SM3-A HANDOUT 7-5 Review the Exponential Function

The "Parent" Exponential Function

$y = b^x$ (exponent ← x; base ← b)

$y = 2^x$ (base _____ exponential function)

$y = 3^x$ (base ____ exponential function)

$y = \left(\frac{1}{2}\right)^x$ (base ____ exponential function)

The base MUST BE _______ and ________ equal _____.

$b =$ ____________

Exponential Function $f(x) = 2^x$

Will the 'y' value ever reach zero (on the left end of the graph)?

As the denominator gets bigger and bigger, the decimal version of the fraction gets smaller and smaller.

x	$2^{(\ )}$	y
-1	$2^{(-1)}$	$1/2$
-2	$2^{(-2)}$	$1/4$
-3	$2^{(-3)}$	$1/8$
-4	$2^{(-4)}$	$1/16$
-5	$2^{(-5)}$	$1/32$

$f(-1) = 1/2$

$f(-2) = 1/4$

$f(-3) = 1/8$

$f(-4) = 1/16$

$f(-5) = 1/32$

'y' gets closer and closer to zero but never reaches zero.

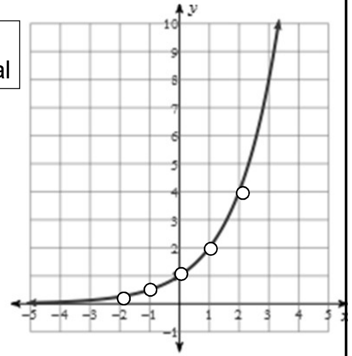
$f(x) = 2^x$

Fill in the output values of the table and graph the points.

Growth Factor is the base of the exponential

x	$2^{(\ )}$	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^0	1
1	2^1	2
2	2^2	4

$* 2$, $* 2$, $* 2$, $* 2$



$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$

"negative exponent property"

$2^0 = 1$

"zero exponent property"

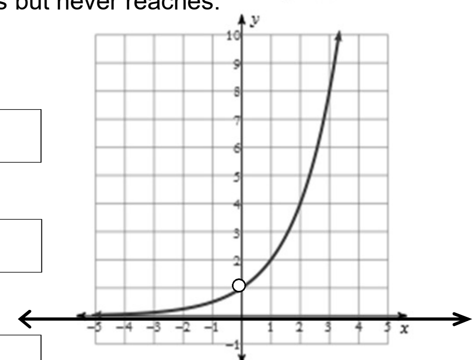
$f(x) = 2^x$

Horizontal Asymptote: a horizontal line the graph approaches but never reaches.

$Domain = ?$

$range = ?$

y-intercept = ?



Exponential Growth: the graph is increasing. Growth occurs when the base of the exponential is greater than 1.

