Math-3A
Lesson 11-8

Solve Systems of Linear Equations
Using
Elimination

Algebraic Methods of Solving Systems of Equations

Substitution: Solve one equation for one of the variables. Substitute the equivalent expression for the variable into the other equation. This results in one equation with one variable.

Elimination: Add the equations (or multiples of the equations) to eliminate one of the variables. Then solve the single variable equation and “back substitute” the result.

<u>Elimination Method</u>: Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

What property allows me to add equations together?

"Property of Equality"

Adding these equations will <u>eliminate</u> the 'x' variable.

$$2x - 3y = 5$$
$$-4x + 3y = 3$$

Adding these equations will <u>eliminate</u> the 'y' variable.

What variable will be eliminated if I add the following equations?

1.
$$\begin{aligned} 2x + y &= -2 \\ -2x + 3y &= -8 \end{aligned}$$

2.
$$\begin{aligned} 4x - 3y &= -2 \\ -2x + 3y &= -8 \end{aligned}$$

3.
$$\begin{aligned} 3x + y &= -1 \\ 2x + 3y &= 18 \end{aligned}$$

Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$x - x - 3y + 5y = 5 + 3$$

$$2y = 8$$

$$y = 4$$

$$x - 3(4) = 5$$

$$x = 17$$

Solution: (17, 4)

Replace ‘y’ with 4 in either of the original equations, then solve for ‘x’.

Check the solution: (using substitution)

If your work indicated the solution to be (17, 4), replace 'x' with 17 and 'y' with 4 in both of the original equations, to see if the ordered pair (17, 4) is a solution to the system of equations.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$(17) - 3(4) = 5$$

Checks!

$$-(17) + 5(4) = 3$$

Checks!

Solution: (17, 4)

<u>Solve</u>

$$2x - 5y = 6$$
$$-x + 5y = 2$$

$$2x - x - 5y + 5y = 6 + 2$$

$$x = 8$$

$$-(8) + 5y = 2$$

$$5y = 10$$

$$y = 2$$

Replace 'x' with 8 in either of the original equations, then solve for 'y'.

Solution: (8, 2)

Solve the equation using “elimination”

4x – 3 y = -2

-2x + 3y = -8

$$2x = -10$$

$$x = -5$$

$$-2(-5) + 3y = -8$$

$$10 + 3y = -8$$

$$3y = -18$$

$$y = -6$$

Least common multiple (of 2 numbers) is the smallest number that is divisible by those two numbers.

2 and 4	LCM = 4
4 and 6	LCM = 12
4, 9	LCM = 36
3, 5	LCM = 15
4, 5	LCM = 20

What if the coefficients are <u>not</u> the same?

$$5x - y = -2$$
$$-2x + 3y = -8$$

What is the LCM for the coefficients of ‘y’?

LCM = 3

You only have to fix one!

$$3*(5x - y) = -2 * 3$$
$$-2x + 3y = -8$$

$$15x - 3y = -6$$
$$-2x + 3y = -8$$

What if the coefficients are <u>not</u> the same?

$$5x - 5y = -2$$
$$-2x + 3y = -8$$

What is the LCM for the coefficients of ‘x’?

LCM = 10

You have to fix both!

$$2*(5x - 5y) = -2 *2 \qquad 10x - 10y = -4$$
$$5*(-2x + 3y) = -8*5 \qquad -10x + 15y = -40$$

$$3x-4y=-10$$
$$6x+3y=-42$$

$(-2)3x - (-2)4y = -10(-2)$

$6x \quad + \quad 3y = -42$

$-6x + 8y = 20$

$6x + 3y = -42$

$$11y=-22$$

$$y=-2$$

$$6x+3(-2)=-42$$

$$6x-6=-42$$

$$6x=-36$$

$$x=-6$$

$$3x + 2y = 6$$
$$x - 4y = -12$$

$$3(0) + 2y = 6$$
$$(0) - 4y = -12$$

$$(2)3x + (2)2y = 6(2)$$
$$x - 4y = -12$$

$$2y = 6$$
$$-4y = -12$$

$$6x + 4y = 12$$
$$x - 4y = -12$$

$$y = 3$$

$$5x = 0$$

$$x = 0$$

Solution is (0, 3)

Linear Equation in 3 Variables:

$$Ax + By + Cz = D \qquad 3x + 2y - z = 5$$

System of Linear Equations: 3 equations, each with the same 3 variables

(3 equations in 3 unknowns)

$$Ax + By + Cz = D$$
$$Ex + Fy + Gz = H$$
$$Jx + Ky + Lz = M$$

Solving by Elimination

Pick two equations and remove one of the variables.

$$Ax + By + Cz = D$$

$$Ex + Fy + G\,z = H$$

$$Jx + K\,y + L\,z = M$$

Pick two other equations and remove the same variable.

$$x + y = G$$

$$x + y = G$$

Solve the system of 2 equations in 2 variables.

Eq#1: $x + 2y - 2z = -15$
Eq#2: $2x + y - 5z = -21$
Eq#3: $x - 4y + z = 18$

Eq#1: $-2(x + 2y - 2z) = (-15)(-2)$
Eq#2 $2x + y - 5z = -21$
$-2x - 4y + 4z = 30$

Eq#1/#2 $-3y - z = 9$

Eq#1: $-1(x + 2y - 2z) = (-15)(-1)$
Eq#3: $x - 4y + z = 18$
$-x - 2y + 2z = 15$

Eq#1/#3 $-6y + 3z = 33$

Eq#1/#2 $-3y - z = 9$

$\div 3(-6y + 3z) = (33)(\div 3)$

Eq#1/#3 $-2y + z = 11$

$-3y - z = 9$

$-5y = 20$

$\boxed{y = -4}$

$-3(-4) - z = 9$

$12 - z = 9$

$\boxed{z = 3}$

$x - 4(-4) + (3) = 18$

$x + 16 + 3 = 18$

$\boxed{x = -1}$

You start your own company to make smartphones. You decide on 3 models; basic, 3G model, and the 4G model.

The basic model is for people who do not have a lot of disposable income. The 3G model has the speed and download capability that most people want. The 4G model has all of the “bells and whistles” and is expandable to meet future needs.

You hire and train your employees to perform all of the basic tasks; assembly, testing, and packaging of each phone.

You analyze your process and employees and decide you have 260 man-hours for assembly in a week, 170 man-hours for testing, and 120 man-hours for packaging.

The table below shows the man-hour totals required for each of the three tasks.

	Basic Model	3G Model	4G Model
Assembly	1 man-hour	3 man-hours	4 man-hours
Testing	1 man-hour	2 man-hours	2 man-hours
Packaging	1 man-hour	1 man-hour	2 man-hours

What are your three constraints?

260 man-hours for assembly $x + 3y + 4z = 260$

170 man-hours for testing, and $x + 2y + 2z = 170$

120 man-hours for packaging. $x + y + 2z = 120$

Write an equation for each of the constraints. Your goal is to figure out how many phones of each type you should build.

Let 'x' be the number of basic phones, 'y' be the number of 3G, and 'z' be the number of 4G phones you will build.