Math-3A
Lesson 1-7

Exponential Function

The "Parent" Exponential Function

$$y = b^x$$

exponent

base

$$y = 2^x$$ **(base 2 exponential function)**

$$y = 3^x$$ **(base 3 exponential function)**

$$y = \left(\frac{1}{2}\right)^{x}$$ **(base 1/2 exponential function)**

The base MUST BE <u>positive</u> and CANNOT equal 1.

$$b = (0,1) \cup (1,\infty)$$

Fill in the output values of the table and graph the points.

$$f(x) = 2^x$$

Growth Factor is the base of the exponential

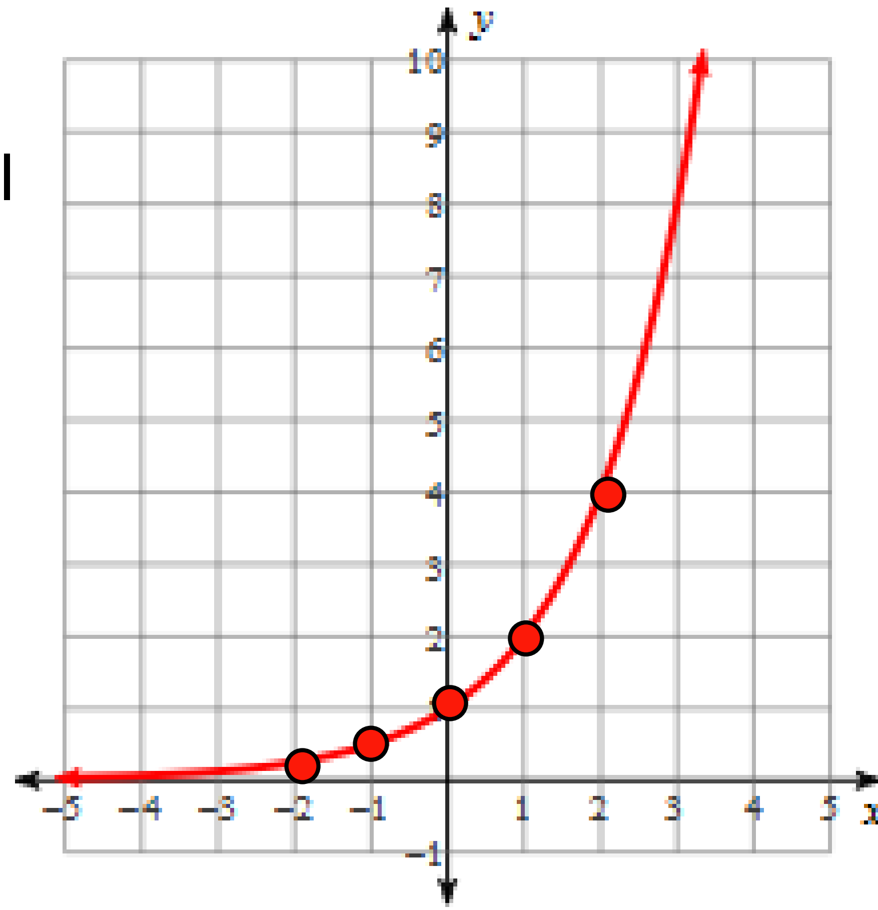
x	$2^{(\ )}$	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^{0}	1
1	2^{1}	2
2	2^{2}	4

$* 2$

$* 2$

$* 2$

$* 2$



$$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4} = 0.25$$

"negative exponent property"

$$2^0 = 1$$

"zero exponent property"

Exponential Function $f(x) = 2^x$

Will the 'y' value ever reach zero (on the left end of the graph)?

As the denominator gets bigger and bigger, the decimal version of the fraction gets smaller and smaller.

x	$2^{(\ )}$	y
-1	$2^{(-1)}$	$1/2$
-2	$2^{(-2)}$	$1/4$
-3	$2^{(-3)}$	$1/8$
-4	$2^{(-4)}$	$1/16$
-5	$2^{(-5)}$	$1/32$

$f(-1) = 1/2$

$f(-2) = 1/4$

$f(-3) = 1/8$

$f(-4) = 1/16$

$f(-5) = 1/32$

'y' gets closer and closer to zero but never reaches zero.

