Math-3A

Lesson 1-9

Modeling Cooling

with the

Exponential Function

What is the equation of the graph? $g(x) = ab^x + k$

1) horizontal asymptote

y = 1

$g(x) = ab^x + k$

$k = 1$

$y = ab^x + 1$

2) y-intercept $(0, 4)$

Substitute (0, 4) into the equation.

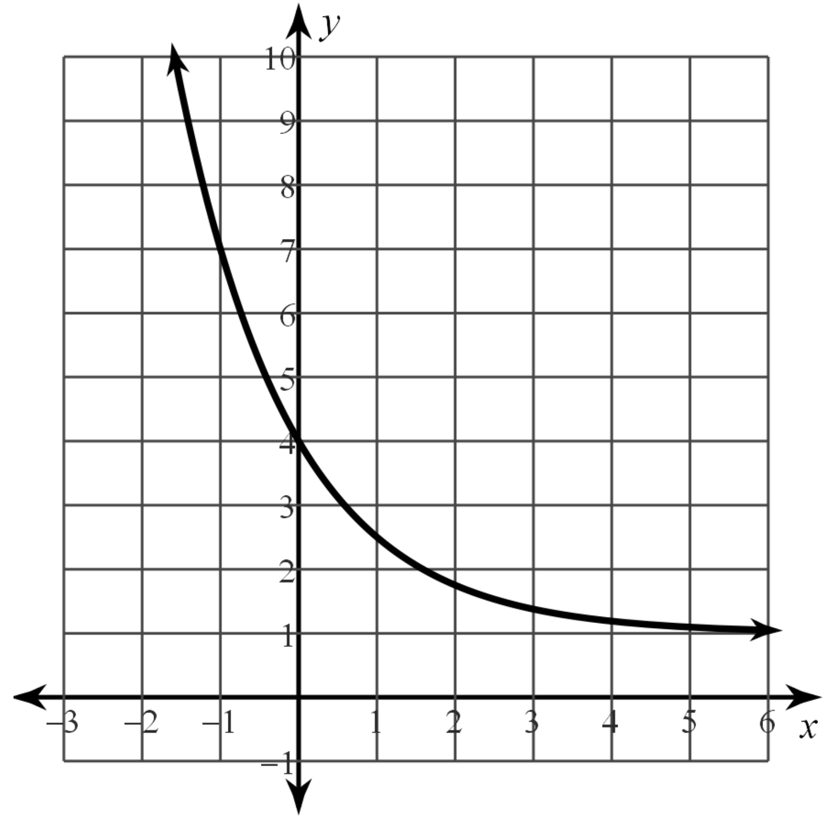
$4 = ab^0 + 1$ $a = 3$

$y = 3b^x + 1$

3) "Nice" x-y pair $(-1, 7)$

Substitute (-1, 7) into the equation.

$7 = 3b^{-1} + 1$



$6 = 3b^{-1}$

$2 = b^{-1}$

$2 = \frac{1}{b}$

$b = \frac{1}{2}$

$y = 3(\frac{1}{2})^x + 1$

$$g(x) = ab^x + k$$

1) Horizontal Asymptote: y = 0

$$g(x) = ab^x + k \qquad k = 0$$

Equation: $y = ab^x$

2) y-intercept: $(0,3)$

$$3 = ab^0 \qquad a = 3$$

Equation: $y = 3b^x$

3) An x-y pair (preferably with x = 1)

