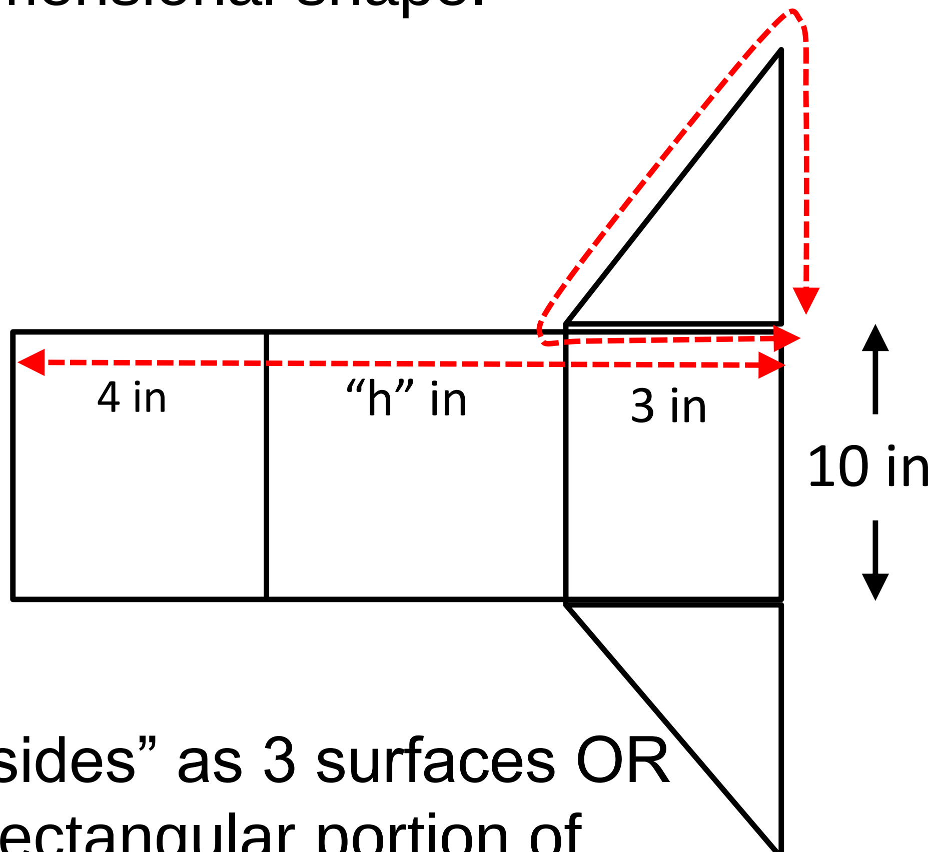
$$\text{Surface area}_{\text{cylinder}} = 16\pi\ \text{in}^2 + 80\pi\ \text{in}^2 + 16\pi\ \text{in}^2$$

$$\text{Surface area}_{\text{cylinder}} = 112\pi\ \text{in}^2$$

What is the "surface area" of the prism?



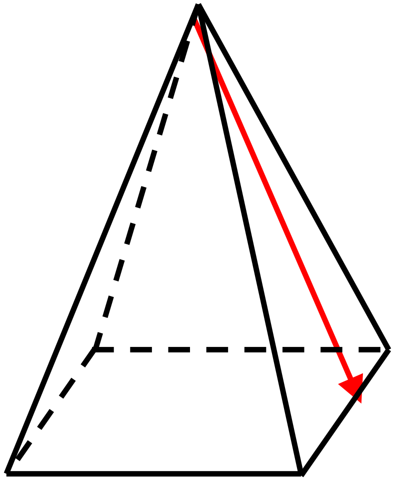
Net a the "flattened" version of a 3-dimensional shape.



You can think of the "lateral sides" as 3 surfaces OR you can think of it as the rectangular portion of the net.

