Math-3A
Lesson 4-7
Holes in the Graph of the Reciprocal Function

Divide

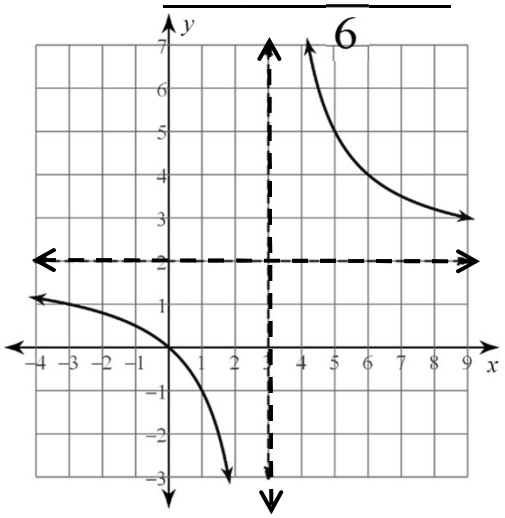
$$g(x) = \frac{2x}{x-3} \qquad x-3\overline{)2x} \qquad \begin{array}{r} 2 \\ x-3\overline{)2x} \\ -(2x-6) \\ \hline 6 \end{array}$$

$$g(x) = 2 + \frac{6}{x-3}$$

$$g(x) = 2 + \frac{6}{x-3}$$

Horizontal Asymptote: y = 2

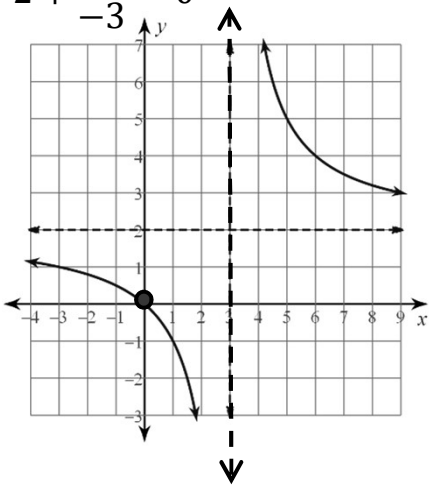
Vertical Asymptote: x = 3



$g(x) = \frac{2x}{x-3}$ $g(x) = 2 + \frac{6}{x-3}$

Zeroes of the numerator are x-intercepts (or are imaginary)

$g(0) = 2 + \frac{6}{0-3}$ $g(0) = 2 + \frac{6}{-3} = 0$



Zeroes of the denominator are "excluded values"

For the reciprocal function, the zeroes of the denominator are vertical asymptotes.

$x = 3$

What are the equations of:

a) Horizontal asymptote? Range = ?

b) Vertical asymptote? Domain = ?

$g(x) = \frac{1}{x} + 7$

a) y = 7, range: y ≠ 7

b) x = 0, domain: x ≠ 0

$f(x) = \frac{-7}{(x+3)} + 5$

a) y = 5, range: y ≠ 5

b) x = -3, domain: x ≠ -3

$h(x) = \frac{1}{3(x-2)} - 4$

a) y = -4, range: y ≠ -4

b) x = 2, domain: x ≠ -2

$f(x) = \frac{x+3}{x-4}$ Domain=? $x \neq 4$

x-intercept? → are "zeroes" of the numerator $(-3,0)$

vertical asymptote? → are "zeroes" of the denominator $x = 4$

y-intercept? → f(0) = ? $f(0) = \frac{0+3}{0-4}$ $(0, -3/4)$

Horizontal asymptote?

→ Is the "quotient" of division

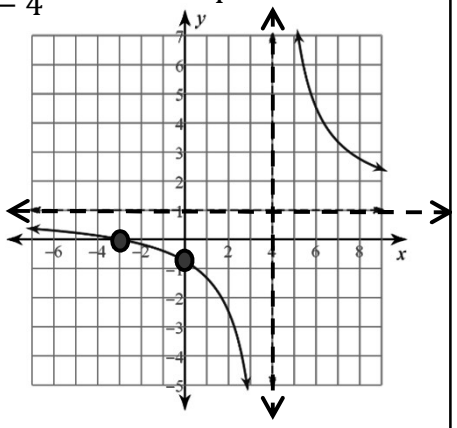
	1	R
x	x	7
-4	-4	

3

$f(x) = 1 + \frac{7}{x-4}$

$f(x) = \frac{7}{x-4} + 1$

$y = 1$



$y = \frac{x^2+5x+6}{x^2-4} = \frac{(x+2)(x+3)}{(x+2)(x-2)}$ Domain=? $x \neq -2, 2$

$y = \frac{(x+3)}{(x-2)}$

x-intercept? → are "zeroes" of the numerator (that do not disappear due to simplification) $(-3,0)$

vertical asymptote? → are "zeroes" of the denominator (that do not disappear due to simplification) $x = 2$

Horizontal asymptote?