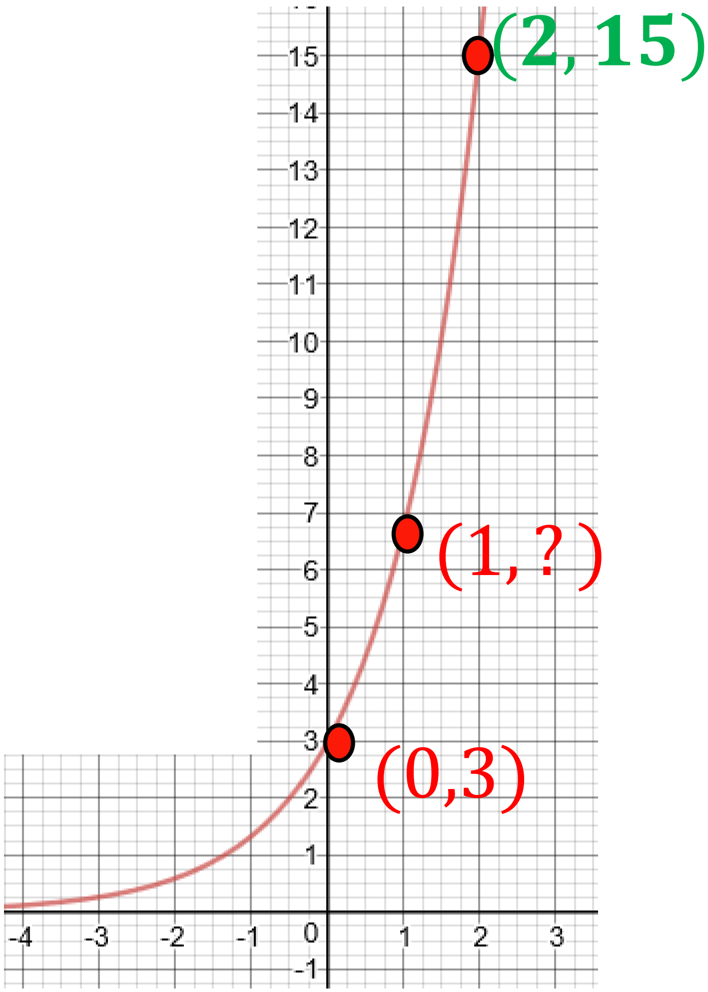
$(2, 15)$

$$15 = 3b^2$$

$$5 = b^2$$

$$\sqrt{b^2} = \sqrt{5}$$

$$b = 2.236$$



$$y = 3(2.236)^x$$

$$g(x) = ab^x + k$$

1) Horizontal Asymptote: $y = 0$

$$g(x) = ab^x + k \qquad k = 0$$

Equation: $y = ab^x$

2) y-intercept: $(0,3)$

$$3 = ab^0 \qquad a = 3$$

Equation: $y = 3b^x$

3) An x-y pair (preferably with x = 1)

