

SM3-A HANDOUT 7-7 (The Logarithm Function)

Find the Inverse: exchange the locations of 'x' and 'y' in the equation then solve for 'y'.

$$f(x) = (x - 2)^2$$

Domain: The input values (that have corresponding outputs)

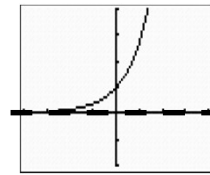
Range: The output values (that have corresponding inputs)

Inverse of a Function: A function resulting from an "exchange" of the inputs and outputs.

$f(x)$ : Domain, Range

$f^{-1}(x)$ : Domain = range of  $f(x)$   
Range = domain of  $f(x)$

Exponential  
Function



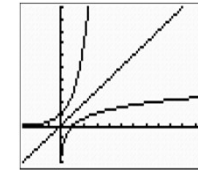
$$f(x) = 10^x$$

Domain = ?

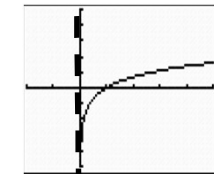
Range = ?

asymptote = ?

Inverse  
Functions



Logarithm  
Function



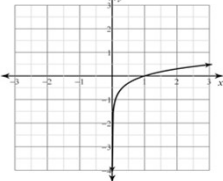
$$f^{-1}(x) = \log_{10}(x)$$

Domain = ?

Range = ?

asymptote = ?

Transformations of the Log Function



$$f(x) = \log x$$

Domain = ?  $(0, \infty)$

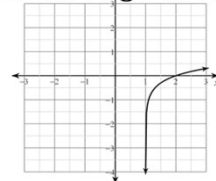
Range = ?  $(-\infty, \infty)$

vertical asymptote = ?  
 $x = 0$

X-intercept = ?  
 $x = 1$

Where increasing = ?

$(0, \infty)$



$$g(x) = \log(x - 1)$$

Right 1 shift

Domain = ?

Range = ?

asymptote = ?

Logarand

$$g(x) = 3 \log(2x - 1) + 5$$

Logarand

Vertical Asymptote: The value of 'x' that makes the logarand equal to zero.

Vertical asymptote = ?

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

Evaluating Logs on your calculator

$\log 8 = ?$        $\log 0 = ?$         Why?

Push buttons:

$\ln 10 = ?$        $\log(-3) = ?$         Why?

Push buttons:

Transformations of the Log Function

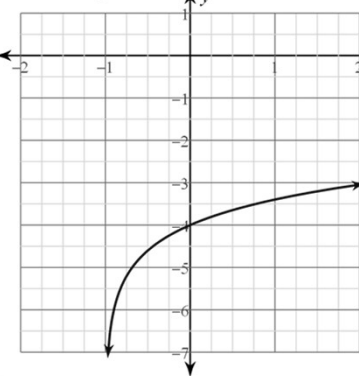
$f(x) = \log x$

$g(x) = 2\log(x+1) - 3$

Domain = ?   

Range = ?   

Asymptote = ?   



$f(x) = \log x$

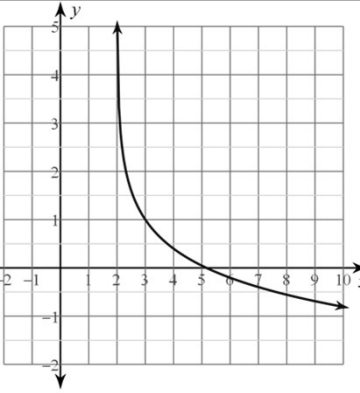
$g(x) = -3\log(x-2) + 1$

Domain = ?   

Range = ?   

Asymptote = ?   

**NOT exponential** (has a vertical asymptote, does NOT have a horizontal asymptote).



What is a logarithm?

A logarithm is another way of writing an exponent.

$2^x = 8$                        $\log_2 8 = x$

x is the exponent              Log = exponent

Both of these equations are saying the same thing:  
 “2 raised to what power is 8?”

Exponential Form	Logarithm Form
$2^3 = 8$ ↗ base	$\text{Log}_{\square} \square = \square$ $\log_2 8 = 3$ ↗ base
"base 2 raised to the 3 <sup>rd</sup> is 8"	"log base 2 of 8 is 3"
$3^x = 9$ What exponent of 3 equals 9?  <b>x = 2</b>	$\log_3 9 = x$ What exponent of 3 equals 9?

