Math-3A
Lesson 11-11
Find Zeroes Using “Long Division”, Synthetic Division, and Box Division

Our goal is to find the x-intercepts of polynomials.

We've learned how to factor:

1) Quadratic form $y = x^4 + 4x^2 + 3$

2) 3rd degree polynomials with a common factor of 'x'

$$y = x^3 + 4x^2 + 3x$$

3) 3rd degree polynomials that have a "nice pattern"

$$y = x^3 + 2x^2 + 3x + 6$$

4) Sum and Difference of 2 "perfect cubes"

$y = x^3 + 8$ $y = x^3 - 27$

Now we learn how to factor Polynomials that *don't* have a "nice pattern".

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\frac{x^3 + 3x^2 + 14x - 18}{(x-1)} = ax^2 + bx + c$$

Divide Evenly: A divisor divides evenly if there is a zero for the remainder.

Polynomial Long Division

$$\begin{array}{rl} & \quad\quad\quad x^2 \\ x-1 & \overline{)\; x^3+3x^2+14x-18} \end{array}$$

1) Look at left-most numbers

2) What # times “left” = “left”?

$$x^3/x = ? = x^2$$

3) Multiply

$$x^2\,(x-1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$
\begin{array}{rl}
 & x^2 \\
x-1 & \overline{)\,x^3+3x^2+14x-18} \\
 & -(x^3-x^2) \\
\hline
 & \quad 4x^2\ \ +14x-18
\end{array}
$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$
\begin{array}{rl}
 & x^2 \boxed{+4x} \\
\cline{2-2}
\boxed{x} - 1 & \big)\, x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 + x^2) \\
\cline{2-2}
 & \boxed{4x^2} + 14x - 18 \\
 & -(4x^2 - 4x) \\
\cline{2-2}
 & 18x
\end{array}
$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times “left” = “left”?

$$\frac{4x^2}{x} = ? \qquad = 4x$$

3) Multiply

$$4x(x-1) \; = 4x^2 - 4x$$

4) Subtract

$$-(4x^2 - 4x)$$

Polynomial Long Division

$$\begin{array}{r l}
 & x^2 \quad +4x \\
x-1 & \overline{\big)\, x^3+3x^2+14x-18} \\
 & -(x^3+x^2) \\
\hline
 & \quad 4x^2+14x-18 \\
 & -(4x^2-4x) \\
\hline
 & \qquad 18x \quad -18
\end{array}$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$
\begin{array}{r l}
 & x^2 \quad +4x \quad \boxed{+18} \\
\boxed{x} - 1 \,\big) & \overline{x^3 + 3x^2 + 14x - 18} \\
 & -(x^3 + x^2) \\ \hline
 & 4x^2 + 14x - 18 \\
 & -(4x^2 - 4x) \\ \hline
 & \boxed{18x} - 18 \\
 & -(18x - 18) \\ \hline
 & 0
\end{array}
$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times “left” = “left”?

