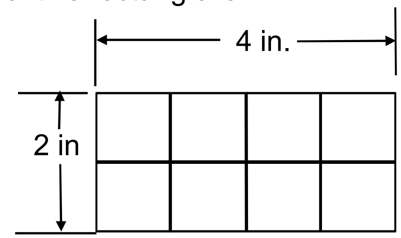
Math-3A
Lesson 9-7

Area of Triangles and Application Problems

The <u>area</u> of this rectangle is….?



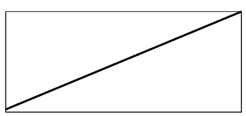
$$Area_{rectangle} = L * W$$
$$Area_{rectangle} = (2\ in)(4\ in)$$
$$Area_{rectangle} = 2 * 4 *\ in *\ in$$
$$Area_{rectangle} = 8\ in^2$$

What are the <u>units</u> of <u>area?</u>

Rectangle area formula. $A_{\text{rectangle}} = L * W$

W = width

L = length



Triangle area formula.

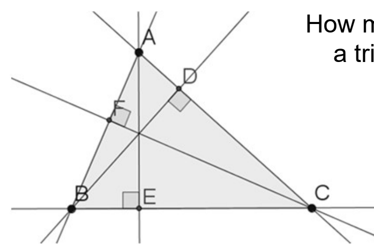
$$A_{\text{triangle}} = \frac{1}{2} * A_{\text{rectangle}} = \frac{1}{2} * L * W$$

$$A_{\text{triangle}} = \frac{1}{2} * B * h$$

<u>Altitude of a triangle</u>: The perpendicular distance from any vertex to its opposite side.

<u>Altitude of a triangle</u>: means the same thing as the <u>height of a triangle</u>.

<u>Height = Altitude</u>



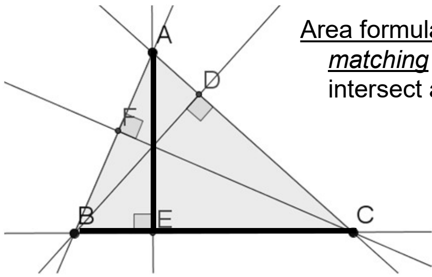
How many "heights" (altitudes) does a triangle have?

<u>three</u>

<u>base = side</u>

$$A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$$

The altitude of a triangle. $A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$

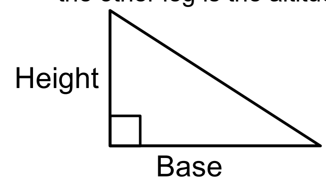


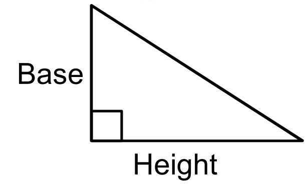
Area formula: requires the use of *matching* heights and sides (that intersect at a 90 degree angle).

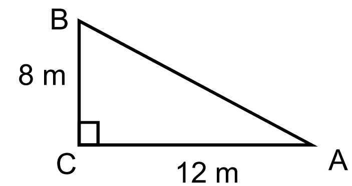
Using segment BC as the base, requires the use of segment AE as the height.

For a right triangle
- one leg is the base
- the other leg is the altitude.

$Area = \frac{1}{2} * \text{base} * \text{height}$







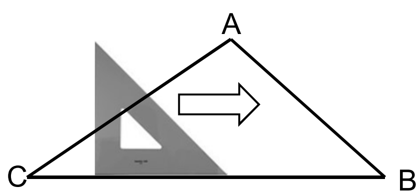
$$A_{\Delta} = \frac{1}{2} * 8 * 12$$

$$A_{\Delta} = 48\ m^2$$

Dropping an altitude: drawing a perpendicular segment from a corner angle to the opposite side.

1) Use the biggest angle.

2) Slide the plastic triangle along one edge of your triangle until the vertical side passes through the biggest angle.

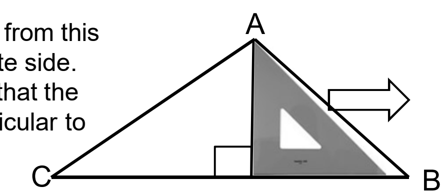


Dropping an altitude: drawing a perpendicular segment from a corner angle to the opposite side.

1) Use the biggest angle.

2) Slide the plastic triangle along one edge of your triangle until the vertical side passes through the biggest angle.

3) “drop an altitude” from this vertex to the opposite side. Make sure to show that the segment is perpendicular to the opposite side.



4) The length of the altitude is the height of the triangle.