<u>Horizontal Asymptote</u>: a horizontal line the graph approaches but never reaches.

$$f(x) = 2^x$$

$$y = 0$$

$$Domain = ?$$

$$x = (-\infty, \infty)$$

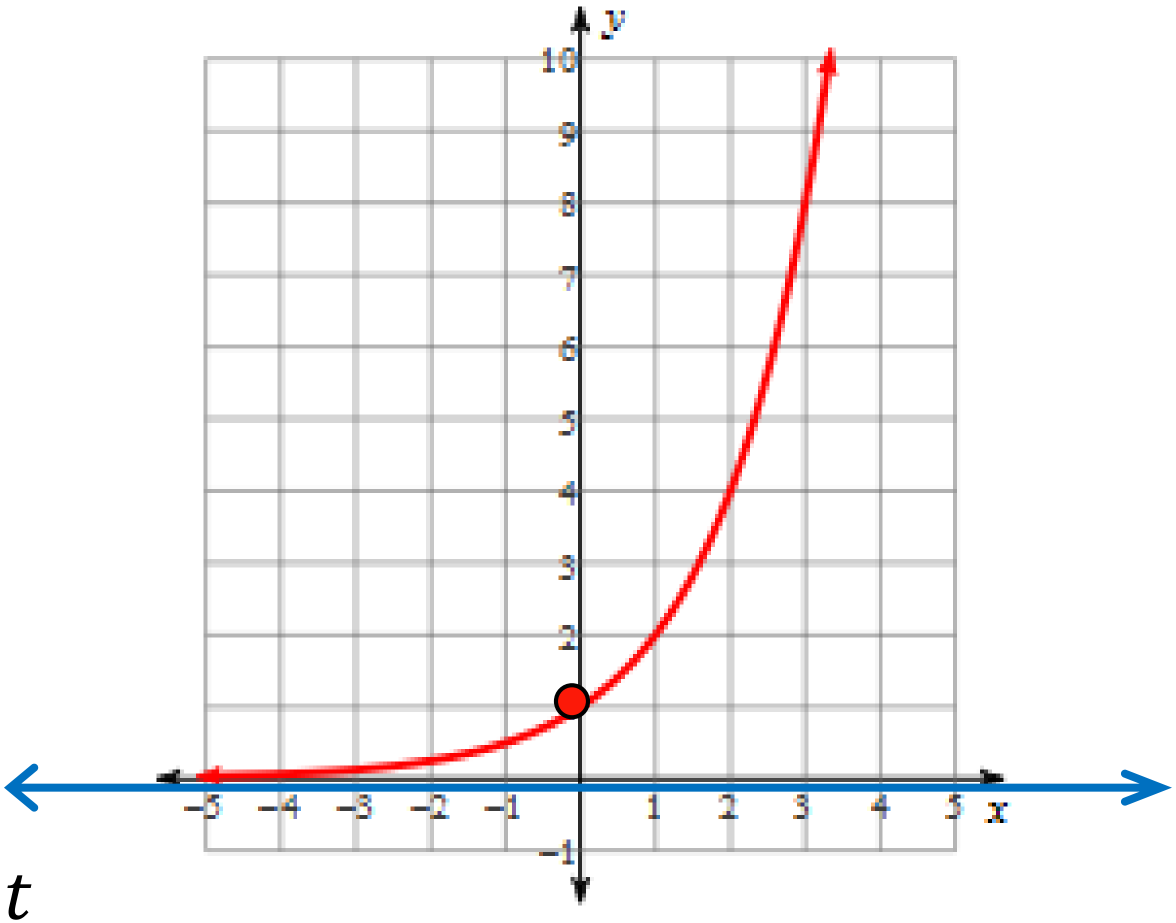
$$range = ?$$

$$y = (0, \infty)$$

y-intercept = ?