$$\frac{18x}{x} = 18$$

3) Multiply

$$18(x-1) = 18x - 18$$

4) Subtract

$$-(18x - 18)$$

$$x^2 \quad +4x \quad -18$$

$$x-1 \overline{\smash{\big)}\; x^3 + 3x^2 + 14x - 18}$$

$$-(x^3 + x^2)$$

$$4x^2 + 14x - 18$$

$$-(4x^2 - 4x)$$

$$18x - 18$$

$$-(-18x + 18)$$

$$0$$

$$x^3 + 3x^2 + 14x - 18 = (x - 1)(x^2 + 4x - 18)$$

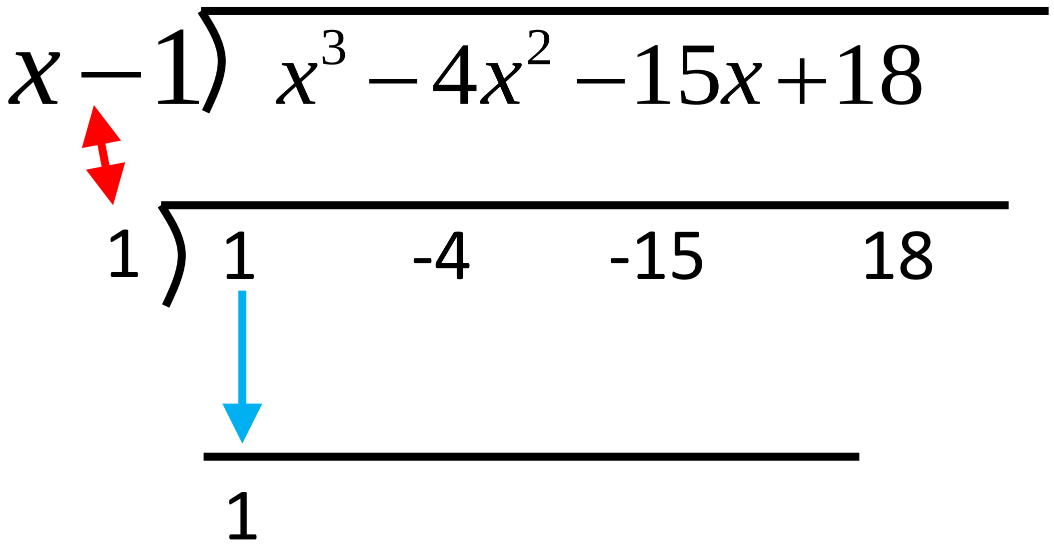
How do we find the zeroes of the unfactorable quadratic factor?

Convert to vertex form and take square roots.

$$x-8\,\overline{\smash{)}\;x^3+2k^2-90x+76}$$

Problem #3 from homework

Synthetic Division



$$x-1\overline{)\,x^{3}-4x^{2}-15x+18}$$

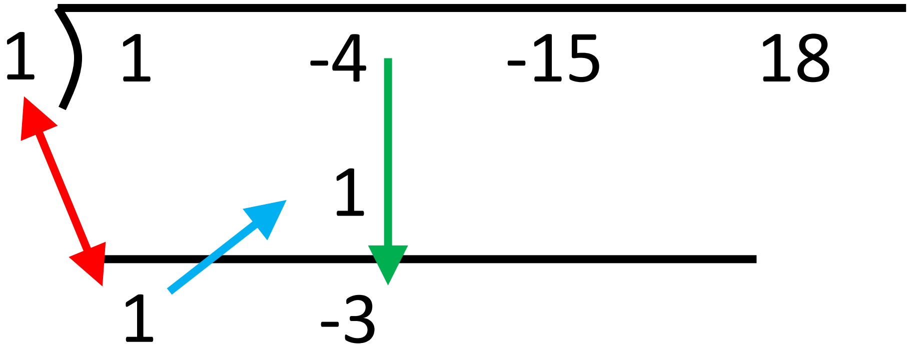
$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & & & \\ \hline & 1 & & & \end{array}$$

1st step: Write the polynomial with only its coefficients.

2nd step: Write the “zero” of the linear divisor.

3rd step: Bring down the lead coefficient

$$x-1\overline{)\,x^3-4x^2-15x+18}$$



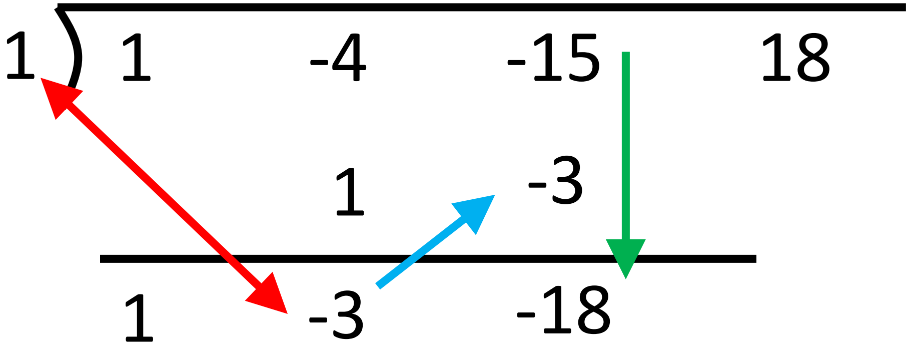
$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & & \\ \hline & 1 & -3 & & \end{array}$$