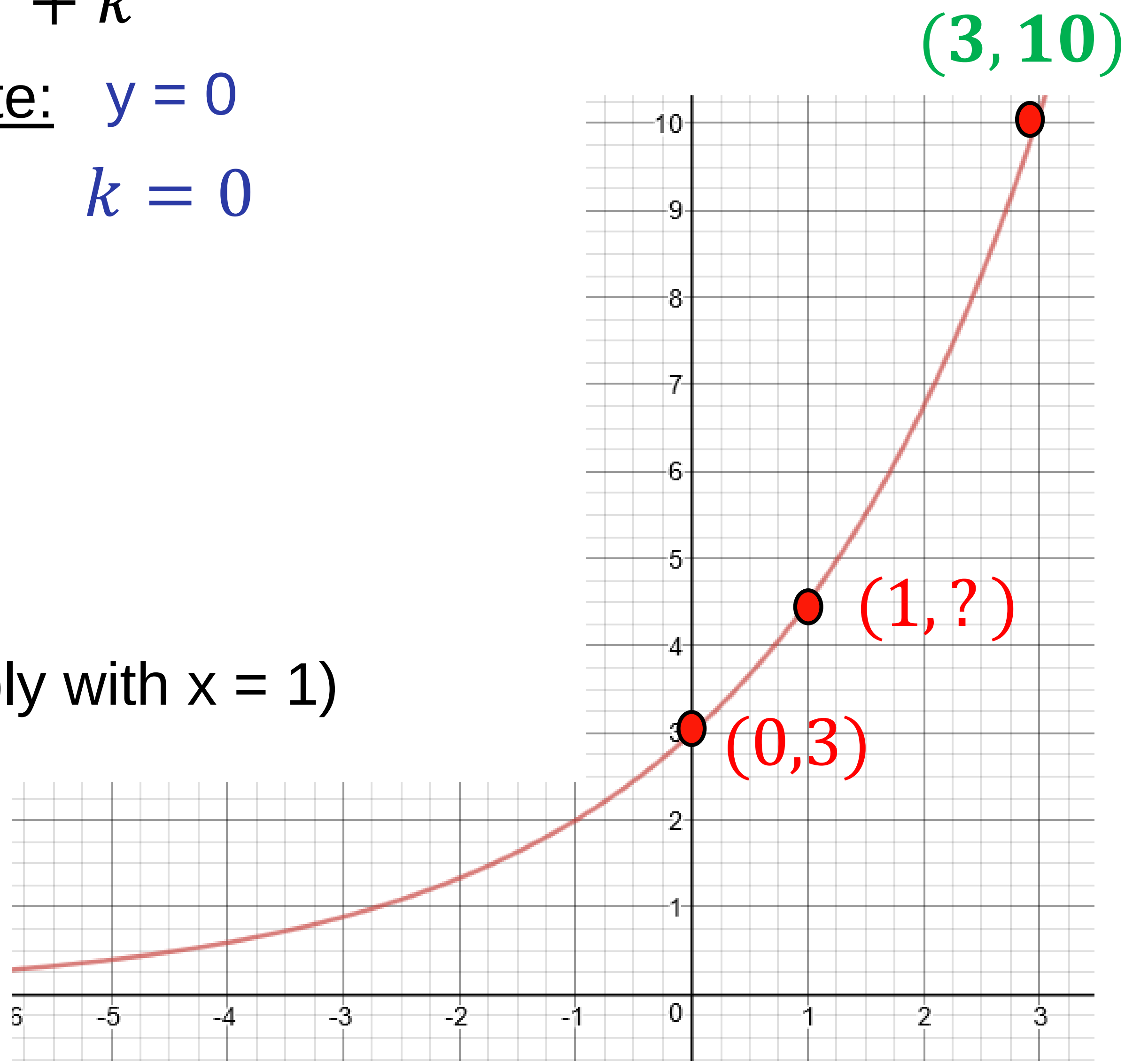
$$(3, 10)$$

$$10 = 3b^3$$

$$3.333 = b^3$$

$$\sqrt[3]{b^3} = \sqrt[3]{3.3333}$$

$$b = 1.4938$$



$$y = 3(1.4938)^x$$

Quantity: a category of measurements in the real world.

Unit of Measure: the unit that is used to measure a quantity.

Examples of quantities:	Examples of units of measure:
Height	(Height) → inches, feet, miles
Weight	(Weight) → pounds, kilograms
Temperature	(Temperature) → degrees Fahrenheit or Celsius

Rate: the change of one quantity compared to the change in another quantity using a fraction.

In pure mathematics we would call this a slope.

$$\frac{\Delta y}{\Delta x} = slope$$

Rate: (a ratio of quantities) becomes a new quantity.

$$\frac{\Delta \text{ temp}}{\Delta \text{ time}} = \text{heatup/cooldown rate}$$

Suppose boiling water (212^o F) is taken off the stove to cool in a room that is at 70 F..

Your turn: draw a graph of what you think the temperature will look like as time passes by (temperature as a function of time).

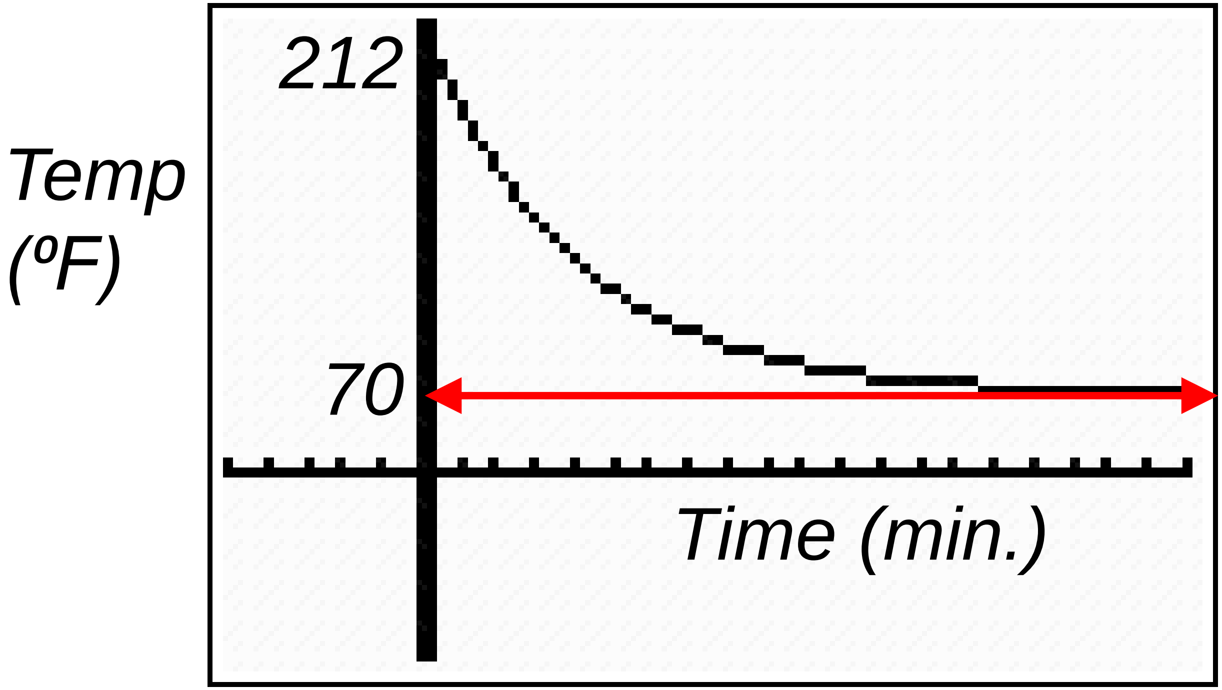
Label the x-axis and y-axis with quantity and unit of measure

At what temperature does it start?