$$f(0) = y\ \ intercept$$

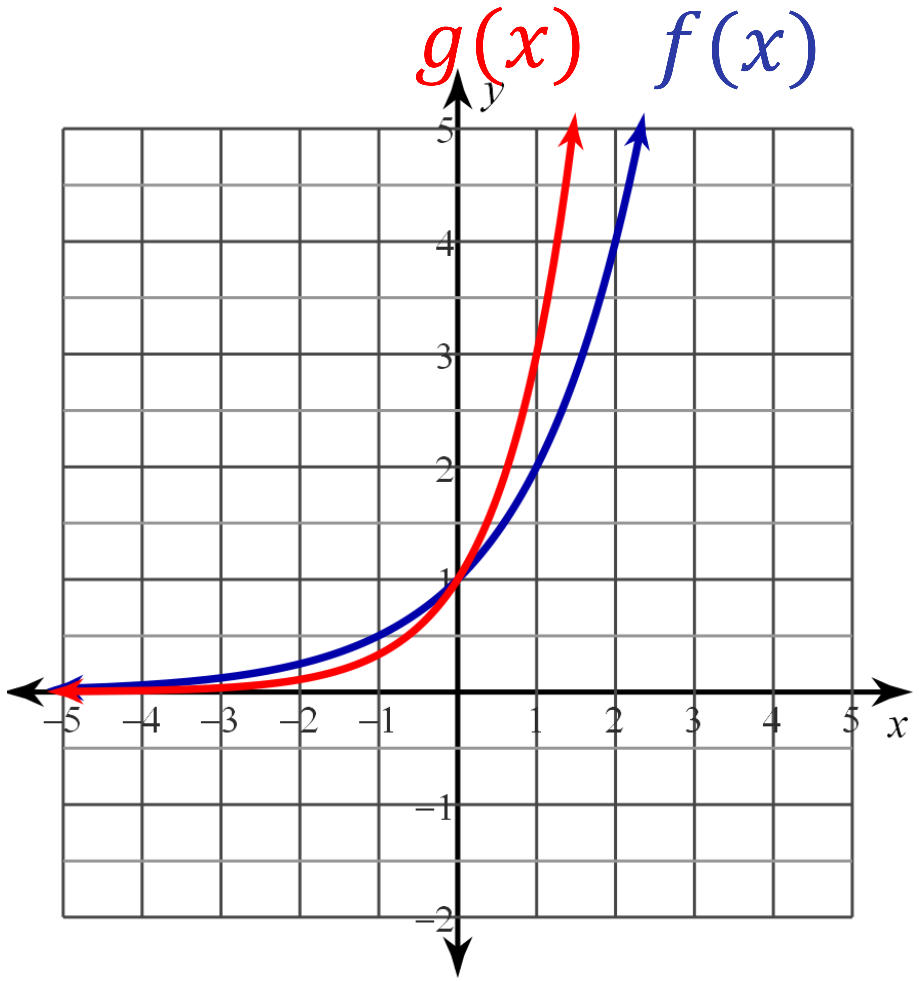
$$f(0) = 2^0 = 1$$



Exponential Growth: the graph is increasing (as you go from left to right the graph goes upward). Growth occurs when the base of the exponential is greater than 1.

$y = b^x$ 'b' > 1 → growth

$f(x) = 2^x$ $g(x) = 3^x$



What do both graphs have the same y-intercept?

$f(0) = 2^0 = 1$

$g(0) = 3^0 = 1$

All exponential "parent" functions have (0, 1) as the y-intercept.