4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

$$x-1 \overline{\big)\, x^3 - 4x^2 - 15x + 18}$$



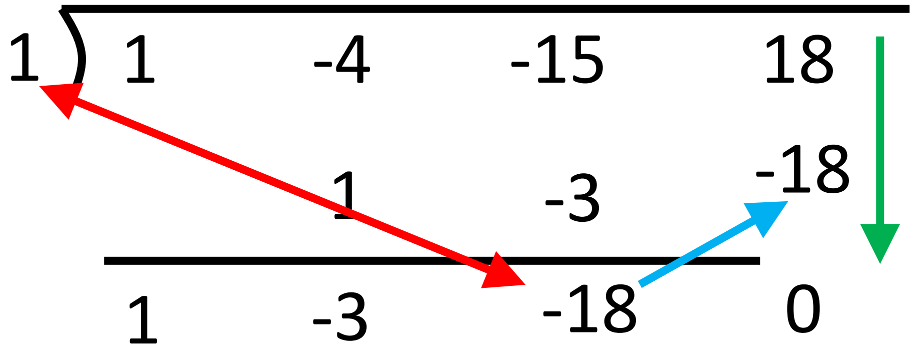
1	1	-4	-15	18
		1	-3	
	1	-3	-18	

7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

$$x-1\overline{)\,x^3-4x^2-15x+18}$$



$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

10th step: Multiply the “zero” by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$x-1\overline{)\,x^3-4x^2-15x+18} \quad = x^2-3x-18$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

This last number is the remainder when you divide:

$$x^3-4x^2-15x+18$$

by

$$x-1$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

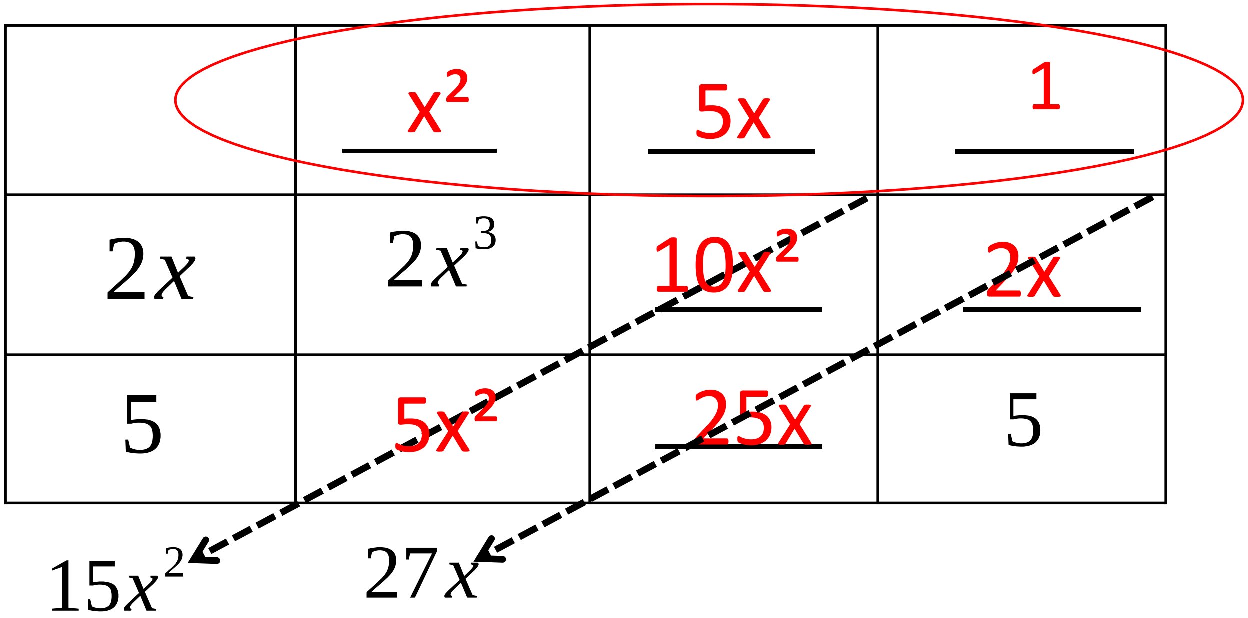
$$x-3\;\overline{\big)\;10x^3-35x^2+17x-7\qquad}$$