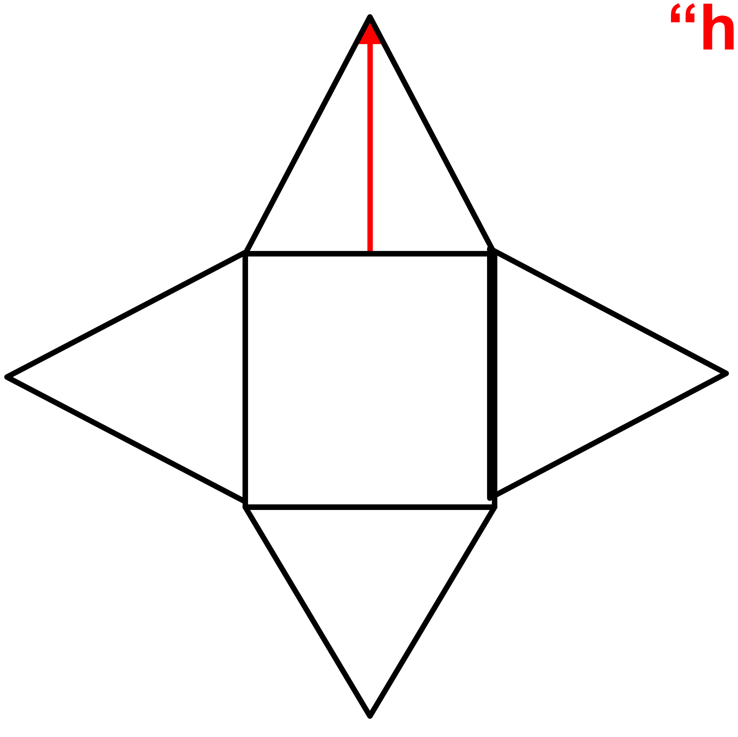
The <u>surface area </u>of a pyramid is….?

The sum of the area of the faces.

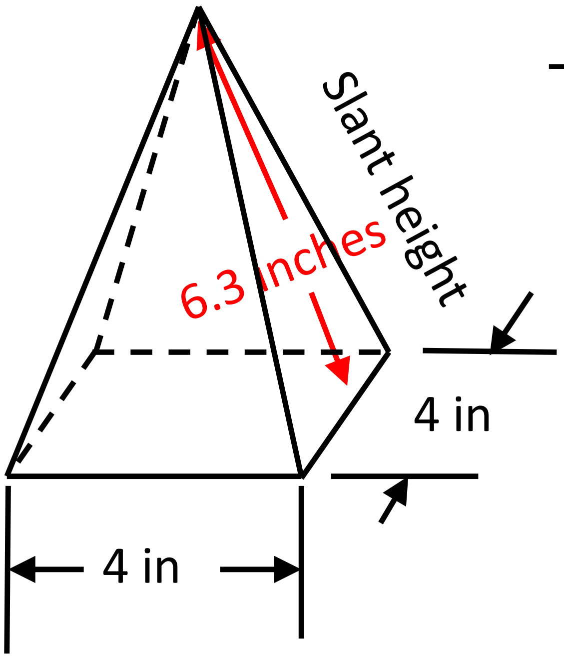


<u>Rectangular Pyramid </u>has a 4-sided base: it has four triangular faces.

The "slant height" of the pyramid is the "height" of the triangular face.



The surface area of a rectangular pyramid is 1 rectangle and 4 triangles.



The sum of the area of the 5 faces.

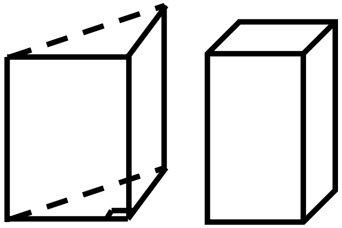
$$\text{area}_{\text{base}} = l * w = 16\,\text{in}^2$$

$$\text{area}_{\text{face}} = \frac{1}{2} * 4\,\text{in} * 6.3\,\text{in}$$

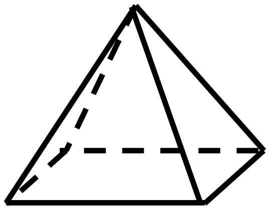
$$\text{area}_{\text{face}} = 12.6\,\text{in}^2$$

$$\text{area}_{\text{total}} = 4(12.6\,\text{in}^2) + 16\,\text{in}^2$$

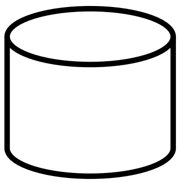
$$\text{area}_{\text{total}} = 66.4\,\text{in}^2$$



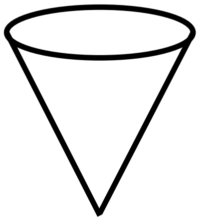
$$\boxed{\text{volume}_{\text{prism}} = (\text{area of base}) * \text{h}}$$



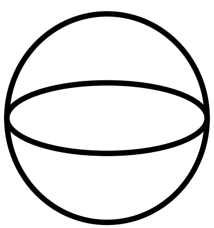
$$\boxed{\text{volume}_{\text{rectangular pyramid}} = \frac{1}{3}(\text{ base area})h}$$



$$\boxed{\text{volume}_{\text{cylinder}} = (\text{area base}) * h}$$



$$\boxed{\text{volume}_{\text{cone}} = \frac{1}{3}(\text{area base}) * h}$$



$$\text{surf. area}_{\text{sphere}} = 4\pi \text{ r}^2$$

$$\text{volume}_{\text{sphere}} = \frac{1}{3} * 4\pi \text{ r}^3$$

What does “f(x)” mean?