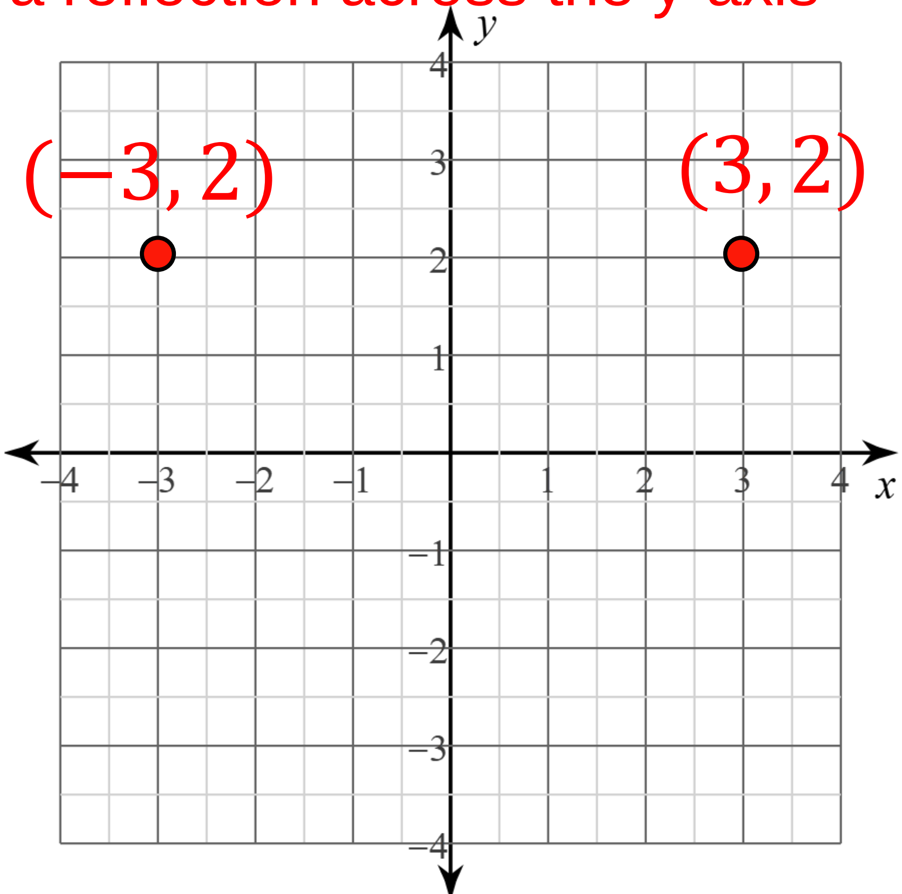
$g(x) = 2^x$

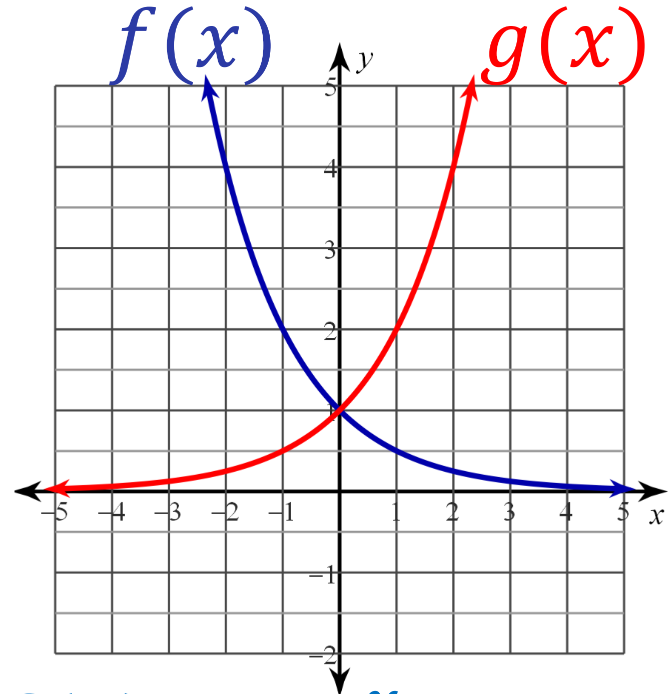
$f(x) = \left(\frac{1}{2}\right)^x$

→ Reflection across the y-axis

If (3, 2) is reflected across the y-axis, where would it be?

→ Replacing ‘x’ with ‘(-x)’ causes a reflection across the y-axis





$f(x) = 2^{-x}$

$f(x) = (2^{-1})^x$

Exponent of a Power Property of Exponents

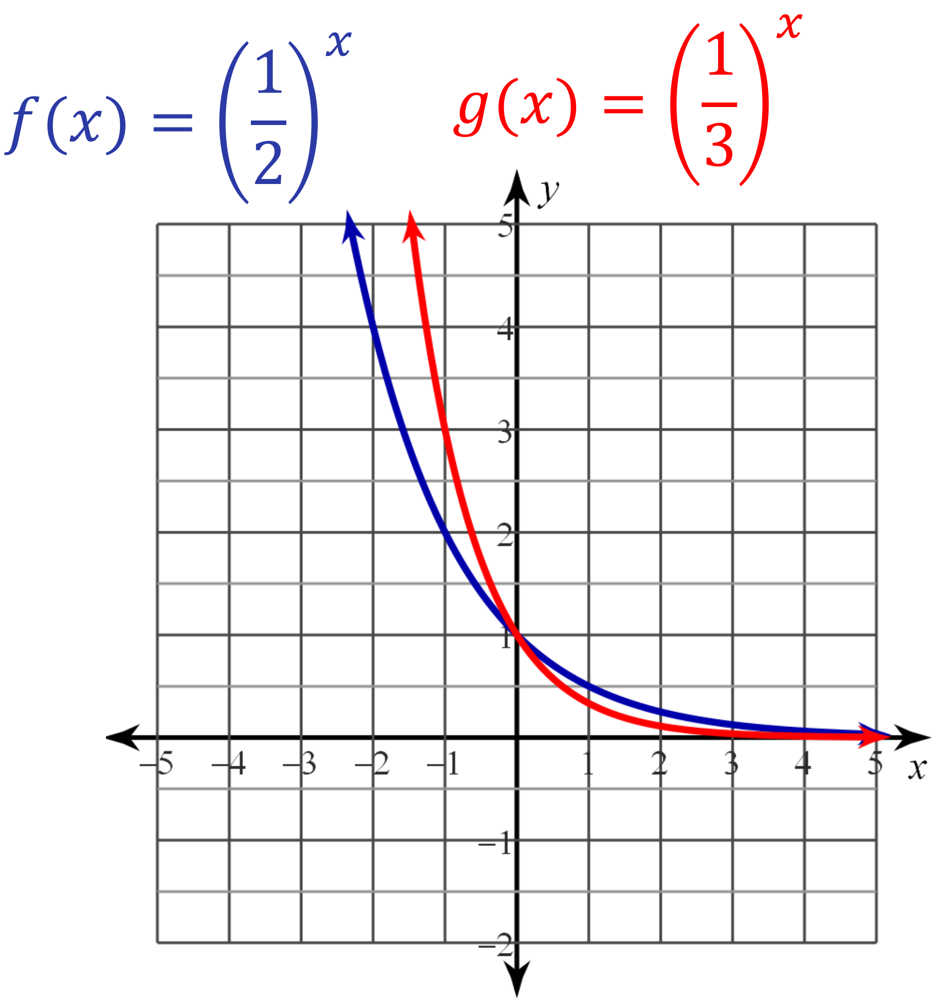
$f(x) = \left(\frac{1}{2}\right)^x$

Negative Exponent Property

<u>Exponential Decay</u>: the graph is decreasing (as you go from left to right the graph goes downward). This occurs when the base of the exponential is between 0 and 1.