Drop an altitude from angle A to opposite side BC.

$$Area = 0.5 * \text{base} * \text{height}$$

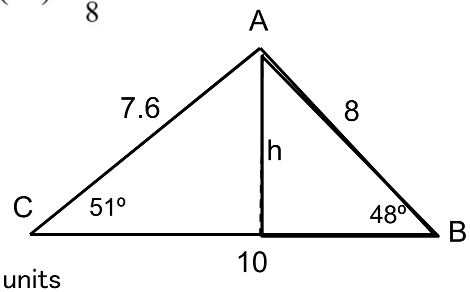
$$\sin A = \frac{opp}{hyp} \qquad \sin(48) = \frac{h}{8}$$

h = 8*Sin(48°)

h = 5.9

Area = ½(10)(5.9)

Area = 29.5 square units



For height "h" we could use the other triangle.

$$Area = 0.5 * \text{base} * \text{height}$$

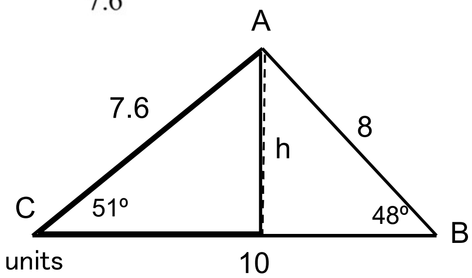
$$\sin A = \frac{opp}{hyp} \qquad \sin(51) = \frac{h}{7.6}$$

h = 7.6*Sin(51°)

h = 5.9

Area = ½(10)(5.9)

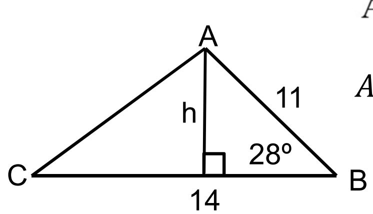
Area = 29.5 square units



Either triangle gives us the same height (and therefore the same area).

B=28, a = 14, c = 11, Find the Area of the Triangle

$$A_{\Delta} = 0.5 * \text{base} * \text{height}$$

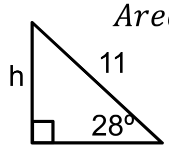


$$Area_{\Delta ABC} = 0.5 * 14 * h$$

$$\sin 28 = \frac{h}{11}$$

$$11(\sin 28) = h$$

$$h = 5.2$$



$$Area_{\Delta ABC} = 0.5 * 14 * (5.2)$$

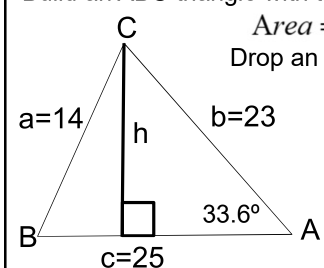
$$Area_{\Delta ABC} = 36.1$$

A=33.6°, a = 14, b = 23 $\quad Area_{\Delta ABC} = ?$

Build an ABC triangle with the longest side on the bottom.

$$Area = 0.5 * \text{base} * \text{height}$$

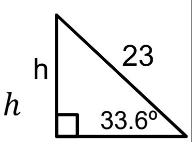
Drop an altitude to the longest side.



$$\sin 33.6 = \frac{h}{23}$$

$$23\ (\sin 33.6) = h$$

$$h = 12.7$$



$$Area_{\Delta ABC} = 0.5 * 25 * (12.7)$$

$$Area_{\Delta ABC} = 159.1$$

Angle of Elevation: angle above the horizon that the eye has to look up to see something.

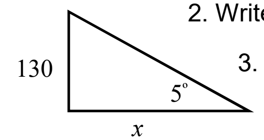
Angle of Depression: angle below the horizon that the eye has to look down at something.

(a) (b)

Using Angle of Elevation

The angle of elevation from the buoy to the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is 5°. Find the distance *x* from the base of the lighthouse to the buoy.

1. Draw the picture



2. Write the equation. $\tan(5^\circ) = \frac{130}{x}$

3. Solve for the unknown variable.

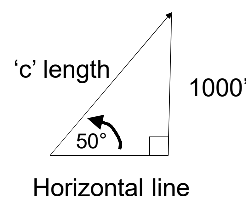
$$x = \frac{130}{\tan(5^\circ)} = 1485.9\,ft$$

Using Angle of Elevation

1. If the height of a building is 470 m and you are standing 100 m away from the building, find the angle of elevation to the top of the building.

2. What is the height of a tree if you are standing 50 feet from it, and that angle of elevation (from your height of eye) is 50 degrees. Assume that your eyes are 5 feet above the ground.

Angle of elevation of the old Bridal Veil Falls tram in Provo Canyon was 50°. The Elevation change from the bottom of the tram to the top was 1000 feet. What was the length of the tram cables?

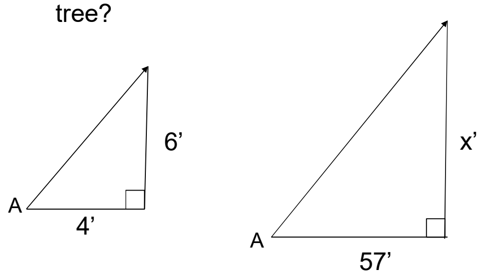


$$\sin\theta = \frac{opp}{hyp} \qquad \sin 50 = \frac{1000}{hyp}$$

$$hyp = \frac{1000}{\sin 50} \qquad hyp = 1305'$$

A tree that is 6 feet tall has a shadow of 4 feet. You want to measure the height of a very large tree (but you don't have a ladder big enough).

1) How can you use similar triangles to solve for the height of the large tree?
2) If the shadow of the large tree is 57 feet, how tall is the tree?



$$\frac{6}{4} = \frac{x}{57}$$

$$\frac{6*57}{4} = x$$

$$85.5\,ft = x$$