Rule name

Input variable

It means that there is a rule, named “f” whose <u>output</u> is a result of “doing math” on the input to the variable ‘x’.

Example: $$f(x) = 2x + 3$$

‘x’ is a <u>place-holder</u> in the rule where we substitute in the <u>input value</u>.

Fill in the blank for each function: "_______" is a function of ____"

$$g(n) = 2n + 3$$

"g is a function of n"

$$A(t) = 10e^{-0.02t}$$

"A is a function of t"

$$k(x) = \sqrt{25 - x^2}$$

"k is a function of x"

$$h(t) = -16t^2 + 100t + 5$$

"h is a function of t"

$$(x-4)^2 + (y+2)^2 = 25$$

neither

$$y = -2 \pm \sqrt{25 - (x-4)^2}$$

"y is a function of x"

$$x = 4 \pm \sqrt{25 - (y+2)^2}$$

"x is a function of y"

Rewriting formulas

We say that one quantity is a function of one or more other quantities.

$$A = \pi r^2 \qquad A = f(r)$$

$$A = L * w \qquad A = f(L, w)$$

$$A_{cylinder} = 2(\pi \ \mathrm{r}^2) + 2\pi rh \qquad A = f(r, h)$$

$$C = 2\pi r \qquad C = f(r)$$

Rewrite the formula so that it is.....

$$C = 2\pi r \qquad r = f(C) \qquad r = C/2\pi$$

$$A = \pi r^2 \qquad r = f(A) \qquad r = \sqrt{A/\pi}$$

$$A = L * w \qquad L = f(A, w) \qquad L = A/w$$

$$A_{cylinder} = 2(\pi \ \mathrm{r}^2) + 2\pi rh \qquad h = f(A, r)$$

$$h = \frac{A - 2\pi r^2}{2\pi r}$$

$$C = \frac{5}{9}(F - 32) \qquad C = f(F)$$

Rewrite the formula so that it is in the form: $F = f(C)$

$$\left(\frac{9}{5}\right)C = \left(\frac{9}{5}\right)\frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

$$F = \frac{9}{5}C + 32$$

Describe the transformation of the parent function:

$$y = -3(x+4)^2 - 5$$

Reflected across x-axis, VSF = 3, left 4, down 5

Solve the equation for ‘x’.

$$y = -3(x+4)^2 - 5$$

$$y + 5 = -3(x+4)^2$$

$$\frac{y+5}{-3} = (x+4)^2$$

$$\frac{-y-5}{3} = (x+4)^2$$

$$\pm\sqrt{\frac{-y-5}{3}} = x + 4$$

$$x = -4 \pm \sqrt{\frac{-y-5}{3}}$$

$$R: \quad -y - 5 \geq 0$$

$$R: \quad -5 \geq y$$

$$R: \quad \mathrm{y} \leq -5$$

Solve the equation for ‘x’

$$y = 5(x-1)^2 + 3$$

$$y - 3 = 5(x-1)^2$$

$$\frac{y-3}{5} = (x-1)^2$$

$$\pm\sqrt{\frac{y-3}{5}} = x - 1$$

$$x = 1 \pm \sqrt{\frac{y-3}{3}}$$

$$R: \quad y - 3 \geq 0$$

$$R: \quad y \geq 3$$

Solve the equation for ‘x’