$$y = b^x$$

0 < ‘b’ < 1 → decay

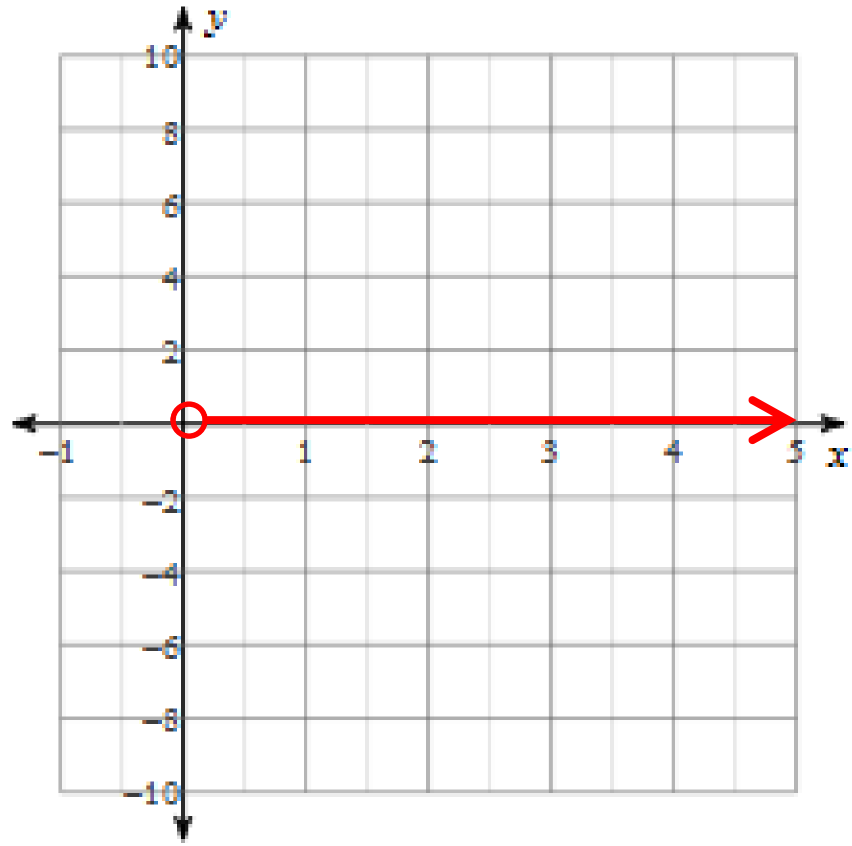


Can the base be <u>zero</u>?

$$f(x) = ab^x$$

$$g(x) = (0)^x$$

x	y	
-1	$\frac{1}{0} = ??$	$\left(\frac{0}{1}\right)^{-1}$
0	$???$	$(0)^0 = 0? \quad = 1?$
1	0	$(0)^1$



b ≠ 0

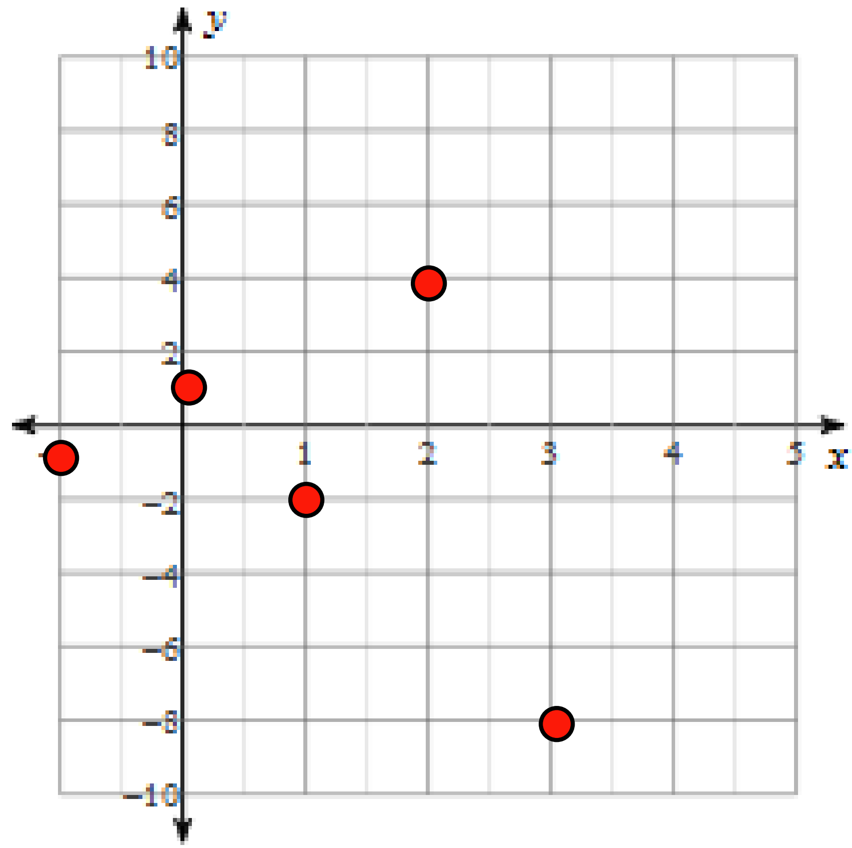
Can the 'base' be negative? $f(x) = ab^x$

