Math-3A

Lesson 13-1
Arithmetic Sequences
and
Linear Functions

What is a <u>sequence</u>?

Sequence: an ordered progression of numbers (a list of numbers that follows a pattern)

Finite sequence: a sequence that has a final term, therefore the total terms in the sequence can be counted.

example 5, 10, 15, 20, 25, 30, 35

5, 10, 15,…, 35 number pattern continues between 15 and 35

Infinite sequence: sequence that does not have a final term, so the number of terms is uncountable.

Example: 2, 5, 8, 11, 14, …

2, 5, 8,… number pattern continues after '8' infinitely

What are the six ways to show a relation between input and output?

example 5, 10, 15, 20, 25, 30, 35

"The 1st term is 5, the 2nd term is 10, the 3rd term is 20 etc.)

What are the input values?

How is a sequence a relation between input and output?

The domain of a sequence is almost always the "natural numbers" (until you get to Pre-Calculus)

k	a_k
1	-9
2	-6
3	-3
4	0
5	3
6	6
7	9
8	12
9	15

The input value refers to the relative position of the term in the sequence (1st, 2nd, 3rd, etc.)

The range is the sequence itself (the set of all the individual numbers of the sequence).

What is the difference between the domain and an individual input value?

Normally, the domain of an infinite sequence is the set of all "natural numbers".

An input value specifies the relative position of an individual term in the sequence.

$\{a_k\}$ Represents all the terms in a sequence named "a" which has subscript "k" to identify the "kth" term of the sequence.

$a_3 = 5$ No French brackets and subscript '3' means the 3rd term of sequence 'a'.

$\{b_n\} = \{2, 4, 6, \ldots, 10\}$ What is the name of the sequence? "b"

What variable is used to define the "counter"? "n"

Is the sequence finite or infinite? finite

Write this relation as a table:

n	1	2	…	5
b_n	2	4	…	10

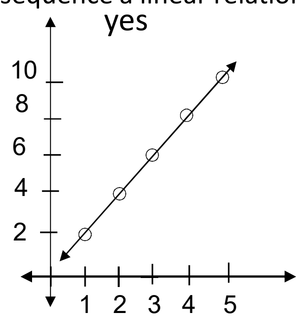
$b_4 = ?$ $b_5 = 10$

$b_4 = 8$

$\{b_n\} = \{2, 4, 6, \ldots\}$ Is a sequence a relation? yes

Graph the relation.

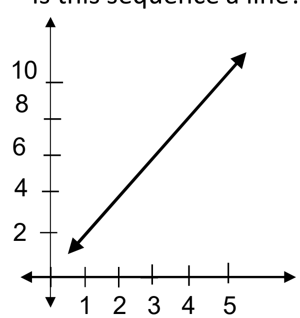
Is this sequence a linear relation? yes



domain = ? Discrete domain

D = {x = 1, 2, 3, 4, 5}

Is this sequence a line?



Domain = ?

D = (-∞,∞)

continuous domain

Discrete set of numbers: a countable set of numbers

Example: {1, 2, 3, 4, 5}

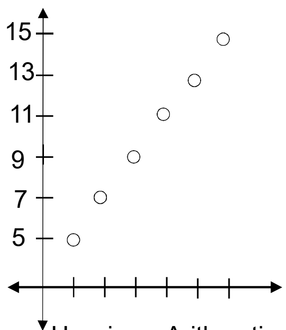
Continuous set of numbers: an infinite set of numbers that includes every number in an interval of numbers (with no "gaps").

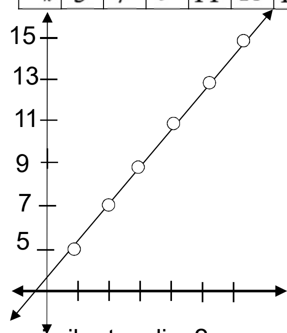
Example (-5, 3]

Arithmetic Sequence: a sequence that is a linear relation.

$\{a_k\} = \{5,7,9,11,13,15\}$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15





How is an Arithmetic sequence similar to a line?
Constant slope (or 1st difference is constant)

How is an Arithmetic sequence different from a line?
Discrete domain vs. continuous domain

$\{a_k\} = \{2k+3 : k = 1,2,3,...\}$ Defines a "rule" so that you can find the "kth" term

We call this method of defining the sequence "set-builder" notation.

Spoken: "the sequence 'a' is defined as 2k +3 with 'k' taking on the values 1, 2, 3, and so forth"

Your turn: make a table of values that contains the input/output pairs for the first 6 numbers of sequence 'a'.

$\{a_k\} = \{2k+3 : k = 1,2,3,...\}$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15

$a_1 = 2(1) + 3 = 5$

$a_2 = 2(2) + 3 = 7$

Your turn: Write a linear equation that contains the ordered pairs in the table below. $y = mx + b$

Slope= $m = \frac{\Delta y}{\Delta x} = 2$ $y = 2x + b$

$\Delta x = 1$

k	1	2	3	...
a_k	5	7	9	...

$\Delta y = 2$

How do you find 'b'?

Method #1: plug in an input—output pair.

$(5) = 2(1) + b$

$3 = b$

$y = 2x + 3$

Method 2: find the "y-intercept" *by going leftwards in the table to the "zero-eth" term*.