Does the temperature go down forever?

What temperature will the water end up at?

Will it take hours, or minutes, or seconds to cool down?



$$T(t) = ab^t + k$$

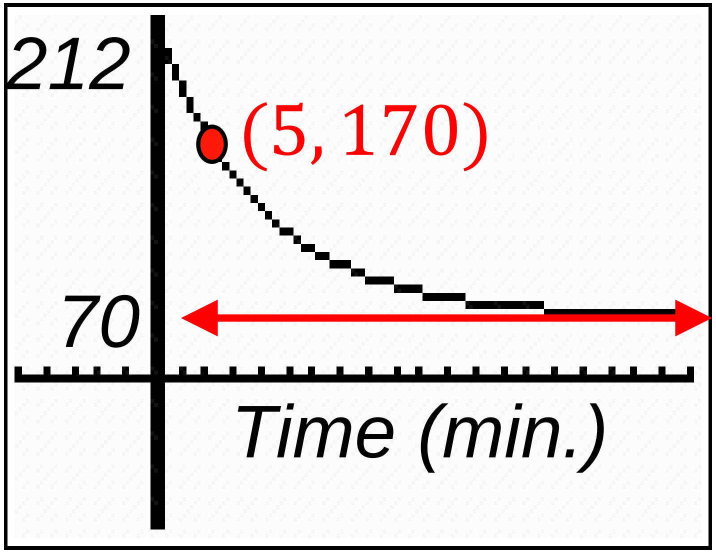
Horizontal asymptote

VSF

Growth (decay)
factor

Initial Value: the y-intercep

Temp (ºF)



$$T(t) = ab^t + k$$

Step 1: horizontal asymptote

$$k = 70$$

$$T(t) = ab^t + 70$$

Step 2: y-intercept → (0, 212)

$$212 = ab^0 + 70$$

$$a = 212 - 70 = 142$$

$$T(t) = 142b^t + 70$$

Step 3: need an x-y pair to plug in

After 5 minutes, the temperature is 170 F. → (5, 170)

$$170 = 142b^5 + 70$$

$$100 = 142b^5$$

$$\frac{100}{142} = b^5$$

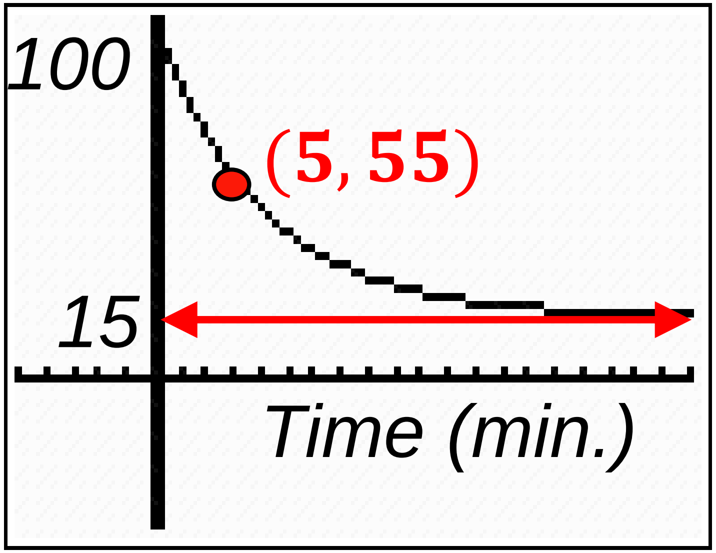
$$b^5 = 0.704$$

$$\sqrt[5]{b^5} = \sqrt[5]{0.704}$$

$$b = 0.932$$

$$T(t) = 142(0.932)^t + 70$$

A hard-boiled egg at temperature 100°C is placed in 15°C water to cool. Five minutes later the temperature of the egg is 55°C. What will be the temperature after 10 minutes?