$g(x) = (-2)^x$

'b' > 1 → growth

0 < 'b' < 1 → decay

x	y	
-1	-0.5	$(-2)^{-1}$
0	1	$(-2)^0$
$\frac{1}{2}$	$i = ?$	$(-2)^{\frac{1}{2}} = \sqrt{-2}$
1	-2	$(-2)^1$
2	4	$(-2)^2$
3	-8	$(-2)^3$



b ≠ negative numbers

Can the base be 1?

$$f(x) = ab^x$$

$$g(x) = (1)^x$$

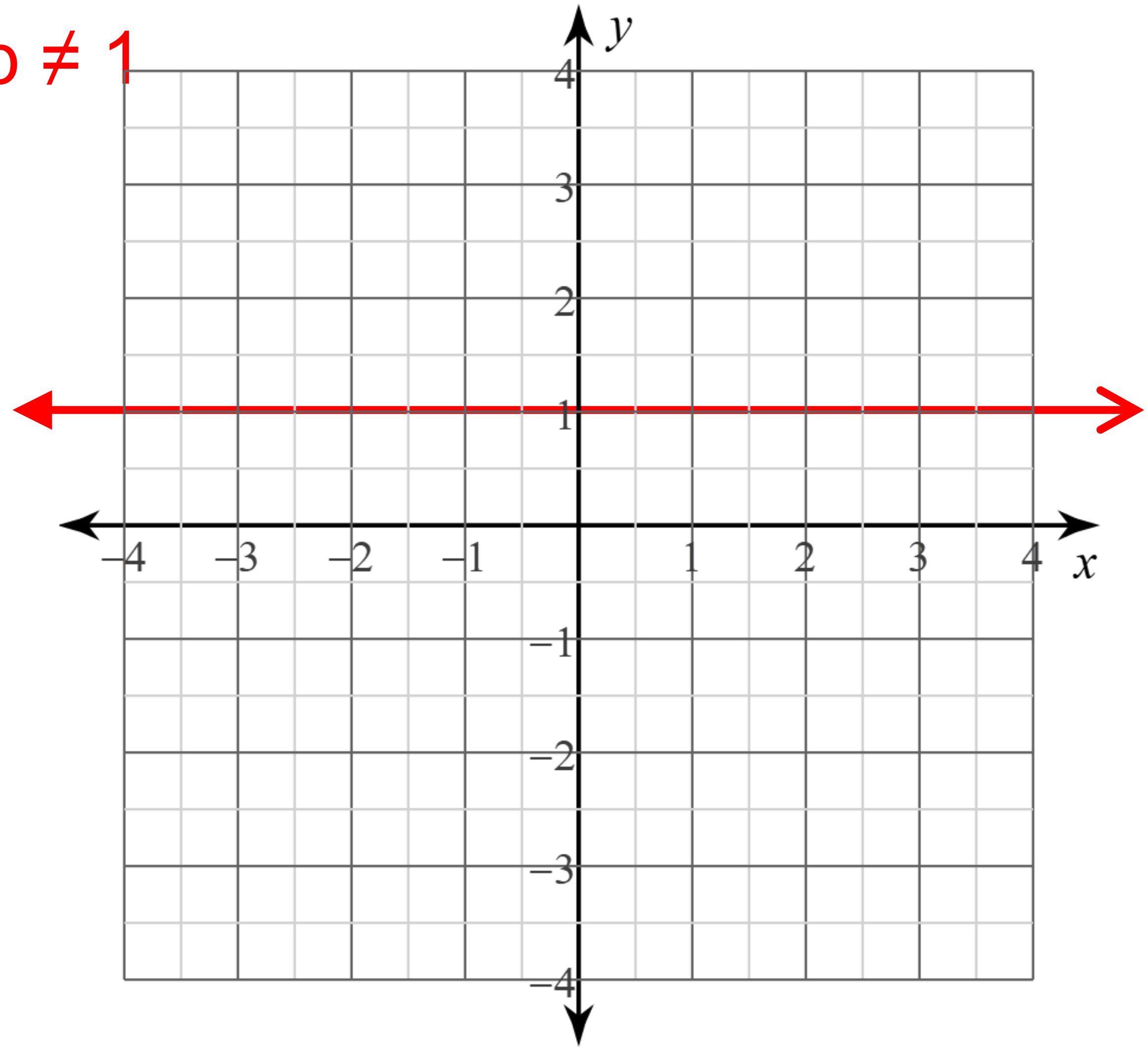
b ≠ 1

x	y
-1	1
0	1
1	1

$(1)^{-1}$

$(1)^0$

1^1



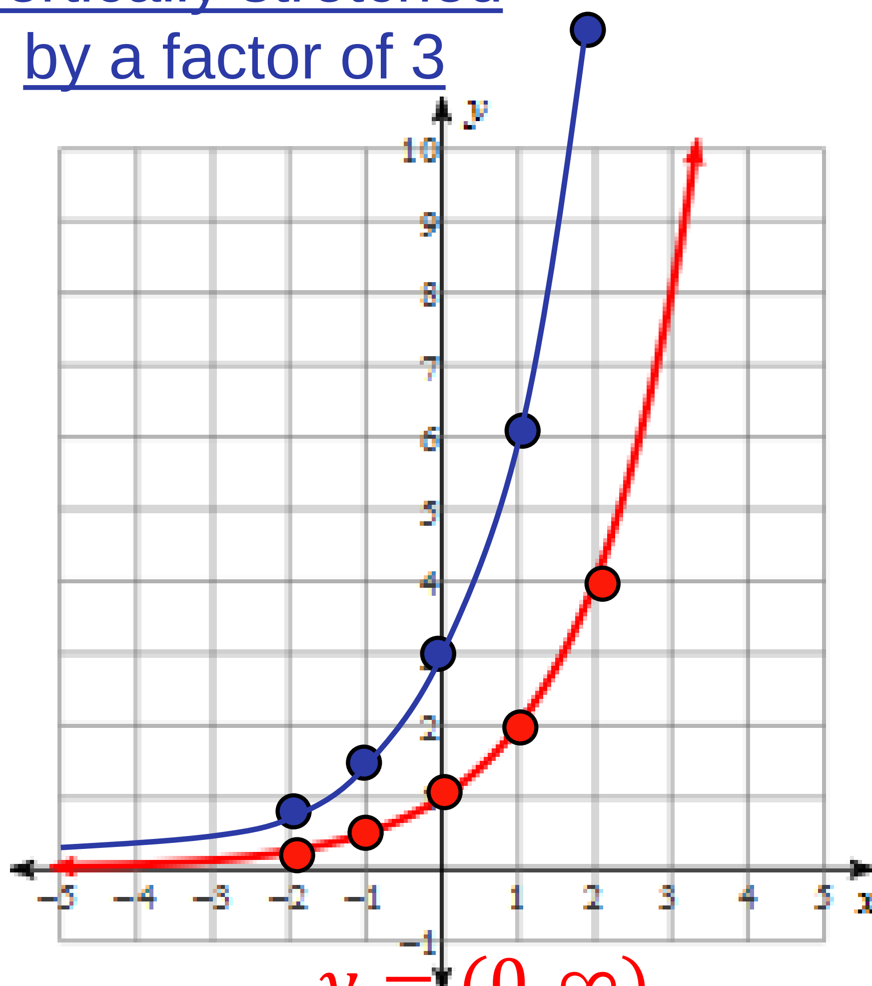
