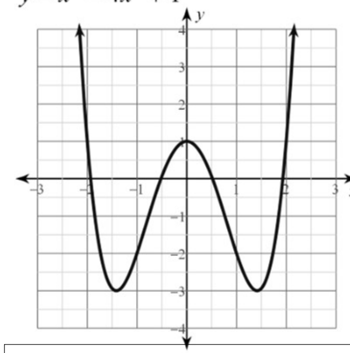
SM3-A HANDOUT 3-1 (Analyzing Polynomials)

Vocabulary:
- Polynomial:
- Lead coefficient:
- Degree:
- Standard Form Polynomial
- Term:
- Number of terms:
- Intercept Form Polynomial
- Linear factors:
- Solve by factoring:
- Find the zeroes:
- "end behavior"

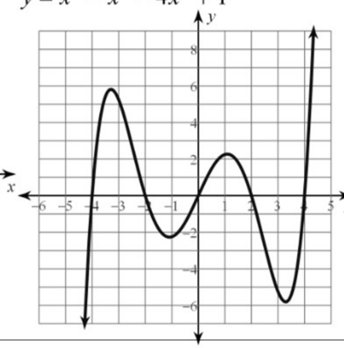
Theorems:
- Fundamental Theorem of Algebra.
- Linear Factorization Theorem:

Max number of x-intercepts?

$y = x^4 - 4x^2 + 1$



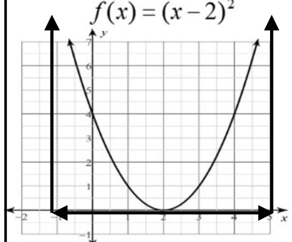
$y = x^5 - x^4 - 4x^2 + 1$



The ____ of the polynomial equals the __________AND gives you the ________________________(real number zeroes).

"End Behavior"

$f(x) = (x-2)^2$



The "end behavior" of a function means:

"on the right end of the graph is the y-value going UP or DOWN?
And
"on the left end of the graph,
is the y-value going UP or DOWN?

In English we could say: "up on right, up on left"

As 'x' gets bigger (right end) 'y' gets bigger (goes upward)

As 'x' gets smaller (left end), 'y' gets bigger (goes upward)

"end behavior"

as $x \to +\infty,\ y \to ?$ as $x \to -\infty,\ y \to ?$

Which of the following transformations affect end behavior? If so, how?

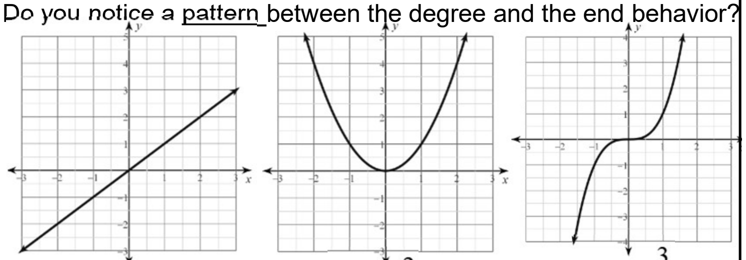
Left or right shift? ________

Up or down shift? ________

Vertical stretching? ________

Reflection across x-axis? ____________________

Do you notice a pattern between the degree and the end behavior?



$y = x$ (up right, down left)

$y = x^2$ (up right, up left)

$y = x^3$ (up right, down left)

Odd degree: ____________

even degree: ___________

How would end-behavior change if the function has been reflected across the x-axis?

Degree vs. End Behavior

$$y = x^5 + x^4 + x^3 + x^2 + x + 1$$

Pick a very large input value: 1,000,000 = 10^6 then compare each term.

$(10^6)^5 = 10^{30}$

$(10^6)^4 = 10^{24}$

$(10^6)^3 = 10^{18}$

$(10^6)^2 = 10^{12}$

$(10^6 = 10^6$

$1 = 10^0$

Compare the largest two powers.

