SM3-A HANDOUT 7-3 (Radicals and Rational Exponents)

$\sqrt{3}$ What number is equivalent to the square root of 3?

$x = \sqrt{3}$ Square both sides of the equation

$(x)^2 = (\sqrt{3})^2 \quad x^2 = 3$

$x = \sqrt{3}$ is an equivalent statement to $x^2 = 3$

$\sqrt{3}$ ≈ 1.732 There is no equivalent number

≈ 1.7321 The decimal, is just an approximation.

≈ 1.73205

≈ 1.732051

$\approx 1.7320508...$

Radicals

$\sqrt{3} \rightarrow \sqrt[2]{3}$

Index number — Radical symbol — Radicand

$x = \sqrt[2]{3}$ The "square root of 3" means:

$x = \sqrt[3]{4}$ The "3rd root of 4" means:

Adding and subtracting radicals

Can these two terms be combined using addition? $3x + 2x$

Write 3x as repeated addition $x + x + x$

Write 2x as repeated addition $x + x$

$$3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$$

When multiplication is written as repeated addition, "like terms" look exactly alike.

$3\sqrt{x} + 2\sqrt{x} \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x}$

$3\sqrt{6} + 2\sqrt{6}$

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Define "like powers" "Same base, same exponent".

$3x^4 + 2x^4 \rightarrow 5x^4$

Define "like radicals" "Same radicand, same index number".

$3\sqrt{6} + 2\sqrt{6} \rightarrow 5\sqrt{6}$

Which of the following are "like radicals" that can be added?

$\sqrt{2} + \sqrt{3}$

$\sqrt[4]{5} + \sqrt[4]{5}$

$2\sqrt{3} + 3\sqrt{2}$

$3\sqrt[5]{2} + 4\sqrt[5]{2}$

$\sqrt[4]{2} + \sqrt[3]{2}$

$6\sqrt[3]{4} + 6\sqrt[4]{4}$

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$\sqrt{3}+\sqrt{2} \rightarrow \sqrt{3+2}=\sqrt{5}$ Are they equivalent?

If this is a property of radicals, it must work for every combination of numbers.

$\sqrt{4}+\sqrt{9} \rightarrow \sqrt{13}$

$\sqrt{4}+\sqrt{9} \rightarrow 2+3 \rightarrow 5 \neq \sqrt{13}$

$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

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$\sqrt{3}*\sqrt{2}$

$\sqrt{3*2} \rightarrow \sqrt{6}$

Will this work?

$\sqrt{3} \approx 1.7321...$ $\sqrt{2} \approx 1.4142...$

$\sqrt{3}*\sqrt{2} \approx 2.4495$

$\sqrt{6} \approx 2.4495...$

Product of Radicals Property

$\sqrt{a}*\sqrt{b} \rightarrow \sqrt{a*b}$

$\sqrt{5}*\sqrt{2}=\sqrt{10}$

$\sqrt{4}*\sqrt{9} \rightarrow \sqrt{4*9}$

$2*3 \rightarrow \sqrt{36}$

$2*3 \rightarrow 6$

$6=6$

Are these equivalent?

$\sqrt{a}*\sqrt{b}=\sqrt{ab}$

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$\sqrt{a}*\sqrt{b}=\sqrt{ab}$

Simplify the following:

$3\sqrt{8}*5\sqrt{2}$

$3*\sqrt{8}*5*\sqrt{2}$

$3*5*\sqrt{8}*\sqrt{2}$

$15*\sqrt{8}*\sqrt{2}$

$15*\sqrt{16}$

$15*4=60$

$2\sqrt{3}*3\sqrt{5}$

$7\sqrt{6}*2\sqrt{5}$

$\sqrt{5}+3\sqrt{5}$

$7\sqrt{6}+2\sqrt{6}$

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Simplify radicals: use the Product of Radicals Property to factor ("break apart") the radical into a "perfect square" times a number. $\sqrt{a}*\sqrt{b}=\sqrt{ab}$

$\sqrt{18} \rightarrow \sqrt{9}*\sqrt{2} \rightarrow 3*\sqrt{2} \rightarrow 3\sqrt{2}$

Simplify $\sqrt{24}$

$3\sqrt{32x^2}$

$\sqrt[3]{x^4}$

$\sqrt[4]{3x^5y}$

Can we add "unlike" radicals?

$$\sqrt{a}*\sqrt{b}=\sqrt{ab}$$

Simplify $7\sqrt{6}+2\sqrt{24}\rightarrow 7\sqrt{6}+(2*\sqrt{4}*\sqrt{6})$

$\rightarrow 7\sqrt{6}+(2*2*\sqrt{6})$

$\rightarrow 7\sqrt{6}+4\sqrt{6}$

$\rightarrow 11\sqrt{6}$

$-3\sqrt{32}+2\sqrt{8}$

Another way to Simplify Radicals Factor, factor, factor!!!