Problem #7 from homework

Division of Polynomials Box Method

$$(2x^3+15x^2+27x+5)\div(2x+5)$$

Only the upper left and bottom right boxes are known.



	x^2	$5x$	1
$2x$	$2x^3$	$10x^2$	$2x$
5	$5x^2$	$25x$	5

$15x^2$ $27x$

Diagonals have “like terms”

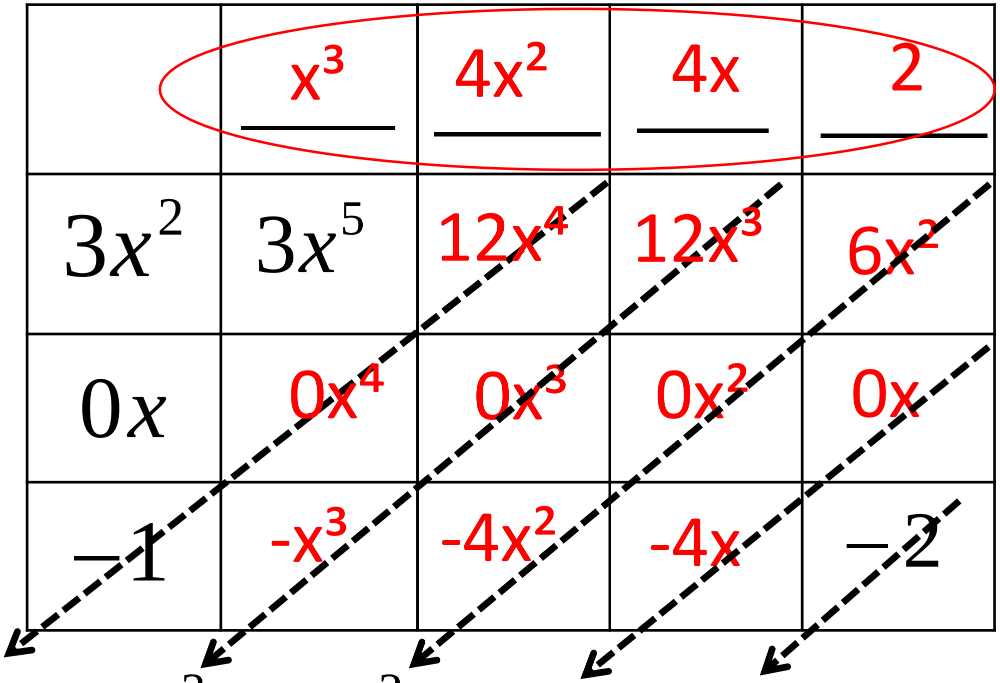
$$(2x^3+15x^2+27x+5)\div(2x+5)=x^2+5x+1$$

Division of Polynomials Box Method

$$(3x^5+12x^4+11x^3+2x^2-4x-2)\div(3x^2-1)$$

Only the upper left and bottom right boxes are known.