$$y = 6\log(x+2) - 4$$

$$y + 4 = 6\log(x+2)$$

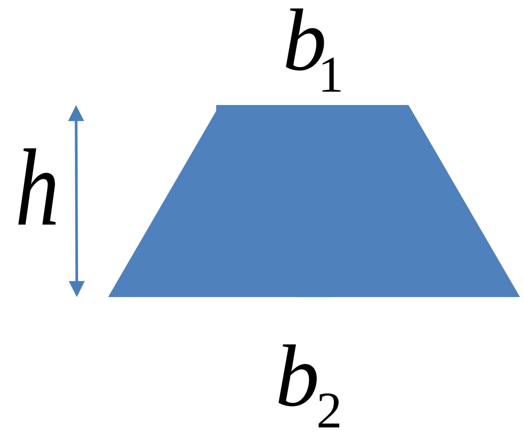
$$\frac{y+4}{6} = \log(x+2)$$

$$10^{\left(\frac{y+4}{6}\right)} = x+2$$

$$x = -2 + 10^{\left(\frac{y+4}{6}\right)}$$

$$A = h * \frac{(b_1 + b_2)}{2} \qquad A = f(h, \mathrm{b}_1, b_2)$$

Rewrite the formula as: $b_1 = f(\mathrm{A}, h, \mathrm{b}_2)$



$$2A = h(b_1 + b_2)$$

$$2A = hb_1 + hb_2$$

$$\frac{2A}{h} = b_1 + b_2$$

$$2A - hb_2 = hb_1$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2A - hb_2}{h}$$

Are the two formulas are equivalent?

Expressions from Phrases, Equations from statements

What is a mathematical expression that represents the following?

Three more than twice a number $2x+3$

Five less than three times a number $3x-5$

The width is 4 times the length. $w=4L$

The area of a rectangle whose width is 4 times its length.

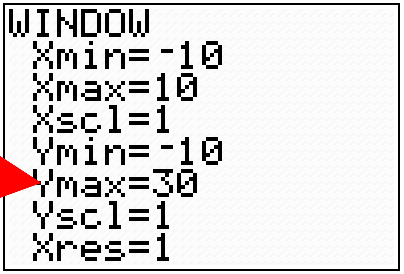
$$A=Lw$$
$$A=L(4L)$$
$$A=4L^2$$

The width of a rectangle is 3 less than twice its length. $w=2L-3$

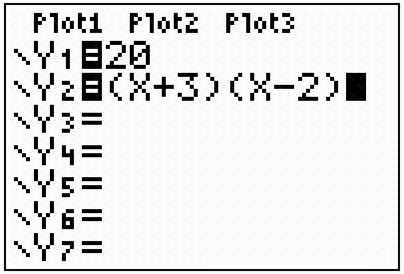
Solve a totally non-recognizable quadratic equation by graphing.

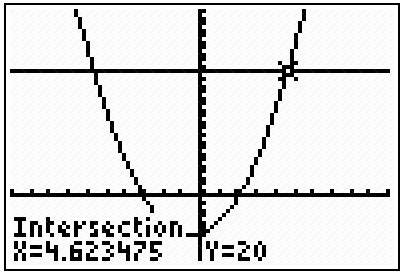
$$20 = (x+3)(x-2)$$

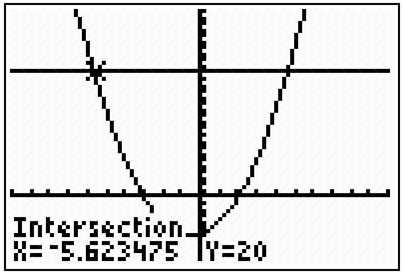
$$y = 20$$



$$y = (x+3)(x-2)$$



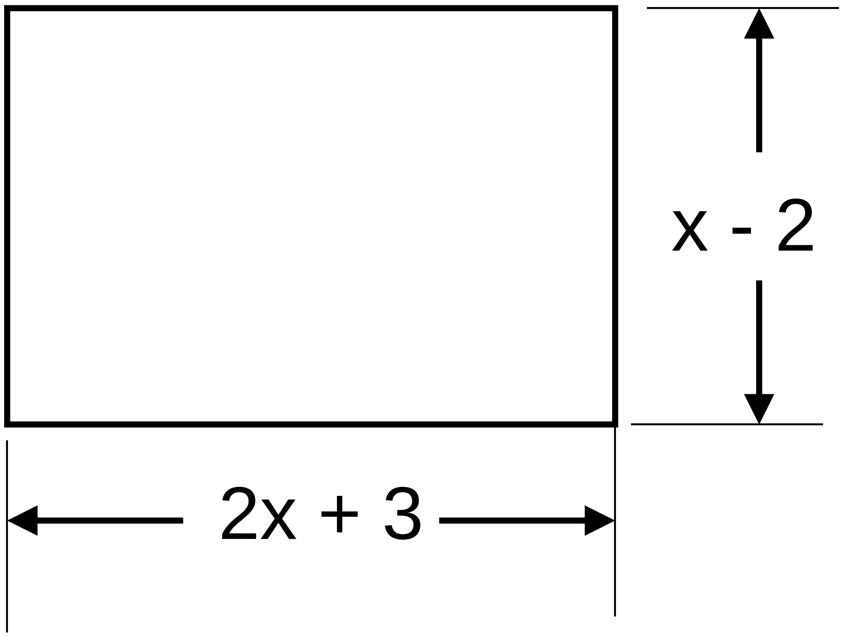




Finding the dimensions of a rectangle.

