SM3-A HANDOUT 8-3 (Solving Log & Exponential Equations)

Linear Equation:

What does "solve" a single variable equation mean?

Find the value of 'x' that ______________________

$3x + 2 = 4x - 5$

"Isolate the variable"

subtract 3x from both sides

add 5 to both sides

Radical Equation:

"Isolate the radical"
"Undo the radical"

$\sqrt{3x-2} - 1 = 3$

add 1 to both sides

square both sides

add 2 both sides

Divide by 3 both sides

Check your solution

Your turn:

"Isolate the radical" "Undo the radical"

Solve: $\sqrt{3x-5} = x - 3$

Solving an Exponential Equation: The easiest problem

$2^x = 2^{4-x}$ Exponents have to be equal to each other!

$x = 4 - x$

+x +x

$2x = 4$

÷x ÷x

$x = 2$

Check your answer!

$2^2 = 2^{4-2}$

$7^{2x+1} = 7^{13-4x}$

$2x + 1 = 13 - 4x$

+4x +4x

$6x + 1 = 13$

-1 -1

$6x = 12$

$x = 2$

Equivalent Powers with *different bases.*

$4^1 = 2^2$

$8^1 = 2^3$

Easy

Harder

$4^2 = (2^2)^2 = 2^4$

$9^2 = (3^2)^2 = 3^4$

$9^{2x} = (3^2)^{2x} = 3^{4x}$

Change the base of the power as indicated:

$27^1 = 3^?$

$16^2 = 4^?$

$25^{2x} = 5^?$

Solving using "convert to same base"

$2^{4x-1} = 8^{x-1}$ "convert to same base"

$2^{4x-} = (2^3)^{x-1}$ Exponent of a power Exponent Property

$2^{4x-1} = 2^{3x-3}$

$\boxed{x = -2}$

$4x - 1 = 3x - 3$
-3x -3x

$x - 1 = -3$
+1 +1

Check your answer!

$2^{4(-2)-1} = 8^{-2-1}$

$2^{-9} = 8^{-3}$

$(\ 2^{-9} = 8^{-3}\ )^{-1}$

$2^9 = 8^3$

$512 = 512$

Solving using "convert to same base"

$9^{2x} = 27^{x-1}$ "convert to same base"

Check your answer!

Solving using "log of a power" property

$9^{2x} = 27^{x-1}$ Take natural log of both side

$\ln 9^{2x} = \ln 27^{x-1}$ "log of pwr property"

$2x \ln 9 = (x-1)\ln 27$
÷ ln 9 ÷ ln 9

$2x = (x-1)\dfrac{\ln 27}{\ln 9}$ simplify

$2x = (x-1)(1.5)$ simplify

$2x = 1.5x - 1.5$
-1.5x -1.5x

$0.5x = -1.5$
*2 *2

$\boxed{x = -3}$

Same solution as previous method!

$(3^2)^{2x} = (3^3)^{x-1}$

$3^{4x} = 3^{3x-3}$

$4x = 3x - 3$

$\boxed{x = -3}$

Solve using "convert to same base"

$8^{x+2} = 4^{x-2}$ 8 and 4 are both powers of 2.

$(2^3)^{x+2} = (2^2)^{x-2}$ Power of a Power property

$2^{3x+6} = 2^{2x-4}$ "undo the power"

$3x+6 = 2x-4$ Did I do a step mentally?

-2x -2x

$x+6=-4$

-6 -6

$\boxed{x=-10}$

Solving using "log of a power" property

$8^{x+2} = 4^{x-2}$ Take natural log of both side

$\ln 8^{x+2} = \ln 4^{x-2}$ "log of pwr property"

$(x+2)\ln 8 = (x-2)\ln 4$

÷ ln 8 ÷ ln 8

$(x+2) = (x-2)\frac{\ln 4}{\ln 8}$ simplify

$x+2 = (x-2)(0.6666666)$ simplify

$x+2 = (x-2)\left(\frac{2}{3}\right)$

$3(x+2) = (x-2)\left(\frac{2}{3}\right)(3)$

$3x+6 = 2x-4$

-2x - 6 -2x - 6

$\boxed{x=-10}$

Sometimes you can't rewrite the exponentials with the same bases so you have no choice. Use log of a power property.