$\sqrt{54}\rightarrow\sqrt[2]{54}\rightarrow\sqrt[2]{2*27}\rightarrow\sqrt[2]{2*3*9}\rightarrow\sqrt[2]{2*3*3*3}$

What is the factor that is used (Index number) '2' times under the radical?

Bring the out factor that is used 2 times.

$\rightarrow 3\sqrt[2]{2*3}\quad\rightarrow 3\sqrt{6}$

Using Properties of Exponents to reduce the writing:

$\sqrt[4]{32x^6}\rightarrow\sqrt[4]{32*x^4*x^2}$

$\rightarrow x\sqrt[4]{32*x^2}$

$\rightarrow x\sqrt[4]{2^4*2^1*x^2}$

$\rightarrow 2x\sqrt[4]{2x^2}$

Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary number) in the denominator into a rational number.

We take advantage of the idea:

$\sqrt{2}*\sqrt{2}=\sqrt{2*2}=\sqrt{4}=2$

$\sqrt{3}*\sqrt{3}=\sqrt{3*3}=\sqrt{9}=3$

$\frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{\sqrt{2}}\rightarrow\frac{\sqrt{2}}{2}$

Identity Property of Multiplication

multiplying by '1' doesn't change the number.

$\frac{2}{\sqrt{6}}*\frac{\sqrt{6}}{\sqrt{6}}\rightarrow\frac{2\sqrt{6}}{6}\rightarrow\frac{2*\sqrt{6}}{2*3}\rightarrow\frac{\sqrt{6}}{3}$

$\frac{25}{\sqrt{15}}$

$\frac{14}{3\sqrt{21}}$

Radicals CAN be written as Powers

$6\sqrt[5]{x}$ — Index, Radicand, Coefficient

$6(x)^{1/5}$ — Exponent, Base, Coefficient

Coefficient → Coefficient

Radicand → Base

Index → Denominator of the Exponent

The index number is the denominator of the exponent.

Are radicals related to powers?

$3^{1/2} = \sqrt[2]{3}$

$5^{1/3} = \sqrt[3]{5}$

$\sqrt[2]{x} = x^{1/2}$

$\sqrt[3]{7} = 7^{1/3}$

None of these have coefficients!

$3\sqrt[2]{y} = 3y^{1/2}$

$5\sqrt[3]{7} = 5(7)^{1/3}$

Multiplication (by a coefficient) is "repeated addition." This explains why coefficients of radicals become coefficients of powers.

$\sqrt{y} = y^{1/2}$

$3\sqrt[2]{y} = \sqrt{y} + \sqrt{y} + \sqrt{y}$

$3y^{1/2} = y^{1/2} + y^{1/2} + y^{1/2}$

What happens if there is a product under the radical?

$\sqrt[2]{xy} = (xy)^{1/2}$

$5\sqrt[3]{3x} = 5(3x)^{1/3}$

$2\sqrt[4]{21mn} = 2(21mn)^{1/4}$

How did we show that the index number applied to the entire product (radicand) when re-written in "power form"?

Power of a product → product inside parentheses with an exponent.

$\sqrt[5]{x^2 y}$ [ ]

$6\sqrt[3]{3m^2}$ [ ]

"Exponential Form" that has both a numerator and denominator

The exponent can be written as a rational number.

$x^{5/2} = \sqrt[2]{x^5}$

Numerator: Exponent of the base.

Denominator: Root of the base.

$\sqrt[3]{2^2} = 2^{2/3}$

Radical Form — Exponential Form

Re-write in power form.

$\sqrt[2]{3m}$ [ ]

$4\sqrt[3]{5y}$ [ ]

$\sqrt[5]{x^3y^2}$ [ ]

Rewrite in “radical form”

$m^{1/5}$ [ ]

$3nm^{1/4}$ [ ]

$2(18n^2)^{1/6}$ [ ]

Multiply Powers Property

$$y^2 * y^3 = ? \quad = y^{2+3} \quad = y^5$$

When multiplying “same based powers” add the exponents.

$$x^{\frac{2}{3}} * x^{\frac{3}{4}} \quad \rightarrow x^{\frac{2}{3}+\frac{3}{4}} \quad \rightarrow x^{\frac{17}{12}}$$

Yes, you must be able to add fractions

Exponent of a Power Property

$$\left(y^2\right)^3 = ? \quad = y^{2*3} \quad = y^6$$

When multiplying “same based powers” add the exponents.

$$\left(y^{1/2}\right)^{2/3} \quad = y^{\frac{1}{2}*\frac{2}{3}} \quad = y^{\frac{1}{3}}$$