## SM3-A HANDOUT 7-1 (Properties of Exponents)

What is a power?
Power: An expression formed by repeated multiplication of the base.


The exponent applies to the number or variable immediately to its left, not to the coefficient !!!

## No Exponent? <br> $3 x$

Usually, we don't write the exponent ' 1 ' (saves ink).

## No Coefficient?


$\square$
Usually, we don't write the coefficient ' 1 ' (saves ink).

Negative? $\quad-x^{2}$


Usually, we don't write the coefficient '-1', we just put the "negative symbol" (saves ink).

Factor: a number that is being multiplied.


Power: is repeated multiplication
multiplication: is repeated addition

$$
3 x=\square
$$

## (adding two terms)

## $3 x+4 x$

## $2 x^{2}+3 x^{2}$

(multiplying two terms)

$$
x^{2} * x^{3}
$$

## Exponents

$$
\begin{aligned}
& \left(\frac{x}{2}\right)^{2}=?=\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)=\frac{x^{2}}{4} \quad \begin{array}{l}
\text { Remember how to } \\
\text { multiply fractions? }
\end{array} \\
& \left(\frac{2}{3 x}\right)^{3}=?=\left(\frac{2}{3 x}\right)\left(\frac{2}{3 x}\right)\left(\frac{2}{3 x}\right) \quad=\frac{8}{27 x^{3}}
\end{aligned}
$$

$$
(4 y)^{2}=? \quad\left(\frac{-2}{x}\right)^{4}=?
$$


$2(5 x)^{2}=$ ?

$$
\left(\frac{x}{2}\right)^{3}=?
$$

Multiply Powers Property
$\left(x^{2}\right)\left(x^{3}\right)=\left(x^{*} x\right)\left(x^{*} x^{*} x\right)$
This is ' $x$ ' used as a factor how many times?
$\left(x^{2}\right)\left(x^{3}\right)=x^{2} x^{3}=x^{2+3}=x^{5}$
' $x$ ' used as a factor five times
When you multiply powers having the same base, you add the exponents.

## Exponent of a Power Property $\left(x^{2}\right)^{3}$

$$
\left(x^{2}\right)^{3}=\left(x^{*} x\right)\left(x^{*} x\right)\left(x^{*} x\right)
$$

This is ' $x$ ' used as a factor how many times?

$$
\left(x^{2}\right)^{3}==x^{6}
$$

' $x$ ' used as a factor six times

$$
\left(x^{2}\right)^{3}=x^{2 * 3}=x^{6}
$$

you multiply the exponents.

## Exponent of a Product Property

$$
\begin{aligned}
&(x y)^{2}=(x y)(x y)=x^{*} y^{*} x^{*} y=x^{*} x^{*} y^{*} y \\
&=x^{2} y^{2} \\
& \quad(x y)^{m}=x^{m} y^{m}
\end{aligned}
$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product!! (It does not apply to the power of a sum.
$(x-y)^{2} \neq x^{2}-y^{2}$
$(x-y)^{2}=(x-y)(x-y)=x^{2}-2 x y+y^{2}$

Combination of

1. Power of a Product
2. Power of a Power

$$
\begin{aligned}
\left(3 x^{3} y^{4}\right)^{2}= & \left(3^{1} x^{3} y^{4}\right)^{2} \\
& =3^{2} x^{6} y^{8}
\end{aligned}
$$

Constants (integer, etc.) usually have an exponent of ' 1 '.
' $x$ ' is a number, we just don't know what it is. You treat all numbers the same (whether they are variables or constants).

$$
3 x^{2}\left(4 x^{3}\right)=?=3 * 4 *\left(x^{2}\right)\left(x^{3}\right)=12 x^{5}
$$

You can re-arrange the order of multiplication.
Coefficients of the powers are handled separately from the base and the exponent.
$\left(x^{2}\right)^{5}=?$

$\left(5 x^{2}\right)\left(2 x^{3}\right)=$ ? $\square$
$(2 x)\left(\frac{1}{2} x^{3}\right)=$ ?

$5(x)^{3} x^{4}=?$

$\left(2 y^{5}\right)^{3}=$ ?


## What is the difference between?

$$
\begin{aligned}
& (x)^{4} \text { and } \mathrm{x}^{4} \\
& \left(x^{2}\right)^{3} \text { and }\left(x^{3}\right)^{2} \\
& x^{4} x^{3} \text { and } x^{3} x^{4} \\
& (x+1)^{2} \text { and }(x+1)(x+1)
\end{aligned}
$$

Watch the negatives!

$$
\left(-x^{3} y^{4}\right)^{2}
$$

$=\left((-1)^{1} x^{3} y^{4}\right)^{2}$ Turn negative signs into multiplication by -1 .
$=(-1)^{2} x^{6} y^{8}$
This way you will be able to tell if the simplified version is positive or negative.
$=x^{6} y^{8}$
$\left(-2 x^{2} y^{6}\right)^{3}$ Negative coefficients have an exponent of ' 1 '.
$=\left((-2)^{1} x^{2} y^{6}\right)^{3}$
$=(-2)^{3} x^{6} y^{18}$
$=-8 x^{6} y^{18}$
A negative number raised to an odd exponent remains negative.
simplify
$\left(-2 x^{2} y^{4} z\right)^{3}$

$2\left(-m^{4} x^{3}\right)^{5}$
$-3\left(-2 x^{2} y z^{3}\right)^{4}$


