## Math-3 Lesson 6-6 (Solving Equation Using Inverse Functions)

Function A : heating by 10 degrees
What is the inverse of this function?
"Cooling something down by 10 degrees"
Function B: cooling by 10 degrees

The temperature of a bowl of soup is 100 degrees.
The temperature of a bowl of soup is 100 degrees. Apply function A then function B (in sequence) to the bowl of soup. What is the final temperature of the soup?

Temperature $=100+10-10$

## Composition of inverse functions

Function A and Function B are inverses of each other.
Function A: "does something" to the input.
Function B: "undoes whatever function A did to the input.
2
Function A "does something" to input value 2


Function B "undoes (whatever A did) to the input value 2

What is the output of function $B$ ?

What function would "undo" a:

1. $f(x)=\left\{x^{4}, x=[0, \infty)\right\} \quad f^{-1}(x)=\sqrt[4]{x}$
2. $g(x)=x^{2 / 3} \quad g^{-1}(x)=x^{3 / 2}$
3. $h(x)=x^{4 / 5} \quad h^{-1}(x)=? \quad=x^{5 / 4}$
4. $k(x)=x^{5} \quad k^{-1}(x)=?=x^{1 / 5}=\sqrt[5]{x}$

$$
\begin{aligned}
& f(x)=(x+1)^{2 / 3} \quad f^{-1}(x)=? \\
& x=(y+1)^{2 / 3} \\
& x^{3 / 2}=\left((y+1)^{2 / 3}\right)^{3 / 2} \\
& x^{3 / 2}=y+1 \\
& y=x^{3 / 2}-1
\end{aligned}
$$

We use compositions of inverse functions to solve equations.

$$
\begin{array}{cl}
(x-3)^{2}+4 & =40 \\
-4 & -4
\end{array} \quad \text { "Isolate the square, undo the square". }
$$

Solve $\quad 13=x^{4}-3 \quad$ Solve $\quad 23=3 x^{3}-1$
$16=x^{4}$
$24=3 x^{3}$ Isolate the power:
$\pm \sqrt[4]{16}=x \quad x= \pm 2$
$8=x^{3}$
Solve $\sqrt{2 x+1}=3 \quad \sqrt[3]{8}=\sqrt[3]{x^{3}}$ undo the power

$$
\begin{aligned}
& (\sqrt{2 x+1})^{2}=3^{2} \quad 2=x \\
& 2 x+1=9 \\
& 2 x=8 \\
& x=4
\end{aligned}
$$

Using an inverse function to solve an equation.
Ticket prices in the NFL can be modeled by:

$$
P=35 t^{0.192} \quad \begin{aligned}
& \text { where ' } \mathrm{t} \text { ' is the number of } \\
& \text { years since } 1995 .
\end{aligned}
$$

(price as a function of time since 1995)
During what year was the price of a ticket $\$ 50.85$ ?
$P=35 t^{0.192}$
$\left(\frac{50.85}{35}\right)^{1 / 0.192}=t$
$\frac{50.85}{35}=t^{0.192}$

$$
t=6
$$

$1995+6=2001$

Solve:
Check your solution.

$$
\begin{array}{cl}
\sqrt{x+3}+5=0 & \sqrt{22+3}+5=0 \\
\sqrt{x+3}=-5 & \sqrt{25}+5=0 \\
x+3=25 & 5+5 \neq 0 \quad \text { Extraneous solution. } \\
x=22 & x=-2,1 \quad \text { Check your solutions. } \\
\sqrt{2-x}=-x & \\
2-x=(-x)^{2} & \sqrt{2-(-2)}=-(-2) \\
2-x=x^{2} & \sqrt{4}=2 \quad \text { Checks. } \\
0=x^{2}+x-2 & \sqrt{2-(1)}=-(1) \\
0=(x+2)(x-1) & \sqrt{1} \neq-1 \quad \text { Extraneous solution. } \\
& x=-2 \\
& x \neq 1 \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
\end{array}
$$

