Math-3
Lesson 12-4
Quadratic Formula

Your turn: Solve by factoring.

1. $y = x^2 - 5x + 4$

2. $y = x^2 - 6x - 27$

Vocabulary

Quadratic Formula: gives the solutions (x-intercepts) to ANY quadratic equation in standard form.

$y = ax^2 + bx + c \quad y = 2x^2 + 3x + 4$

a = 2 b = 3 c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How did we get the quadratic formula?

We covert the following standard form quadratic equation to vertex form. $y = ax^2 + bx + c$

Which of the letters are variables? x and y

Set y = 0 so that we are finding "zeroes" of the equation.

$0 = ax^2 + bx + c$ x-coordinate of vertex: x = -b/2a

The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right) = a\left(\frac{-b}{2a}\right)^2 + b(\frac{-b}{2a}) + c$$

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c$$

The x-coordinate of the vertex is: x = -b/2a
The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right)=\frac{ab^2}{4a^2}-\frac{b^2}{2a}+c \quad =\frac{b^2}{4a}-\frac{b^2}{2a}+c$$

Obtain common denominators then Add fractions

$$f\left(-\frac{b}{2a}\right)=\frac{b^2}{4a}-\frac{2b^2}{4a}+\frac{4ac}{4a} \qquad =\frac{-b^2+4ac}{4a}$$

Write in Vertex form:

$$0=\left(x+\frac{b}{2a}\right)^2-\left(\frac{-b^2+4ac}{4a}\right)$$

Vertex form: $0=a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a}\right)$

Isolate the square, undo the square

$$\frac{b^2-4ac}{4a}=a\left(x+\frac{b}{2a}\right)^2 \qquad \pm\frac{\sqrt{b^2-4ac}}{2a}=x+\frac{b}{2a}$$

$$\frac{b^2-4ac}{4a^2}=\left(x+\frac{b}{2a}\right)^2$$

$$\pm\sqrt{\frac{b^2-4ac}{4a^2}}=x+\frac{b}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

Your turn: $y=ax^2+bx+c$

Identify 'a' 'b' and 'c' for each of these standard form quadratic equations.

$$y=x^2-4x+3$$

$$y=-2x^2+3x-7$$

$$y=3x^2-10$$

$$y=-x^2+3x$$

What if it's not in standard form?

$$2x=3x^2-5$$

Use math properties to get it into standard form (same thing left/right, combine like terms, etc.)

$0=3x^2-2x-5$ a = 3 b = -2 c = -5

a = ? b = ? c = ?

Determine the following values: a = ?, b = ?, c = ?

$$y=3-12x^2-4x$$

$$5x=3x^2-5x+1$$

Solve using the Quadratic formula.

$y = ax^2 + bx + c$

$y = x^2 - 6x + 4$

a = 1 b = -6 c = 4

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{36 - 16}}{2}$

$x = \dfrac{6 \pm \sqrt{20}}{2}$

$x = \dfrac{6 \pm \sqrt{4}\sqrt{5}}{2}$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

$x = \dfrac{6}{2} \pm \dfrac{2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$

If the quadratic CANNOT be factored, the solutions are "ugly."

Can you "plug" 'a', 'b', and 'c' into the Quadratic formula?

$y = ax^2 + bx + c$

Identify 'a', 'b', and 'c' in the standard form equation.

$y = x^2 + 3x + 2$

a = 1 b = 3 c = 2

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Replace the 'a', 'b', and 'c' with parentheses!!!

$x = \dfrac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$

Put the numerical values of 'a', 'b', 'c' into the parentheses.

$x = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(1)(2)}}{2(1)}$

$y = ax^2 + bx + c$

$y = x^2 + x - 1$

a = 1 b = 1 c = -1

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$

"Gotcha" parts of the formula.

$y = ax^2 + bx + c$

$y = x^2 - 15x - 1$

a = 1 b = -15 c = -1

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$

If you don't use the parentheses, it would be easy to write this as -15

With the parentheses, -(-15) simplifies to +15!!!

If you don't use the parentheses, it would be easy to write this as -15^2 instead of $(-15)^2$

$-15^2 = -225$

$(-15)^2 = +225$

"Gotcha" parts of the formula.

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$$

It is easy to make mistakes with the negatives.

If you use parentheses, then you can type the expression under the radical into your calculator and it will be correct. $(-15)^2 - 4(1)(-1) = 229$

$$x = \frac{15 \pm \sqrt{229}}{2}$$

Plug in and simplify

$$y = ax^2 + bx + c$$

$$y = x^2 + x - 1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{-1}{2} + \frac{\sqrt{5}}{2}$$

$$x = \frac{-1}{2} - \frac{\sqrt{5}}{2}$$

Another one (be careful of the "gotcha's")

$$y = ax^2 + bx + c$$

$$y = x^2 - 4x + 3$$

a = 1 b = -4 c = 3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

Can you simplify the result?

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16-12}}{2}$$

$$x = \frac{4 \pm \sqrt{4}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4}{2} \pm \frac{2}{2}$$

$$x = 2 \pm 1$$

$$x = 1,3$$