## Math-3A

Lesson 9-6
Radian Measure,
Measures of Arcs and Sectors

Negative Angles: The terminal side opened in the clockwise
direction from the initial side of the angle.

Angles outside of the range: $0 \leq \theta \leq 360$ : The terminal side may make more than one revolution. An angle whose terminal side makes one complete evolution and then an additional 30 degree would have a

measure of 390
degrees.

Positive Angles: The terminal side opened in the counter-clockwis direction from the initial side of the angle.

Angle measure: is put at the end of the terminal side rather than inside the angle.


Co-terminal Angles: may have different measures but they share both the initial and terminal sides.


In general: co-terminal angles have measures defined by: $m \angle \theta=m \angle \beta \pm 360 n$
where ' $n$ ' is the number of revolutions beyond the $1^{\text {st }}$ one.

$$
\begin{gathered}
\theta=? \\
\tan \theta=3 / 4
\end{gathered}
$$

"Undo" the tangent function

$$
\begin{gathered}
\tan ^{-1} \tan \theta=\tan ^{-1}(3 / 4) \\
\theta=\tan ^{-1}(3 / 4) \\
\theta=36.86^{\circ}
\end{gathered}
$$

Another way to think about it:
$\tan \theta=o p p / a d j \quad \theta=\tan ^{-1}(o p p / a d j)$

$$
\begin{gathered}
\tan \theta=5 / 9 \\
\cos \theta=?
\end{gathered}
$$



Tangent ratio gives 2 sides of a right triangle. To find the cosine ratio, you need to find the hypotenuse.

$$
\begin{aligned}
h=\sqrt{5^{2}+9^{2}} & \cos \theta=\frac{9}{\sqrt{106}} \\
h=\sqrt{25+81} & \\
h=\sqrt{106} & \cos \theta=\frac{9 \sqrt{106}}{106}
\end{aligned}
$$

## Degrees and Radians

What is the circumference of a circle whose radius $=1$ ?

$$
C=2 \pi r \quad r=1 \quad C=2 \pi
$$

We say that the "radian measure" of a circle is $2 \pi$.

$$
\text { radian measure }=\frac{\operatorname{arc} \text { length }}{\text { radius }}
$$

radian measure $($ of a circle $)=$ circumference
radian measure (of a circle) $=\frac{2 \pi f}{y}=2 \pi$
What is the radian measure of an angle that is $1 / 2$ of the circle?

$$
180^{\circ}=\pi \text { (radians) }
$$

Radian Measure: The ratio of the arc length and the


Units of radians = inches/inches
Radian measure has no units! (nice)

## Convert between radians and degrees using a "proportion".

$$
\frac{\text { angle }_{\text {degrees }}}{360}=\frac{\text { angle }_{\text {radians }}}{2 \pi}
$$

$$
\frac{7}{8} \pi \quad \frac{\text { angle }_{\text {degrees }}}{360}=\frac{7 / 8 \pi}{2 \pi}
$$

$$
\begin{gathered}
360 * \frac{\text { angle }_{\text {degrees }}}{360}=0.4375 * 360 \\
\text { angle }_{\text {degrees }}=157.5^{\circ}
\end{gathered}
$$

## Convert between radians and degrees using a "unit

 conversion factor".Unit Conversion factor: a ratio of equal measurements in different units that allow conversion of a one type of unit to another
(feet $\rightarrow$ inches, degrees $\rightarrow$ radians, radians $\rightarrow$ degrees etc.)

$$
\begin{array}{ccc}
\frac{180^{\circ}}{180^{\circ}}=\frac{\pi}{180^{\circ}} & 180^{\circ}=\pi \text { radians } & \frac{180^{\circ}}{\pi}=\frac{\pi}{\pi} \\
1=\frac{\pi}{180^{\circ}} & & 1=\frac{180^{\circ}}{\pi} \\
\left(\frac{\pi}{180^{\circ}}\right) & \text { "conversion face } \\
\text { These " }
\end{array}\left(\frac{180^{\circ}}{\pi}\right) .4
$$

When you multiply a number by one of these factors, (you are multiplying by " 1 ") but the units are converted.

## Converting from Degrees to Radian Measure

$$
140 \%\left(\frac{\pi}{180^{\prime}}\right)=\frac{140}{180} \pi=\frac{14}{18} \pi=\frac{7}{9} \pi
$$

Converting from Radian Measure to Degrees

$$
\frac{t}{2}\left(\frac{180^{\circ}}{\not t}\right)=90^{\circ}
$$





