

SM3-A Lesson 13-13 The "Box Problem"

"The whole is greater than the sum of its parts."

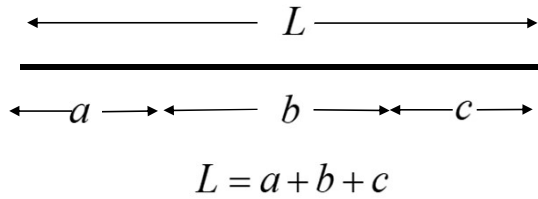
Aristotle



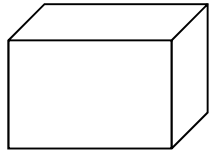
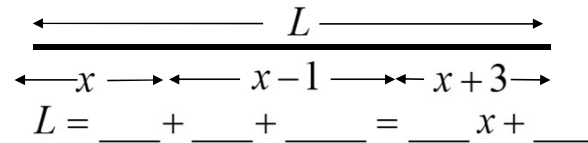
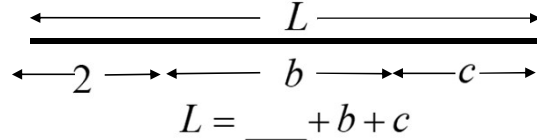
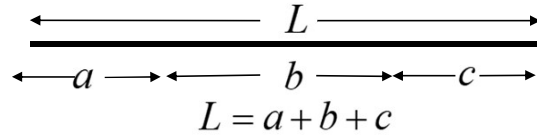
This is a good definition for *synergism*.

It doesn't make sense for math.

The length of a line segment is the sum of its parts.

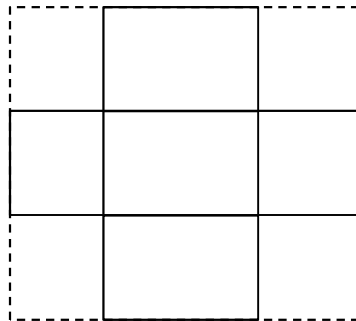


Rewrite the relation between the total length of the segment and its parts.



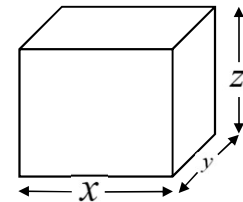
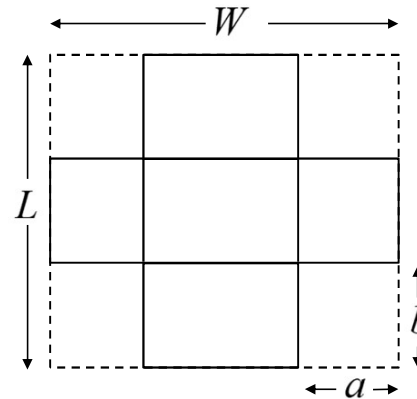
Start with an open-topped box.

Build the "net" for the box.



Describe how you can use a single flat piece of cardboard to build the box.

Dimensions "a" and "b" on the net correspond to which dimensions on the box?



$a = b = z$

Write an expression for length 'y' (the width of the box) as a function of the original cardboard width "W" and your chosen height of the box 'x'.

$y = w_{\text{box}} = W_{\text{cardboard}} - 2x$

Write an expression for length 'z' (the length of the box) as a function of the original cardboard width "W" and your chosen height of the box 'x'.

$z = l_{\text{box}} = L_{\text{cardboard}} - 2x$

Write an expression for the volume of the box as a function of the original length (L) and width (W) of the cardboard and your chosen height of the box ('x').

$V_{\text{box}} = x * y * z$

$V_{\text{box}} = x(L - 2x)(W - 2x)$

Corners are cut out of a 9" x 21" piece of cardboard. Then the sides are folded up along the dotted lines.

$21 - 2x$

$9 - 2x$

Write a formula for the volume of the box as a function of its height "x" only.  $V_{box} = x(L-2x)(W-2x)$

$V_{box} = x(9-2x)(21-2x)$

What is the implied domain for the height of the box (that starts out as a piece of cardboard measuring 9" x 21')?

Hint: what is the minimum distance you can fold the sides of the cardboard up?  $height > 0$ "

Hint: What is the  $height < 4.5$ " max distance?

implied domain:  
 $0 < x < 4.5$ "  
 or  
 $x = (0, 4.5)$ "

Why is the maximum height determined by the shortest side of the cardboard?

Graph your equation for the volume of the box.  
 $V = x(9-2x)(21-2x)$       $V = L * w * h$

### Maximizing Volume of a Box

$V(h) = h(9-2h)(21-2h)$

$y = x(9-2x)(21-2x)$

Why does the "implied domain" make sense now that you can see the graph of volume as a function of height?

What is the maximum volume of the box?

What height corresponds to the maximum volume of the box?