$$0 < b < 1, \text{ OR } \ b > 1$$

$$b = (0,1) \cup (1,\infty)$$

$f(x) = 2^x$ $g(x) = 3(2)^x$

Vertically stretched by a factor of 3

x	$2^{(\ )}$	f(x)	$g(x)$
-2	2^{-2}	0.25	0.75
-1	2^{-1}	0.5	1.5
0	2^{0}	1	3
1	2^{1}	2	6
2	2^{2}	4	12



Horizontal asymptote: $y = 0$ $y = 0$

$range = ?$ $y = (0, \infty)$ $y = (0, \infty)$

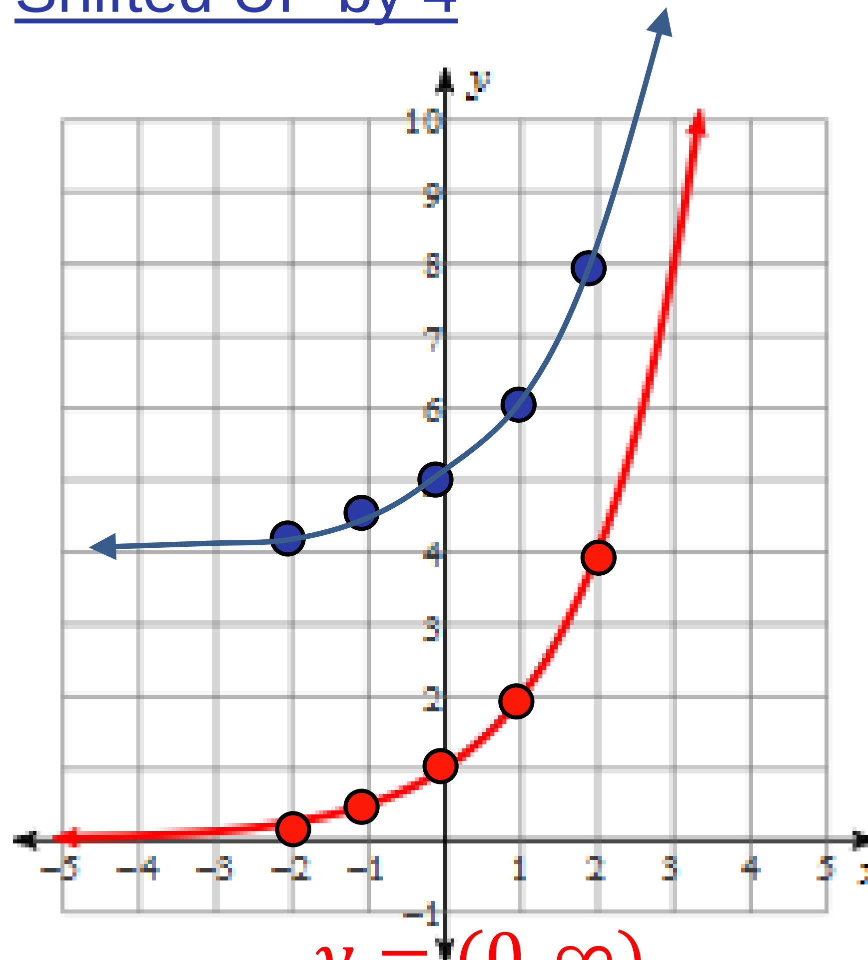
$Domain = ?$ $x = (-\infty, \infty)$ $x = (-\infty, \infty)$