$$T(t) = ab^t + k$$

Step 1: horizontal asymptote

$$k = 15$$

$$T(t) = ab^t + 15$$

Step 2: y-intercept → (0, 100)

$$100 = ab^0 + 15$$

$$a = 100 - 15 = 85$$

$$T(t) = 85b^t + 15$$

Step 3: an x-y pair to plug in → (5, 55)

$$55 = 85b^5 + 15$$

$$40 = 85b^5$$

$$\frac{40}{85} = b^5$$

$$b^5 = 0.471$$

$$\sqrt[5]{b^5} = \sqrt[5]{0.471}$$

$$b = 0.86$$

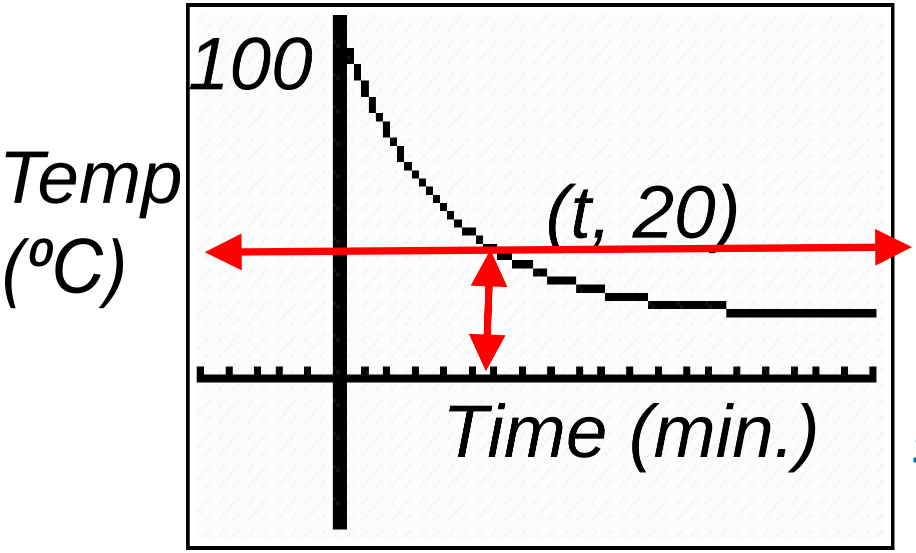
$$T(t) = 85(0.86)^t + 15$$

$$T(10) = 85(0.86)^{10} + 15$$

$$T(10) = 33.8\ C^0$$

When will the temperature reach 20 C?

$$T(t) = 85(0.86)^t + 15$$



Solve by graphing

Step 1: graph your equation

$$y = 85(0.86)^x + 15$$

Step 2: graph the equation

$$y = 20$$

Step 3: find the point of intersection

Time $= 18.8\ minuts$

Money in a bank account grows exponentially. The equation for the growth of money is:

Initial deposit

$$A(t) = A_0(1+r)^t$$

Years after the deposit

Amount of $$ In the account at time = "t"

"A-subscript 0" means "amount at t = 0"

Annual interest rate

A $100 initial deposit in a bank account grows at a rate of 3% per year. What is the equation to predict the value of the account in "*t*" years?

$$A(t) = \$100(1.03)^t$$

A bank gives you 3% interest per year. If they pay you (for the privilege of using your money) at the end of the year, what will be the growth factor for the year?

$$(1 + 0.03) = 1.03$$

The exponential growth function will be: $A(t) = A_0(1.03)^t$

A bank gives you 5% interest per year. If they pay you (for the privilege of using your money) at the end of the year, what will be the growth factor for the year?

$$(1 + 0.05) = 1.05$$

The exponential growth function will be: $A(t) = A_0(1.05)^t$

A bank pays 3% interest per year, and they pay you each month, what is the monthly interest rate?

$$\frac{0.03}{\cancel{year}} * \frac{\cancel{year}}{12\ months} \rightarrow \frac{0.03}{12\ month} \rightarrow \frac{0.03}{12}\ per\ month$$

$$\rightarrow 0.0025\ per\ month$$

A bank pays 5% interest per year, and they pay you each month, what is the monthly interest rate?

$$0.05\ \ per\ year \rightarrow \frac{0.05}{12}\ per\ month \rightarrow 0.0042\ \ per\ month$$

The exponential growth equation for money in a bank for account where the bank pays you more frequently than at the end of the year is:

Amount of $$ in the account as a function of time

Initial value

Annual interest rate

Years after the deposit

# of times the bank pays you each year

$$A(t) = A_0(1 + r/k)^{k*t}$$

"Compounding period" → the number of times the bank pays you each year.

"A bank pays 3% per year compounded monthly."

$$A(t) = A_0(1 + 0.03/12)^{12*t}$$

Values of "k"	
Words to look for	K
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365