Math-3A

Lesson 12-6

Add, Subtract, Multiply, and Divide
Rational Expressions

Adding Fractions

We can add "like fractions"

$$\frac{2}{3}+\frac{1}{3}+\frac{4}{3}$$

Combine the numerator over a common denominator.

$$=\frac{2+1+4}{3} \quad =\frac{7}{3}$$

Vocabulary

A rational number can be written as a ratio of integers. $\frac{2}{5},\frac{3}{1}$

A rational expression can be written as a ratio of expressions.

$$\frac{x}{(x+1)}$$

We will be looking at ratios of polynomial expressions.

Excluded Value: the value for 'x' that results in division by zero

$$\frac{x}{(x+1)}, \quad x \neq -1$$

Simplifying Fractions

You must FACTOR the fractions.

$$\frac{32}{44}=\frac{4*8}{4*11} \qquad \frac{x^2-4}{x^2-3x+2}=\frac{(x-2)(x+2)}{(x-2)(x-1)}$$

Break them apart into the product of fractions.

$$=\frac{4}{4}*\frac{8}{11} \qquad =\frac{(x-2)}{(x-2)}*\frac{(x+2)}{(x-1)}$$

Notice the fractions that equal '1'

$$=1*\frac{8}{11} \quad =\frac{8}{11} \qquad =1*\frac{(x+2)}{(x-1)}=\frac{(x+2)}{(x-1)}$$

Adding/Subtraction Rational Expressions

The easy problem: $\frac{2}{7}+\frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$

Combine the numerator over a common denominator.

The easy problem: $\frac{4}{(x-5)}+\frac{3x}{(x-5)} = \frac{4+3x}{(x-5)}$

Combine the numerator over a common denominator.

Can you combine 4 and 3x ?

Why not?

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{\cancel{2}(x-1)}{\cancel{2}*x^2} = \frac{(x-1)}{x^2}$$

What property are we using?

Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{\cancel{x}+\cancel{2}}{\cancel{2}*\cancel{x}*x}+\frac{\cancel{x}-(\cancel{2}*2)}{\cancel{2}*\cancel{x}*x} = \frac{1}{x}+\frac{-2}{x}$$

Why not? You CANNOT use the Inverse Property of Multiplication on addends.

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{2(x-1)}{2*x^2}$$

I will not allow you to simplify using the Inverse Property of Multiplication until you have factored it into two fractions.

$$= \frac{\cancel{2}}{\cancel{2}}*\frac{(x-1)}{x^2} = \frac{(x-1)}{x^2}$$

Only then will you be able to see how the Inverse Property of Multiplication changes the rational expression into multiplication by one.

Your turn: add/subtract

$$\frac{(2x-7)}{x^2+2}-\frac{(x-4)}{x^2+2}$$

Which one is correct?

$$= \frac{2x-7-x-4}{x^2+2}$$

$$= \frac{2x-7-x+4}{x^2+2}$$

$$\frac{(2x-7)-(x-4)}{x^2+2} = \frac{2x-7-x-(-4)}{x^2+2} = \frac{x-3}{x^2+2}$$

Subtract every term in the right side numerator! (Half of you will make this mistake on the HW and on the Test).

Can you factor this into two fractions multiplied together?

One third of you will miss this on the test.

Multiplying Rational Expressions

<u>Simplify</u> before you <u>multiply</u>.

$$\frac{(x-1)}{2(x+3)} * \frac{x^2-9}{(x+1)} = \frac{(x-1)}{2(x+3)} * \frac{(x+3)(x-3)}{(x+1)}$$

$$= \frac{\cancel{(x+3)}}{\cancel{(x+3)}} * \frac{(x-1)(x-3)}{2(x+1)} = \frac{(x-1)(x-3)}{2(x+1)}$$

DON'T multiply the simplified version of the product, just leave it in factored form.

Your turn:

Multiply the expressions

$$\frac{3\cancel{(x-4)}}{\cancel{(x-3)}} * \frac{\cancel{(x-2)}}{\cancel{(x-4)}} * \frac{\cancel{(x-3)}}{6\cancel{(x-2)}} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{(x+3)}{(x-5)} * \frac{(x^2-16)}{(x+4)} = \frac{(x+3)(x-4)\cancel{(x+4)}}{(x-5)\cancel{(x+4)}}$$

$$= \frac{(x+3)(x-4)}{(x-5)}$$

Your turn:

Multiply the expressions

(Solutions on this slide)

$$\frac{x^2+x-12}{x^2-9} * \frac{x^2-2x-15}{x^2-16}$$

$$\frac{\cancel{(x-3)}\cancel{(x+4)}}{\cancel{(x-3)}\cancel{(x+3)}} * \frac{(x-5)\cancel{(x+3)}}{\cancel{(x+4)}(x-4)} = \frac{(x-5)}{(x-4)}$$

Your turn:

Multiply the expressions

$$\frac{2x^2-8x-24}{x^2+2x-3} * \frac{x^2+7x+12}{x^2-2x-24}$$

$$\frac{2(x^2-4x-12)}{\cancel{(x+3)}(x-1)} * \frac{\cancel{(x+3)}\cancel{(x+4)}}{(x-6)\cancel{(x+4)}} = \frac{2(x^2-4x-12)}{(x-1)(x-6)}$$

$$= \frac{2\cancel{(x-6)}(x+2)}{(x-1)\cancel{(x-6)}} = \frac{2(x+2)}{(x-1)}$$

Divide Rational Expressions

$$\frac{2}{3} \div \frac{5}{7}$$

What do we do? Multiply by the reciprocal

$$\frac{2}{3} * \frac{7}{5} = \frac{14}{15}$$

Dividing Rational Expressions

$$\frac{x+3}{x^2+x-6} \div \frac{x-8}{x-2} = ? \quad = \frac{x+3}{x^2+x-6} * \frac{x-2}{x-8}$$

simplify then multiply!

$$= \frac{x+3}{(x+3)(x-2)} * \frac{x-2}{x-8} \quad = \frac{(\cancel{x+3})(\cancel{x-2})}{(\cancel{x+3})(\cancel{x-2})(x-8)}$$

$$\frac{1}{(x-8)} \quad \text{OR} \quad (x-8) \quad ?$$

Your turn:

$$\frac{x+3}{x^2+x-6} \div \frac{x-8}{x^2+x-6}$$

$$\frac{x+3}{x^2+x-6} * \frac{x^2+x-6}{x-8} = \frac{(x+3)}{\cancel{(x^2+x-6)}} * \frac{\cancel{(x^2+x-6)}}{(x-8)} = \frac{(x+3)}{(x-8)}$$

Your turn:

$$\frac{x^2+2x-35}{x^2-4x-12} \div \frac{x^2-2x-15}{x^2+9x+14}$$

$$\frac{x^2+2x-35}{x^2-4x-12} * \frac{x^2+9x+14}{x^2-2x-15} = \frac{(x+7)\cancel{(x-5)}}{(x-6)\cancel{(x+2)}} * \frac{\cancel{(x+2)}(x+7)}{\cancel{(x-5)}(x+3)}$$

$$= \frac{(x+7)(x+7)}{(x-6)(x+3)}$$