Math-3-A

Lesson 3-4

Factoring “NICE” 3rd Degree Polynomials

Find the zeroes of the following 3rd degree Polynomial

$$y = x^3 + 5x^2 + 4x$$

Set y = 0

$$0 = x^3 + 5x^2 + 4x$$

Factor out the common factor.

$$0 = x(x^2 + 5x + 4)$$

Factor the quadratic

$$0 = x(x+1)(x+4)$$

Identify the zeroes

0, -1, -4

"Nice" (factorable) 3rd Degree Polynomials

$$y = ax^3 + bx^2 + cx + d$$

If it has no constant term, it will look like this:

$$y = ax^3 + bx^2 + cx$$

This can easily be factored (by taking out the common factor 'x').

$$y = x(ax^2 + bx + c)$$

Resulting in 'x' times a quadratic factor.

We have been factoring quadratics for quite a while!

"Nice" 3rd Degree Polynomial (with no constant term)

$$y = x^3 + 5x^2 + 4x$$

It has no constant term so it can easily be factored into 'x' times a quadratic factor.

$$y = x(x^2 + 5x + 4)$$

If the quadratic factor is "nice" we can factor that into 2 binomials.

$$y = x(x+1)(x+4)$$

This is now "intercept form" so we can "read off" the x-intercepts. What are they?

0, -1, -4

"Nice" 3rd Degree Polynomial (with no constant term)

$$y = x^3 + 6x^2 + 4x + 0$$
$$0 = x^3 + 6x^2 + 4x$$

It has no constant term so it can easily be factored into 'x' times a quadratic factor. $0 = x(x^2 + 6x + 4)$

x = 0

What if the quadratic factor is not factorable?

Convert the quadratic factor into vertex form and solve.

$$x = \frac{-b}{2a} = \frac{-(6)}{2(1)}$$

$x = -3$

$$y = f(-3) = (-3)^2 + 6(-3) + 4$$

$y = -5$

$$0 = (x+3)^2 - 5$$
$$x = -3 \pm \sqrt{5}$$

Zeroes:
$x = 0, -3 \pm \sqrt{5}$

Factor the following "nice" 3rd degree polynomials then find the "zeroes" of the polynomial.

$$y = x^3 + 5x^2 - 14x$$

$$0 = x^3 + 5x^2 - 14x$$

$$0 = x(x^2 + 5x - 14)$$

$$0 = x(x+7)(x-2)$$

0, -7, 2

$$y = 3x^3 - 24x^2 + 6$$

$$0 = 3x(x^2 - 8x + 2)$$

x = 0

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)}$$

$$x = 4$$

$$y = f(4) = (4)^2 - 8(4) + 2$$

$$y = -14 \quad 0 = (x+4)^2 - 14$$

$$x = -4 \pm \sqrt{14}$$

Another "Nice" 3rd Degree Polynomial

$$y = ax^3 + bx^2 + cx + d$$

This has the <u>constant</u> term, but it has a very useful feature:

$$y = 1x^3 + 2x^2 + 2x + 4$$

What pattern do you see?

$$3rd/1st = 2/1 \qquad 4th/2nd = 4/2 = 2/1$$

“factor by grouping” if it has this nice pattern.

$$y = 1x^3 + 2x^2 + 2x + 4$$

Group the 1st and last pair of terms with parentheses.

$$y = (x^3 + 2x^2) + (2x + 4)$$

Factor out the common term from the first group.

$$y = x^2(x + 2) + (2x + 4)$$

Factor out the common term from the last group.

$$y = x^2(x + 2) + 2(x + 2)$$

Common factor of (x + 2)

$$y = x^2(x+2) + 2(x+2)$$

These two binomials are now common factors of

$$x^2 \text{ and } 2$$

Factor out the common binomial term.

$$y = (x+2)(x^2+2)$$

Apply the Complex Conjugates Theorem

$$x = -2, \mathrm{i}\sqrt{2}, -\mathrm{i}\sqrt{2}$$

An easier method is "box factoring" if it has this nice pattern.