y-intercept = ? $(0, 1)$ $(0, 3)$

$f(x) = 2^x$ $\quad$ $\mathrm{k}(x) = 2^x + 4$

Shifted UP by 4

x	$2^{(\ )}$	f(x)	$k(x)$
-2	2^{-2}	0.25	4.25
-1	2^{-1}	0.5	4.5
0	2^0	1	5
1	2^1	2	6
2	2^2	4	8



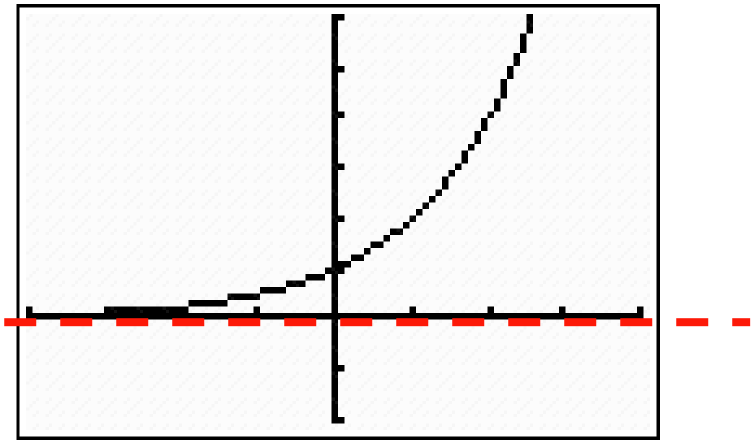
Horizontal asymptote: $y = 0$ $\quad$ $y = 4$

$range = ?$ $\quad$ $y = (0, \infty)$ $\quad$ $y = (4, \infty)$

$Domain = ?$ $\quad$ $x = (-\infty, \infty)$ $\quad$ $x = (-\infty, \infty)$

y-intercept = ? $\quad$ $(0, 1)$ $\quad$ $(0, 5)$

Transformations of the Exponential Function

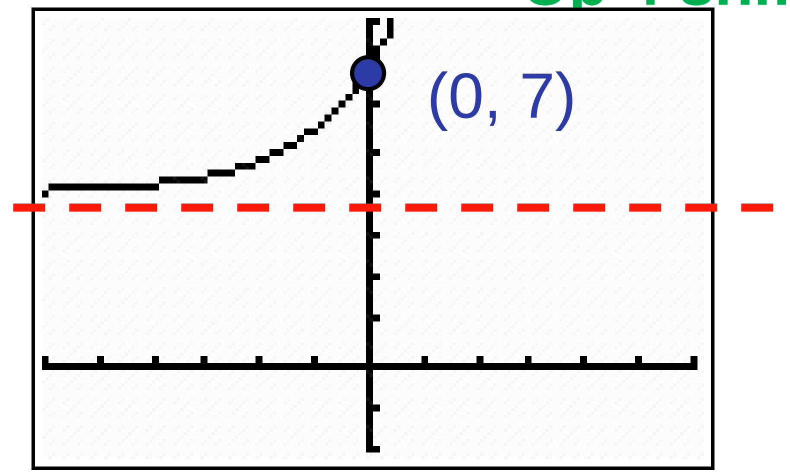


$f(x) = 2^x$ Base-2 Exponential Parent Function

VSF= 3

$$h(x) = 3(2)^x + 4$$

Up 4 shift



Transformation Form of the Exponential Function

$$y = ab^x + k$$

vertical shift and horizontal Asymptote

VSF:

y-intercept: (0, a + k)

Growth Factor (the base of the exponential)

$$h(0) = 3(2)^0 + 4$$

$$h(0) = 7$$