$5^x = 7^{2x-1}$

$\ln 5^x = \ln 7^{2x-1}$

$x\ln 5 = (2x-1)\ln 7$

÷ ln 5 ÷ ln 5

$x = (2x-1)\frac{\ln 7}{\ln 5}$

$x = (2x-1)(1.21)$

$x = 2.42x - 1.21$

+1.21 +1.21

$1.21 + x = 2.42x$

-x -x

$1.21 = 1.42x$

÷ 1.42 ÷ 1.42

$\boxed{x=0.85}$

Solve using the "log of a power property"

$5^{x+2} = 4^{x-2}$

Solve for ‘x’ (how do you get the exponent ‘x’ all by itself?

$8^x - 2 = 5$ “Isolate the exponential”

+2 +2

$8^x = 7$ “convert to a log”

$x = \log_8 7$

$x = 0.9358$

Change of base formula

$x = \frac{\log 7}{\log 8}$

$x = 0.9358$

Solve using “undo the exponential”

$3^{2x-1} + 5 = 7$ “Isolate the exponential”

The easiest log equation.

$\log(x+3) = \log(2x-1)$

$x + 3 = 2x - 1$ $\rightarrow x = 2$

Why does this work?

y = y

$y = \log(x+3)$ $10^y = x + 3$

$y = \log(2x-1)$ $10^y = 2x - 1$

Substitution Property

$x + 3 = 10^y = 2x - 1$

$x + 3 = 2x - 1$

Some functions don’t have domain of all real numbers → equations of these types may have extraneous solutions

Extraneous solution: an apparent solution that does not work when plugged back into the original equation.

You MUST check the solutions in the original equation for any equation this is of the function type that has a restricted domain.

Square root equations: Radicands cannot be negative; Square roots do not equal negative numbers

Rational equations: Denominators cannot equal zero

Log equations: Logarands cannot be zero or negative

Solve: $\log(x-5) = \log(2x+3)$

Remember to check for extraneous solutions by plugging the solution for 'x' back into the original equation.

Can you have a negative logarand?

$\log_2 5^x = 5$ Power property of logarithms

$x \log_2 5 = 5$ → Change of base $x\frac{\ln 5}{\ln 2} = 5$

$2.32192x = 5$

÷2.32192 ÷2.32192

$x = 2.1534$

Use inverse property of multiplication

$x = 5\frac{\ln 2}{\ln 5}$

$x = 2.1534$

Solve:

$\log_2 5^x = 4$

$\log_3 4^{5x} = 6$

<u>Solving Logs requiring condensing the product.</u>

$\log 2x + \log(x-5) = 2$ "condense the product"

$\log 2x(x-5) = 2$ "undo the logarithm"

$10^2 = 2x(x-5)$ Quadratic → put in standard form

$100 = 2x^2 - 10x$

$2x^2 - 10x - 100 = 0$ Divide both sides by '2'

$x^2 - 5x - 50 = 0$ factor

$(x-10)(x+5) = 0$ Zero product property

x = 10, -5

Check for extraneous solutions: x = 10, ~~-5~~

$\log 2x + \log(x-5) = 2$

$\log(2*10) + log(10-5) = 2$

$\log(20) + log(5) = 2$ All logarands are positive ☺

$log100 = 2$ "Condense the product" $10^2 = 100$ Checks

$\log(2)(-5) + log(-5-5) = 2$

$\log(-10) + log(-10) = 2$ Negative logarands ☹

x = 5 is an extraneous solution.

$\ln 5^{x+2} + \ln 5^2 = 2$

$\log_2 4x + \log_2 3 = 6$

More complicated Logarithmic Equations

$2 + \log_2 5^{x-2} = 7$ "Isolate the logarithm"

-2 -2

$\log_2 5^{x-2} = 5$ "undo the logarithm"

$(x-2)\log_2 5 = 5$

$\div \log_2 5 \quad \div \log_2 5$

$x - 2 = 2.15338$

$+2 \quad +2$ Add '2' to both sides.

$x = 4.1524$

$3 + \log_4 3^{2x-1} = 6$

$-7 + 2\ln 4^{x-3} = 5$

$\log_4(5x-1) = 3$