$y = b^x$

‘b’ = 1 → no growth

‘b’ > 1 → growth

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (1.1)^x$

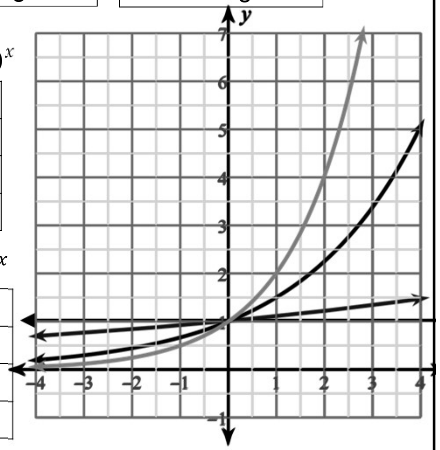
x	g(x)
-1	0.91
0	1
1	1.1

$h(x) = (1.5)^x$

x	h(x)
-1	0.67
0	1
1	1.5

$k(x) = (2)^x$

x	k(x)
-1	0.5
0	1
1	2

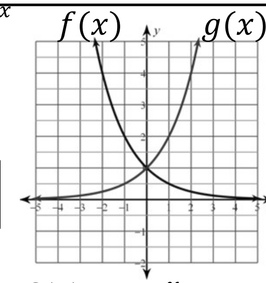


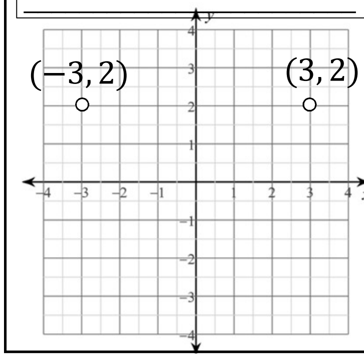
$g(x) = 2^x$ $f(x) = \left(\frac{1}{2}\right)^x$

→ Reflection across the y-axis

If (3, 2) is reflected across the y-axis, where would it be?

→ Replacing ‘x’ with ‘(-x)’ causes





$f(x) = 2^{-x}$

$f(x) = (2^{-1})^x$

Exponent of a Power Property of Exponents

$f(x) = \left(\frac{1}{2}\right)^x$

Negative Exponent Property

Exponential Decay: the graph is decreasing. decay occurs when the base of the exponential is between 0 and 1.

$y = b^x$

‘b’ = 1 → no growth

0 < ‘b’ < 1 → decay

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (0.9)^x$

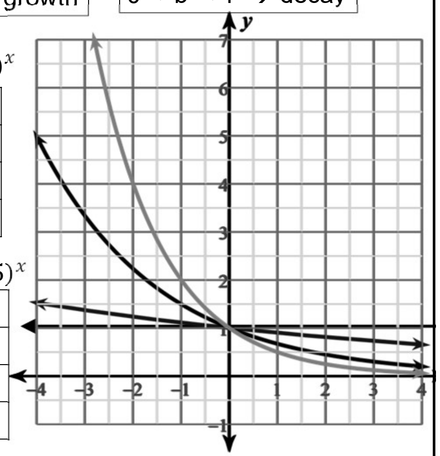
x	g(x)
-1	1.1
0	1
1	0.9

$h(x) = (0.67)^x$

x	h(x)
-1	1.5
0	1
1	0.67

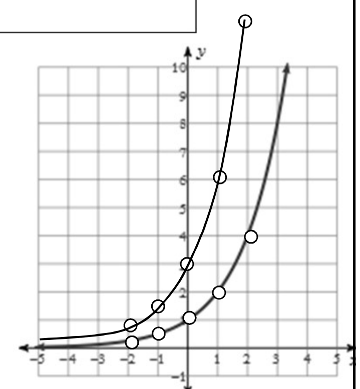
$k(x) = (0.5)^x$

x	k(x)
-1	5
0	1
1	0.2



$f(x) = 2^x$ $g(x) = 3(2)^x$

x	$2^{(\ )}$	f(x)	$g(x)$
-2	2^{-2}	0.25	
-1	2^{-1}	0.5	
0	2^0	1	
1	2^1	2	
2	2^2	4	



Horizontal asymptote: $y = 0$

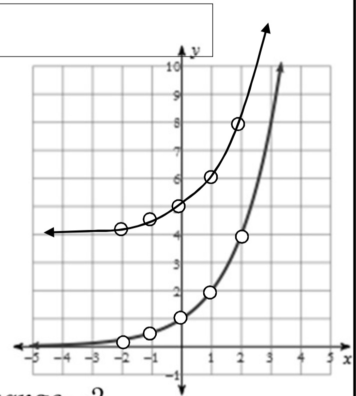
$Domain = ?$ $x = (-\infty, \infty)$

$range = ?$ $y = (0, \infty)$

y-intercept = ? $(0, 1)$

$f(x) = 2^x$ k$(x) = 2^x + 4$

x	$2^{(\ )}$	f(x)	$k(x)$
-2	2^{-2}	0.25	
-1	2^{-1}	0.5	
0	2^0	1	
1	2^1	2	
2	2^2	4	



Horizontal asymptote:

$y = 0$

$Domain = ?$

$x = (-\infty, \infty)$

$range = ?$

$y = (0, \infty)$

y-intercept = ?