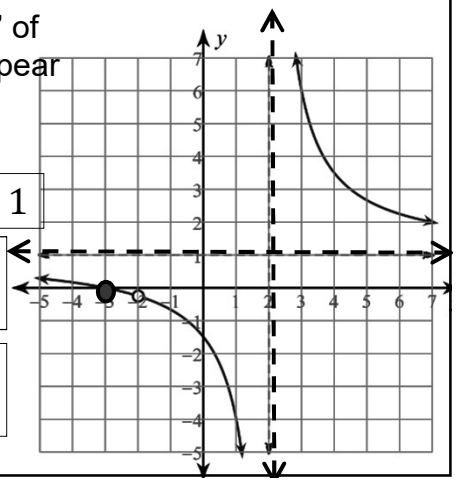
→ Is the "quotient" of division $y = 1$

	1	R
x	x	5
-2	-2	

3

$y = 1 + \frac{5}{x-2}$

$f(x) = \frac{5}{x-2} + 1$



$$y = \frac{x^2 + 5x + 6}{x^2 - 4} \qquad \boxed{y = \frac{(x+3)}{(x-2)}}$$

Domain=?

$x \neq 2, -2$

$x = 2 \qquad f(-2) = ?$

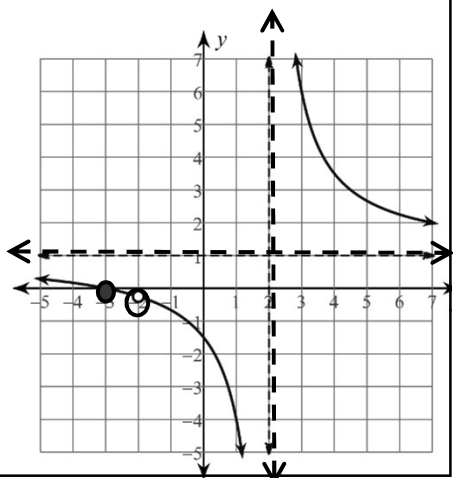
vertical asymptote? → are "zeroes" of the denominator (that *do not* disappear due to simplification)

Hole (in the graph) → are "zeroes" of the denominator (that *do* disappear due to simplification)

$$f(-2) = \frac{(-2)+3}{(-2)-2}$$

$$f(-2) = -\frac{1}{4}$$

There is a hole in the graph at (0, -1/4)



$$y = \frac{2x^2 - 8x - 24}{x^2 - 2x - 8} \qquad y = \frac{2(x^2 - 3x - 12)}{(x-4)(x+2)} \qquad y = \frac{2\cancel{(x+2)}(x-6)}{\cancel{(x+2)}(x-4)}$$

$$\boxed{y = \frac{2x - 12}{(x-4)}}$$

Domain=? $x \neq -2, 4$

x-intercept? $(6,0)$

vertical asymptote? $x = 4$

Horizontal asymptote?

→ Is the "quotient" of division

	2	R
x	$2x$	-5
-4	-8	

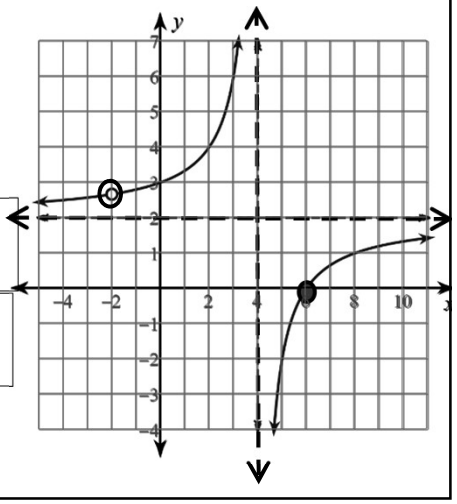
$\boxed{y = 2}$

$\boxed{y = 2 - \frac{5}{x-4}}$

$\boxed{y = \frac{-5}{x-4} + 2}$

-12 hole =?

$f(-2) = 2\frac{5}{6}$ $\qquad f(-2) = \frac{-5}{-2-4} + 2$



Your turn: Find the domain, x-intercepts, holes, vertical asymptotes, and the horizontal asymptote: (HINT: simplify first..

$$g(x) = \frac{x^2 - 1}{x^2 - 6x - 7}$$

$$g(x) = \frac{(x+1)(x-1)}{(x+1)(x-7)}$$

x-intercept: x = 1

hole: x = -1

Vertical asymptote: x = 7

Horizontal asymptote: y = 1

Domain: $x \neq \pm 1$

$$g(x) = \frac{(x-1)}{(x-7)}$$

$$g(x) = \frac{x^2 - 1}{x^2 - 6x - 7} = \frac{(x-1)}{(x-7)}$$

Domain: $x \neq \pm 1$

hole: x = -1 → (-1, ¼)

x-intercept: x = 1

Vertical asymptote: x = 7

Horizontal asymptote: **y = 1**