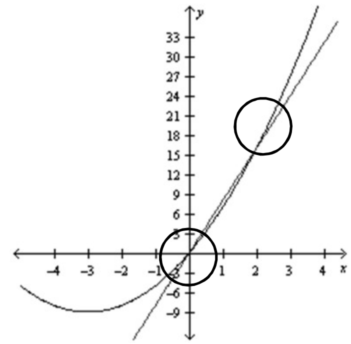
Math-3A
Lesson 11-6
Solving Systems of Equations Graphically

A solution of a system of two equations in two variables is an ordered pair of real numbers that lies on both graphs (where the graphs intersect).



Depending upon the types of equations in the system, there can be more than one solution!

Methods of Solving Systems

1. Substitution: a method using algebra and properties of algebra (we learned this in Math-2)

2. Elimination: another algebraic method that we learned in Math-2.

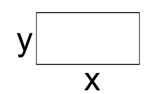
3. Graphing: The points of intersection of the graphs are the solutions.

Graphing is extremely useful for very difficult equations!

Garden Problem: Find the dimensions of a rectangle with a perimeter of 200 m and an area of 500 m².

$$2x + 2y = 200$$

$$xy = 500$$



Rewrite each equation by solving for 'y'.

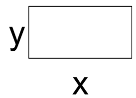
$$y = \frac{(200 - 2x)}{2} \qquad y = 500/x$$

Solve by graphing.

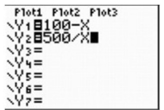
Garden Problem: Find the dimensions of a rectangle with a perimeter of 200 m and an area of 500 m².

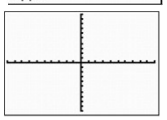
$$y = \frac{(200-2x)}{2} = 100 - x$$

$$y = 500/x$$



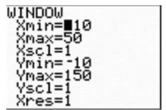
Solve by graphing.

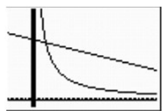


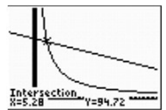


Oops, window is too small.

Adjust window.







Solution: (5.3, 94.7)

Width = 5.3 m

Length = 94.7 m

Solving a single variable equation

$$4 = 3\log(x-4) + 2$$

left side of the equation equals right side

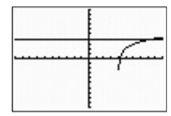
Set left side equal to y

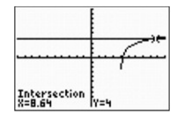
Set right side equal to y

$$y = 4 \qquad y = 3\log(x-4) + 2$$

We know that: y = y

The solution to the two equations will be **(x, 4)** where 'x' is the solution to the single variable equation.





Check: 3log(8.64 – 4) + 2 = 3.99955

How could you turn the following single variable equation into a system of equations in two variables?

$$7 = 3 + 5\sqrt{x-4}$$

$$y = 7 \qquad y = 3 + 5\sqrt{x-4}$$

Solve by graphing

Convert the following single variable equations into a system of equations in two variables.

$$5 = x^2 + 4x - 10$$

$$y = 5$$

$$y = x^2 + 4x - 10$$

Solve by graphing

Convert the following single variable equations into a system of equations in two variables.

$$-4 = 6 - 2\log(3x-7)$$

$$y = -4$$

$$y = 6 - 2\log(3x-7)$$

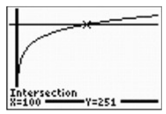
Solve by graphing

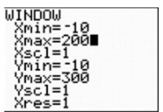
<u>Formula</u> relating distance (d) that a tornado travels and the wind speed (s) inside the cone of the tornado.

$$s = 93\log d + 65$$

The wind speed of a tornado was 251 mi/hr. How far did it travel on the ground?