$(0, 1)$

Summary

1) Start with $g(x) = ab^x + k$

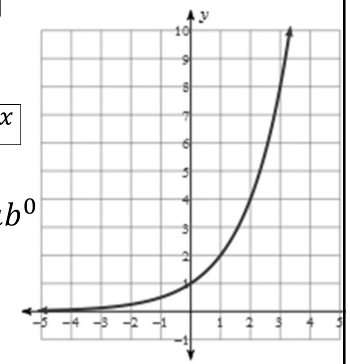
2) Find the value of 'k' (horizontal asymptote). $k = 0$

$g(x) = ab^x + k$ → $y = ab^x$

3) Substitute the y-intercept

(0, 1) → $y = ab^x$ → $1 = ab^0$

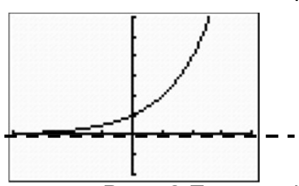
→ a = 1 → $y = b^x$



4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 2) → $y = b^x$ → $2 = b^1$ → b = 2 → $y = 2^x$

Transformations of the Exponential Function

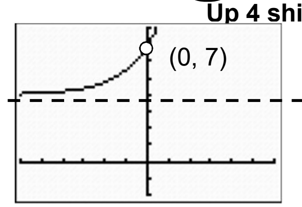


$f(x) = 2^x$ Base-2 Exponential Parent Function

VSF= 3

$h(x) = 3(2)^x + 4$

Up 4 shift



Transformation Form of the Exponential Function

$y = ab^x + k$

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

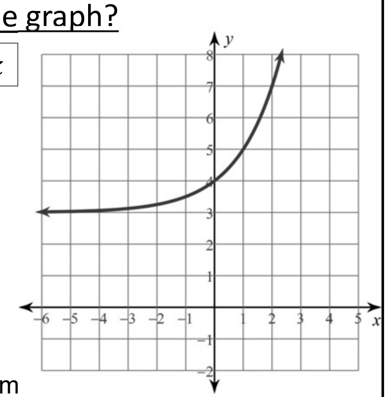
2) Find 'k'

Horizontal asymptote:

3) Substitute the y-intercept

(0, 4) →

→



4) Substitute a "nice" x-y pair from the graph into the equation.

→

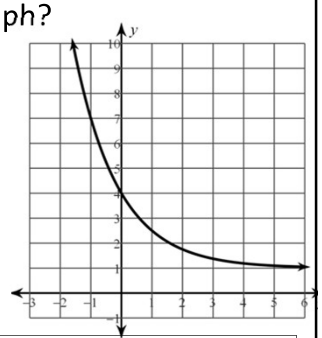
What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

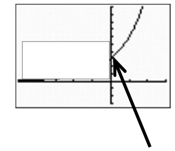
2) horizontal asymptote

3) y-intercept $(0, 4)$

4) "Nice" x-y pair $(-1, 7)$



Initial Value: (of the exponential) is the vertical stretch factor (for problems with no up/down shifts)



$f(0) = 3$

"Initial Value" is a term that is applicable to modeling of real world situations.

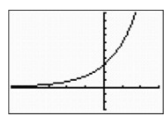
$P(t) = 500(1.03)^t$

$A(t) = \$2500(1.032)^t$

$C(t) = 0.5\ gm/liter(0.73)^t$

$A(t) = 10\ gm\ (0.999879)^t$

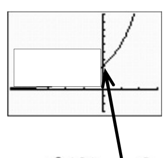
Initial Value: (of the exponential) is the vertical stretch factor (for problems with no up/down shifts)



If in input is time ("stopwatch time") the initial value occurs when t = 0.

$f(t) = 3(2)^t$ Domain: x = [0, ∞)

$f(0) = 3(2)^0 = ?$



$f(0) = 3$