Diagonals have "like terms"



	x^3	$4x^2$	$4x$	2
$3x^2$	$3x^5$	$12x^4$	$12x^3$	$6x^2$
$0x$	$0x^4$	$0x^3$	$0x^2$	$0x$
-1	$-x^3$	$-4x^2$	$-4x$	-2

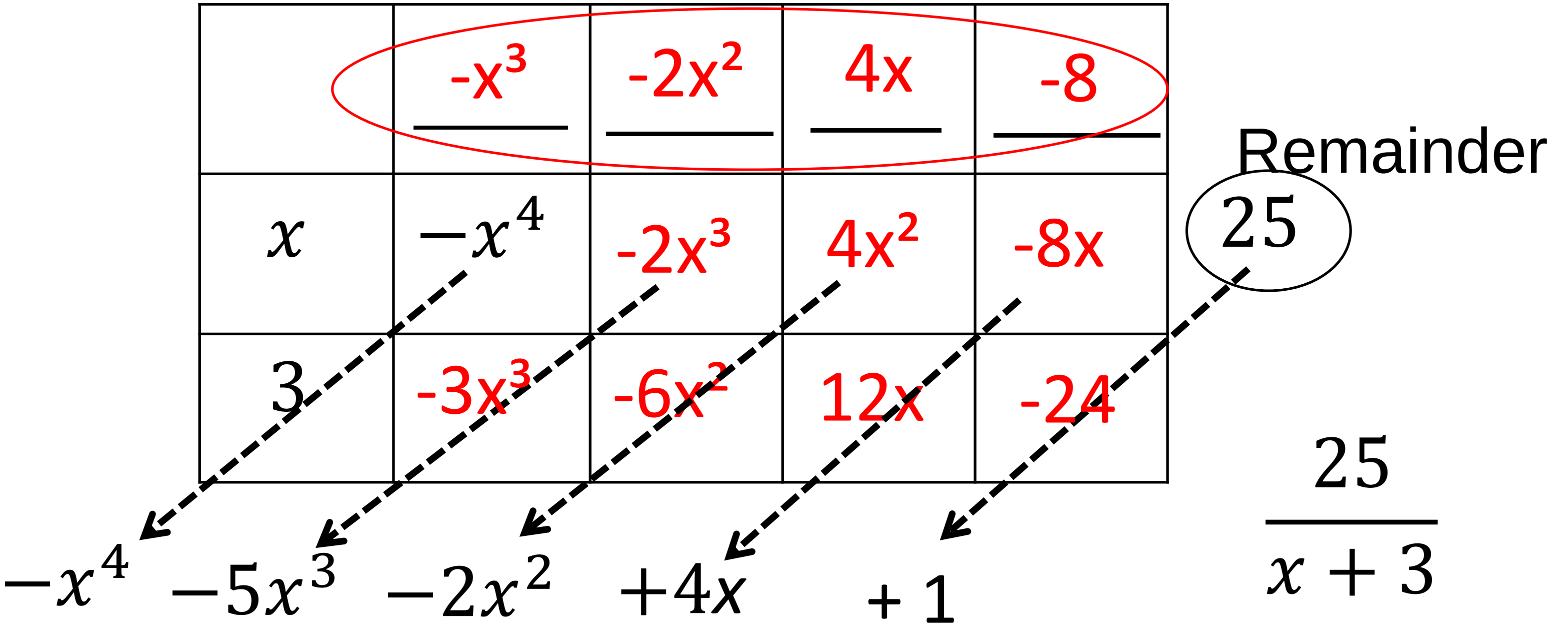
$12x^4$ $11x^3$ $2x^2$ $-4x$ -2

$$(3x^5+12x^4+11x^3+2x^2-4x-2)\div(3x^2-1)=x^3+4x^2+4x+2$$

Division with remainders

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

Only the upper left and bottom right boxes are known.



	$-x^3$	$-2x^2$	$4x$	-8
x	$-x^4$	$-2x^3$	$4x^2$	$-8x$
3	$-3x^3$	$-6x^2$	$12x$	-24

Remainder

25

$$-x^4 \quad -5x^3 \quad -2x^2 \quad +4x \quad +1$$

$$\frac{25}{x+3}$$

Diagonals have “like terms”

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

$$\boxed{= (-x^3 - 2x^2 + 4x - 8 + \frac{25}{x+3})}$$

Divide.

$$k^3 + 12k^2 + 19k - 72 \quad \div k + 9$$

Problem #9 from homework