$y = mx + b \qquad y = 2x + b \qquad \boxed{y = 2x + 3}$

$\Delta x = 1$

k	0	1	2	3	4	5	6
a_k	3	5	7	9	11	13	15

$\Delta y = 2$

y-intercept = (0, b)
y-intercept = (0, 3)

Your turn: 1. Fill in the table.
2. Write a linear equation that contains the ordered pairs in the table below.

$\{b_m\} = \{2(m-1) + 5 : m = 1, 2, 3, \ldots\}$

$y = mx + b \qquad y = 2x + b \qquad \boxed{y = 2x + 3}$

$\Delta x = 1$

k	0	1	2	3	4	5	6
a_k	3	5	7	9	11	13	15

$\Delta y = 2$

$\text{slope} = \frac{\Delta y}{\Delta x} = 2$

y-intercept = (0, b)
y-intercept = (0, 3)

Whoa, they're the same sequence!!!

$\{a_k\} = \{2k + 3 : k = 1, 2, 3, \ldots\}$

$\{b_m\} = \{2(m-1) + 5 : m = 1, 2, 3, \ldots\}$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15
b_m	5	7	9	11	13	15

Is this true? $2(x-1) + 5 = 2x + 3$ yes

$2x - 2 + 5 = 2x + 3$

$2x + 3 = 2x + 3$

Whoa, they're the same sequence!!!

$\{a_k\} = \{2k + 3 : k = 1, 2, 3, \ldots\}$ Emphasizes "0th" term

$\{b_m\} = \{2(m-1) + 5 : m = 1, 2, 3, \ldots\}$ Emphasizes 1st term

x	0	1
a_x	3	
b_x		5

Arithmetic Sequence: a sequence where there is a "constant difference" between each of the adjacent terms.

$\{a_k\} = \{2k + 3 : k = 1,2,3,...\}$ $\quad y = 2x + 3$

k	1	2	3	4	5	6
a_k	5	7	9	11	13	15

$a_2 - a_1 = 2$ $\quad a_4 - a_3 = 2$

Arithmetic Sequence: each pair of adjacent terms has the same "common difference"

Note: the "common difference" becomes the slope of the linear equation passing through each ordered pair because the 1st difference of the input is always '1'.

Explicitly Defined Arithmetic Sequence: an equation (in "sequence notation" that allows you to find any term in the sequence without having to find previous values in the sequence.

$$a_k = 3(k-1) + 4$$

$$a_k = d(k-1) + a_0$$

common difference $\quad$ first term of the sequence

n	a_n
1	-9
2	-6
3	-3
4	0
5	3
6	6
7	9
8	12
9	15

Your turn: Define the following sequence of numbers using "set builder" notation.

-9, -6, -3, 0, 3, 6, 9, 12, 15

$$a_n = \{3n - 12 : \text{for } n = 1,2,3,...,9\}$$

$$a_n = \{3(n-1) - 9 : \text{for } n = 1,2,3,...,9\}$$

Examples Sequences

Running total: if your car payment is \$200/month, a running total would look like:

Month	1	2	3	4
Total (\$)	200	400	600	800

Your Turn:

1. Is this an arithmetic sequence?
2. Define this sequence explicitly and name it "C".

$C_m = 200(m-1) + 200$ (m = 1,2,3,4)

$C_m = 200m$

Recursively Defined Sequence: a sequence defined by the first term, and how to find the "next term."

$b_1 = 4$ ← 1st term

$b_n = b_{n-1} + 2$ ← "next" term = "previous" term + 2

For all n > 1 ← 'n' acts as a counter for the sequence since 1st term is already defined.

From this information, could you write the 1st four terms of the sequence?

4, 6, 8, 10

From the recursive definition, could you write the explicit definition (formula) of the sequence? $a_k = d(k-1) + a_0$

common difference (d); first term of the sequence (a_0)

$$b_n = 2(n-1) + 4$$

Recursively Defined Sequences

$b_1 = 4$ $\quad b_n = b_{n-1} + 2$ $\quad$ For all n > 1

Your turn: Find the 3rd term of the sequence.

$b_2 = b_1 + 2$ $\quad b_3 = b_2 + 2$

To find the 3rd term, we need to find out what the 2nd term is:

$b_2 = b_1 + 2$ $\quad b_2 = 4 + 2 = 6$ $\quad b_3 = 6 + 2 = 8$

4, 6, 8, 10, 12,…

Is this an arithmetic sequence? yes

What is the common difference? 2

What is explicit formula for the sequence?

$a_k = d(k-1) + a_0$ $\quad a_k = 2(k-1) + 4$

Two ways to write a formula to define the numbers in a sequence.

Explicitly Defined Sequence

Recursively Defined Sequence

Define the following sequence of numbers recursively.

-9, -6, -3, 0, 3, 6, 9, 12, 15

$b_1 = -9$ $\quad b_n = b_{n-1} + 3$ $\quad$ For 1 < n ≤ 9

Define the sequence explicitly: $b_n = d(k-1) + b_0$

$$b_n = 3(k-1) - 9$$

Define the following sequence of numbers recursively.

-20, -16, -12, -8, -4, 0, 4, 8, 12, 16, 20

$b_1 = -20$ $b_n = b_n + 4$ For $1 < n \leq 11$

If the sequence was an infinite sequence, what would be the 27th term?

Define the sequence explicitly: $b_n = d(k-1) + b_0$

$b_n = 4(n-1) - 20$

$b_{27} = 4(27-1) - 20$

$b_{27} = 84$