The length of one side of a rectangle is three more than two times a number. What is the expression for the length of the side?

The width of the rectangle is two less than the number. What is the expression for the length of the side?



If the area of the rectangle is 400 square inches, what is the length and width of the rectangle?

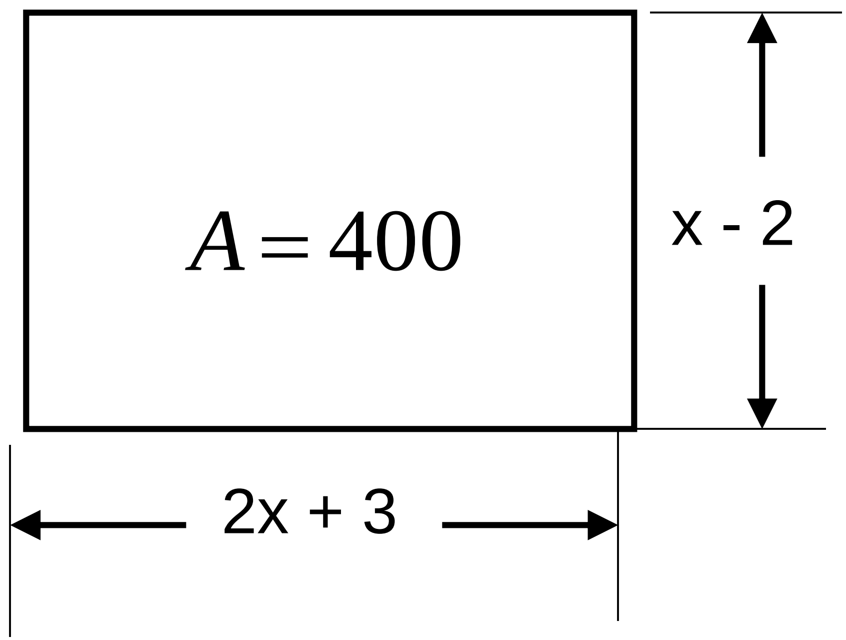
Finding the dimensions of a rectangle.

A = 400,   length = ?   Width = ?

$$A = L * W \qquad L = 2x + 3 \qquad w = x - 2$$

By substitution:

$$400 = L * W$$



By substitution:

$$400 = (2x + 3) * W$$

By substitution:

$$400 = (2x + 3)(x - 2)$$

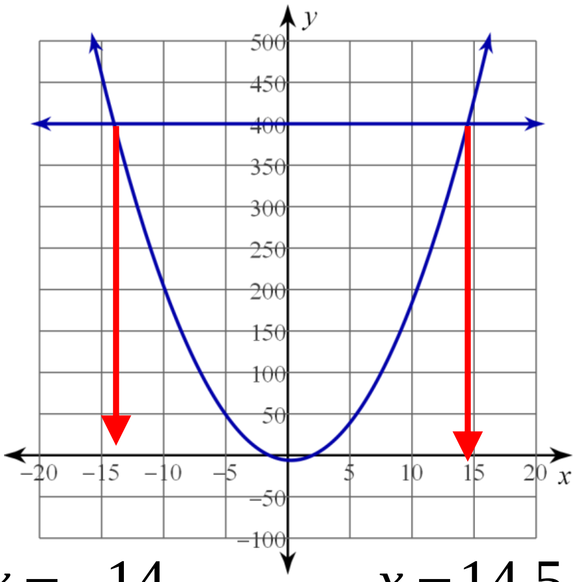
$$400 = (2x+3)(x-2)$$

Solve by graphing→ system of equations.

$$y = 400$$

$$y = (2x+3)(x-2)$$

Do both values of ‘x’ give you an area that is a positive number?



$x = -14$ $x = 14.5$

$$w = x - 2 \qquad L = 2x + 3$$

$$w = 14.5 - 2 \qquad L = 2(14.5) + 3$$

$$w = 12.5 \qquad L = 32$$

check

$$400 = (32)(12.5)$$