Convert to <u>logarithm form</u>	Log $\square = \square$
What is the solution?	
<input type="text"/> $5^x = 25$	<input type="text"/>
<input type="text"/> $4^x = 64$	<input type="text"/>
$x = ???$ $b^x = y$	<input type="text"/>
<input type="text"/> $9^x = 81$	<input type="text"/>
<input type="text"/> $10^x = 1000$	<input type="text"/>

Convert to <u>exponential form</u>	Log $\square = \square$
What is the solution?	
<input type="text"/> $\log_{10} 100 = x$	<input type="text"/>
<input type="text"/> $\log_3 27 = x$	<input type="text"/>
<input type="text"/> $\log_9 1 = x$	<input type="text"/>
<input type="text"/> $\log_4 x = 2$	<input type="text"/>
<input type="text"/> $\log_2 x = 5$	<input type="text"/>

Common Logarithm: has a base of 10.

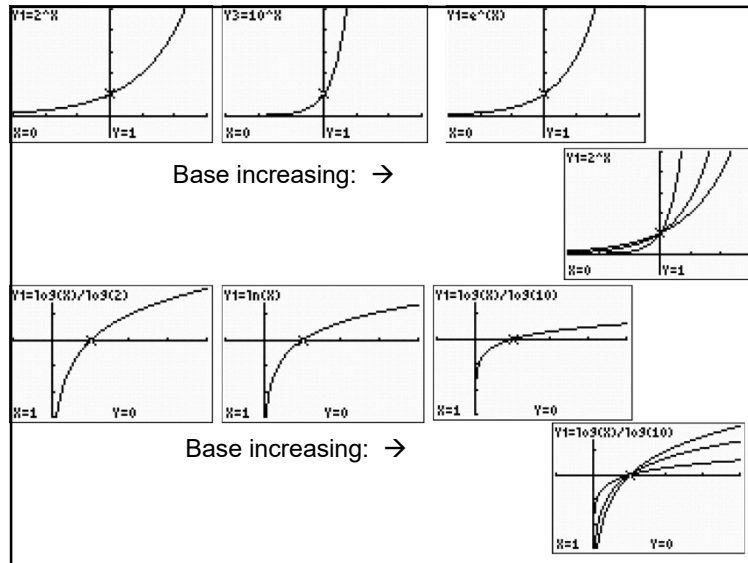
$\log_{10} 100 = x$

We usually write it in this form:  $\log 100 = x$

Natural Logarithm: has a base of e.

$\log_e 2.718 = 1$

We always write it in this form:  $\ln 2.718 = 1$



What is the base?

$\log_2 8 = x$        $\ln 5 = x$        $\log 20 = x$

What is the Solution?

$\frac{1}{100} = \log_{10}(x)$      

$x = \log_2 \sqrt{2}$      

$x = \log_5 \frac{1}{\sqrt[3]{5}}$      

Estimate the value of the log:  $\log(8)$

$\log 8 = x$        $10^x = 8$

$x \approx 0.8$        $x \approx 0.9$

Find  $\log 8$  on your calculator.       $\log 8 = 0.903$

Estimate the value of the log:  $\log_2 17$

$\log_2 17 = x \rightarrow 2^x = 17$

$x \approx 4.1 ?$

Find  $\log_2 17$  on your calculator.       $\log_2 17 = 4.09$

Estimate the value of the log (without using your calculator)

$\log_3 30$        $\log_5 30$        $\log_6 30$

Finding the Inverse  $f^{-1}(x) = ?$

$$f(x) = 3^x \quad \text{Shift 'x' and 'y'}$$

$$x = 3^y \quad \text{"Undo the Exponential" (Convert it to a log)} \\ \text{"A log is an exponent"}$$

$$y = \log_3 x \quad \boxed{f^{-1}(x) = \log_3 x}$$

Finding the Inverse  $f^{-1}(x) = ?$

$$f(x) = (3)^{x-1} + 2$$

Finding the Inverse  $f^{-1}(x) = ?$

$$f(x) = (3)^{\textcircled{x-1}} + \textcircled{2} \quad f^{-1}(x) = \log_3(\textcircled{x-2}) + \textcircled{1}$$

**Right 1 → up 2**

**Right 2 → up 1**

Finding the Inverse  $f^{-1}(x) = ?$

$$f(x) = 2 \log_2(x+1)$$