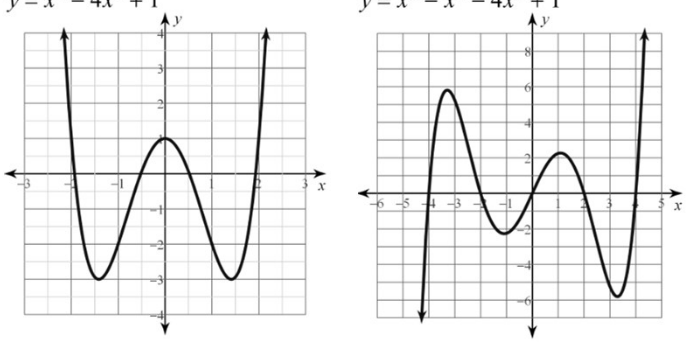
$$10^{30} = 10^{24} * 10^6$$

For x = 1,000,000 → x^6 is 1,000,000 times larger than x^5

→ At the right and left ends of the graph (where the input value is a *gigantic* positive or negative number) the largest power in the polynomial has the largest effect on the output value (y-value) so this term dominates the effect on end-behavior.

Polynomial Degree → End Behavior?

$y = x^4 - 4x^2 + 1$ $\qquad$ $y = x^5 - x^4 - 4x^2 + 1$



All even degree polynomials have the ________________

All odd degree polynomials have the __________________

4th degree polynomial → "4 zeroes"

$y = (x - 1)(2x + 8)(x - 3)(x + 2)$

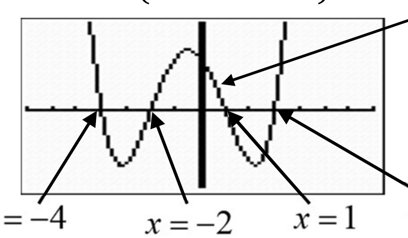
Lead term: "left*left*left*left" $= 2x$

$y = (x - 1)(2x + 8)(x - 3)(2x + 2)$

Constant term: (y-intercept) "right * right * right * right" = ? $= 48$

y-intercept = 48

$$y = 2x + \ldots \text{(other terms)} \ldots + 48$$



$x = -4$ $\quad$ $x = -2$ $\quad$ $x = 1$ $\quad$ $x = 3$

Complex Conjugates Theorem

If f(x) is a polynomial and if (x + bi) is a factor (–bi is a zero) then its complex conjugate, (x - bi) is also a factor (and +bi is a zero) of f(x).

Example: $0 = x^2 + 4 \rightarrow 0 = (x - 2i)(x + 2i)$

x = 2i, x = 2i

Example: $0 = x^4 + 5x^3 + 13x^2 + 45x + 36$

$0 = (x + 4)(x + 1)(x - 3i)(x + 3i)$

x = -4, -1, *3i*, *-3i*

Irrational Roots Theorem

If f(x) is a polynomial and if $(x - \sqrt{b})$ is a factor of the polynomial ($\rightarrow \sqrt{b}$ is a zero) then its irrational conjugate $(x + \sqrt{b})$ is also a factor of the polynomial ($\rightarrow \sqrt{b}$ is also a zero).

Example: $0 = x^2 - 3 \quad \rightarrow 0 = (x - \sqrt{3})(x + \sqrt{3})$

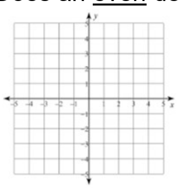
$x = \sqrt{3}, \quad -\sqrt{3}$

Example: $0 = x^4 - x^2 - 20$

$\rightarrow 0 = (x + 2i)(x - 2i)(x - \sqrt{5})(x + \sqrt{5})$

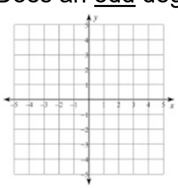
$x = -2i, \quad 2i, \quad \sqrt{5}, \quad -\sqrt{5}$

Does an even degree polynomial necessarily cross the x-axis?



All zeroes can be ___________for even-degree polynomials.

Does an odd degree polynomial necessarily cross the x-axis?



Since the end-behavior is down left/ up right,

______________ so it has

____

Describe the (1) end behavior $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

Positive, even degree Up on left/right

(2) number of real zeroes and/or imaginary zeroes

Degree	Real zeroes	Imaginary Zeroes
4	0	4
	1	3 Not possible
	2	2
	3	1 Not possible
	4	0

Describe the
(1) end behavior

$f(x) = -8x^2 + 14x + 8$

____

(2) number of real zeroes and/or imaginary zeroes

Make a table of the possible zeroes by category

Degree	Real zeroes	Imaginary Zeroes

Since "multiplicities" are counted separately, it's easier to just count the number of zeroes without specifying them as multiplicities.

$f(x) = 4x^3 + 14x + 8$

Degree	Real zeroes	Imaginary Zeroes
		Not possible
		Not possible

Why not?