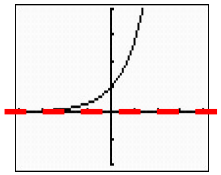
Math-3A
Lesson 8-1

Properties of Logarithmic Functions
(Product of Logs
Log of a Power)

Exponential Function



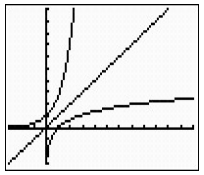
$f(x) = 10^x$

Domain = ? (-∞, ∞)

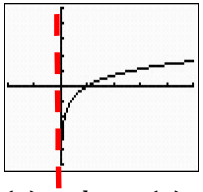
Range = ? (0, ∞)

Horizontal asymptote = ?
y = 0

Inverse Functions



Logarithm Function



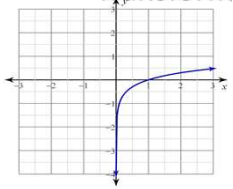
$f^{-1}(x) = log_{10}(x)$

Domain = ? (0, ∞)

Range = ? (-∞, ∞)

Vertical asymptote = ?
x = 0

Transformations of the Log Function



$f(x) = \log x$

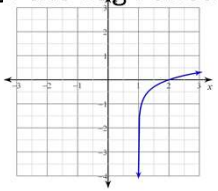
Domain = ? (0, ∞)

Range = ? (-∞, ∞)

vertical asymptote = ?
x = 0

X-intercept = ?
x = 1

Where increasing = ?
(0, ∞)



$g(x) = \log(x-1)$

Right 1 shift

Domain = ? (1, ∞)

Range = ? (-∞, ∞)

asymptote = ? x = 1

Logarand

$$g(x) = 3\log(2x-1)+5$$

Logarand

Vertical Asymptote: The value of 'x' that makes the logarand equal to zero.

Vertical asymptote = ? 2x – 1 = 0

x = ½

Convert to logarithm form Log□ = □

What is the solution?

x = 2	$5^x = 25$	$\log_5 25 = x$
x = 3	$4^x = 64$	$\log_4 64 = x$
x = ???	$b^x = y$	$\log_b y = x$
x = 2	$9^x = 81$	$\log_9 81 = x$
x = 3	$10^x = 1000$	$\log_{10} 1000 = x$

Convert to exponential form Log□ = □

What is the solution?

x = 2	$\log_{10} 100 = x$	$10^x = 100$
x = 3	$\log_3 27 = x$	$3^x = 27$
x = 0	$\log_9 1 = x$	$9^x = 1$
x = 16	$\log_4 x = 2$	$4^2 = x$
x = 32	$\log_2 x = 5$	$2^5 = x$

$f(x) = 5^{2x+4}$ Find $f^{-1}(x)$

$y = 5^{2x+4}$ Replace f(x) with 'y'

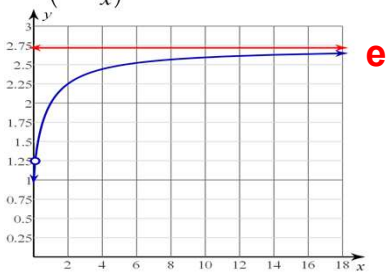
$x = 5^{2y+4}$ exchange 'x' and 'y'

$\log_5 x = 2y + 4$ Log is the exponent (remember how to convert between the two?)

$-4 + \log_5 x = 2y$ Solve for 'y'

$\frac{-4 + \log_5 x}{2} = y$ $\boxed{y = -2 + \frac{1}{2}\log_5 x}$

$y = \left(1 + \frac{1}{x}\right)^x$



e ~ 2.718...

$A(t) = (1 + r/k)^{kt}$ **If we compound continuously (k → ∞) then**

$A(t) = e^{rt}$

$$2^3 * 2^2 = 2^5$$

The product of powers → add the exponents

$$2^3 * 2^2 = 2^5$$

Logarithm: another way of writing the exponent

Convert each exponent above into a log:

$$\log_2 8 + \log_2 4 = \log_2 32$$

$$3 + 2 = 5$$

This is the logarithm equivalent of the multiply powers property of exponents.

Log of a Product Property

$$\log_2 15 = \log_2 (3*5)$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

$$\log_b (RS) = \log_b R + \log_b S$$

log of a product = sum of the logs of the factors.

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log_3 xy \qquad \log_3 x + \log_3 y$$

$$\log_3 45 \qquad \log_3 3 + \log_3 3 + \log_3 5$$

$$45 = 3*3*5 \qquad 2\log_3 3 + \log_3 5$$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log(3xy^2) = \log 3 + \log x + \log y^2$$

$$= \log 3 + \log x + \log y + \log y$$

$$= \log 3 + \log x + 2\log y$$

$$\log_4 6 = \log_4 3 + \log_4 2$$

$$\ln 2xyw = \ln 2 + \ln x + \ln y + \ln w$$

Condense the Logarithm: apply properties of logarithms to rewrite the log expression as a single log.

$\log_2 7 + \log_2 5 = \log_2 (7*5) = \log_2 35$

$\log 5 + \log x = \log 5x$

$\log_7 5 + \log_5 7$ "unlike logs" → can't condense

"Condense the Log"

$\log_5 2 + \log_5 7 = \log_5 14$

$\log 9 + \log 4 = \log 36$

$\log_5 6 + \log_8 4$ "unlike logs" → can't condense

"Expand the Log"

$\log_2 9 = \log_2 (3*3)$

$= \log_2 3 + \log_2 3$

$= 2\log_2 3$

Notice something interesting

$\log_2 9 = \log_2 (3)^2 = 2\log_2 3$

"Expand the Product"

$\log_3 16 = \log_3 (4*4)$

$= \log_3 4 + \log_3 4$

$= 2\log_3 4$

Notice something interesting

$\log_3 16 = \log_3 (4)^2 = 2\log_3 4$

"Expand the Product"

$\log_5 10^2$ Log of a product is the sum of the logs of the factors.

$= \log_5 10 + \log_5 10$ Combine "like terms"

$= 2\log_5 10$

$\log_5 10^{2} = 2\log_5 10$

New property: "log of a power"

Use Log of a Power simplify

$\log x^3 = 3\log x$

$\ln 8 = \ln 2^3 = 3\ln 2$

$\log \sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$

Gotcha'

$\log 3y^5$ $= 5\log 3y$ Which one?

$= \log 3 + \log y^5$

$5\log 3y = \log(3y)^5 = \log 3^5 y^5$

Log of a Power

$\log_b R^{c} \rightarrow c\log_b R$

A potential error is this:

$\log_2 6x^3 = 3\log_2 6x$

What is the error ? '3' is an exponent of the base 'x' not '6x'

Correct the error.

$\log_2 6x^3 = \log_2 6 + \log_2 x^3$

$= \log_2 3 + \log_2 2 + 3\log_2 x$