$$y = 1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Rewrite in intercept form.

$$y = 1x^3 + 2x^2 + 2x + 4$$

$$y = (x^2 + 2)(x + 2)$$

Find the "zeroes."

$$0 = (x^2 + 2)(x + 2)$$

$$0 = x^2 + 2 \qquad 0 = x + 2$$

$$-2 = x^2 \qquad x = -2$$

$$x = \pm i\sqrt{2}$$

Which of the following 3rd degree polynomials have the “nice pattern” we saw on the previous slide?

$$y = 2x^3 + 3x^2 + 4x + 6 \qquad {}^{3^{rd}}\!/_{1^{st}} = {}^{4^{th}}\!/_{2^{nd}} = 2$$

$$y = 4x^3 - 5x^2 + 12x - 15 \qquad {}^{3^{rd}}\!/_{1^{st}} = {}^{4^{th}}\!/_{2^{nd}} = 3$$

$$y = 4x^3 + 8x^2 + 5x + 10 \qquad {}^{3^{rd}}\!/_{1^{st}} = {}^{4^{th}}\!/_{2^{nd}} = {}^{5}\!/_{4}$$

$$y = -6x^3 + 7x^2 - 18x + 21 \qquad {}^{3^{rd}}\!/_{1^{st}} = {}^{4^{th}}\!/_{2^{nd}} = 3$$

All of them!

Find the zeroes using "box factoring"

$$y = 4x^3 - 5x^2 + 12x - 15$$

	$4x$	-5
x^2	$4x^3$	$-5x^2$
3	$12x$	-15

$$0 = (x^2 + 3)(4x - 5)$$

$$0 = x^2 + 3 \qquad 4x - 5 = 0$$

$$-3 = x^2 \qquad 4x = 5$$

$$x = \pm i\sqrt{3} \qquad x = {}^5/_4$$

$$x = i\sqrt{3},\ -i\sqrt{3},\ {}^5/_4$$

What have we learned so far?

"Nice" Common Factor 3rd degree polynomial:

$$y = x^3 + 3x^2 + 2x \qquad = x(x^2 + 3x + 2)$$

$$= x(x+1)(x+2)$$

"Nice" Factor by box 3rd degree polynomial:

$$y = 2x^3 + 3x^2 + 4x + 6$$

“Nice” Difference of Squares (of higher degree):

$y = x^4 - 81$ Use “m” substitution Let $m^2 = x^4$

Then $m = x^2$

$y = m^2 - 81$

$y = (m + 9)(m - 9)$

Use “m” substitution

$y = (x^2 + 9)(x^2 - 9)$

$y = (x + 3)(x - 3)(x + 3i)(x - 3i)$

Find the *zeroes*. $x =$ -3, 3, -3i, 3i

Convert to standard form:

$$y = (x-3)(x^2+3x+9)$$

$$y = x^3 - 27$$

There are NO x^2 terms
and NO ‘x’ terms

	x	-3
x^2	x^3	$-3x^2$
$3x$	$3x^2$	$-9x$
9	$9x$	-27

$0x^2$ $0x$

<u>The Difference of cubes</u>: factors as the cubed root of each term multiplied by a 2nd degree polynomial. $y = x^3 - 8$

$$y = (x-2)(ax^2+bx+c)$$

$$y = (x-2)(x^2+2x+4)$$

	x	-2
x^2	x^3	$-2x^2$
$2x$	$2x^2$	$-4x$
4	$4x$	-8

$0x^2$ $0x$

The Sum of cubes: factors as the cubed root of each term multiplied by a 2nd degree polynomial. $y = x^3 + 64$

$$y = (x+4)(ax^2 + bx + c)$$

	x	4
x^2	x^3	$4x^2$
$-4x$	$-4x^2$	$-16x$
16	$16x$	64

$0x^2$ $0x$

$$y = (x+4)(x^2 - 4x + 16)$$