$$251 = 93\log d + 65$$

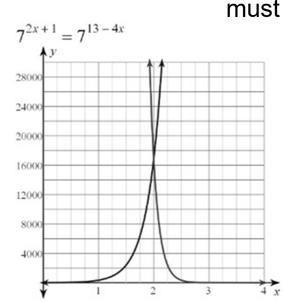




$$d = 100 \text{ miles}$$

$$7^{2x+1} = 7^{13-4x}$$

If you can't remember that the exponents must equal each other (in this case)…

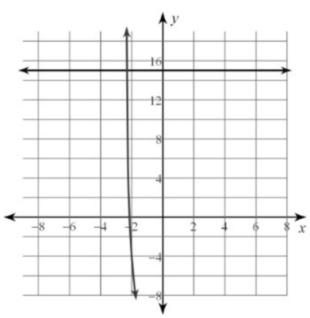


$$y = 7^{2x+1}$$
$$y = 7^{13-4}$$

$$x = 2$$

$$15 = -5\log_2(3x+7) - 4$$

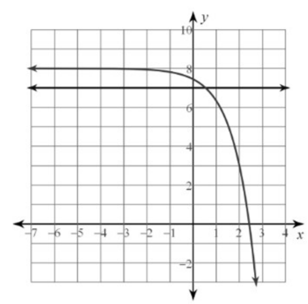
$$y = 15$$
$$y = -5\log_2(3x+7) - 4$$



$$x = -2.309$$

$$7 = -5(3)^{(x-2)} + 8$$

$$y = 7$$
$$y = -5(3)^{(x-2)} + 8$$



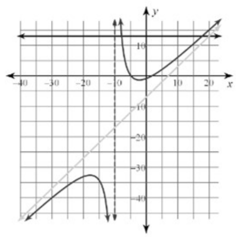
$$x = 0.535$$

$$13 = \frac{x^2 + 3x - 9}{x + 10}$$

This has an oblique asymptote since x + 10 doesn't divide evenly.

$$y = x - 7 - \cancelto{0}{\frac{79}{x+10}}$$

$$y = x - 7$$



$$y = 13$$

$$y = \frac{x^2 + 3x - 9}{x + 10}$$

$$\begin{array}{r|rrr} -10 & 1 & 3 & -9 \\ & & -10 & -70 \\ \hline & 1 & -7 & -79 \end{array}$$

$$x = -7.81$$

$$x = 17.81$$

Finding the equation of a circle:

What are the (x, y) coordinates of the point?

What is the radius of the circle?

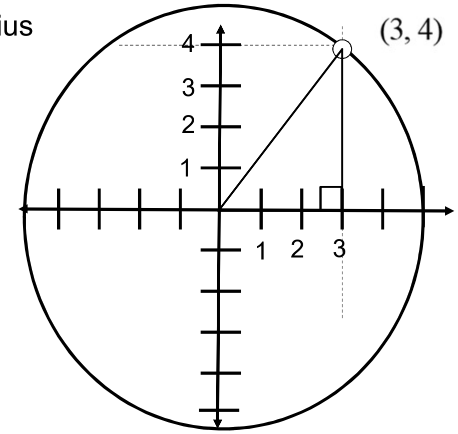
$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$9 + 16 = 25$$

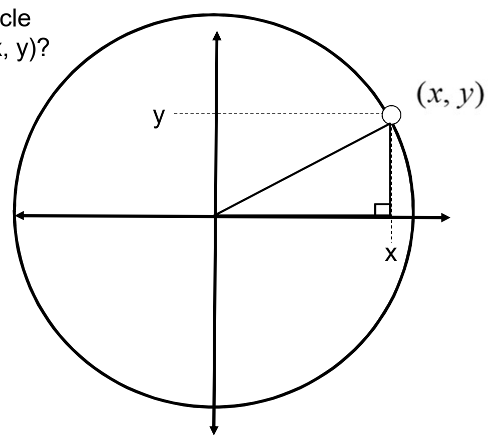


Now I will pick a random point on the circle.

What is the equation for the radius of the circle when the point is (x, y)?

$$x^2 + y^2 = r^2$$

This is the general equation for a circle centered at (0, 0)!!!

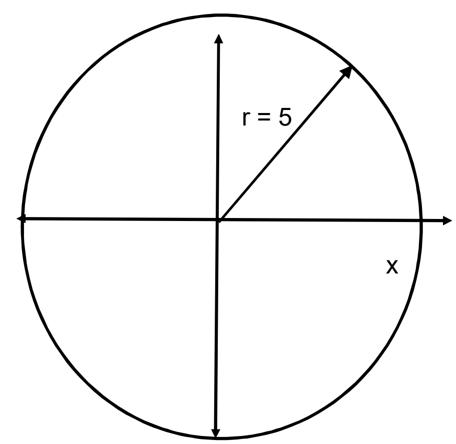


This is the equation of a circle centered at the origin whose radius is 5.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 5^2$$

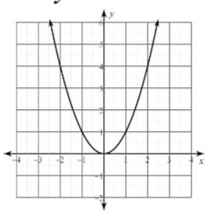
$$x^2 + y^2 = 25$$



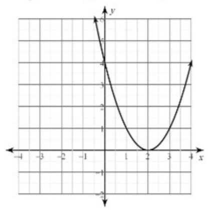
Graphical Transformations

<u>Parent Function</u>: The <u>simplest</u> function in a family of functions (lines, parabolas, cubic functions, etc.)

$y = x^2$



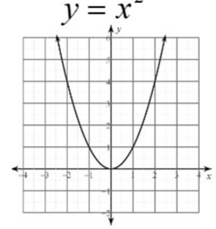
$y = (x-2)^2$



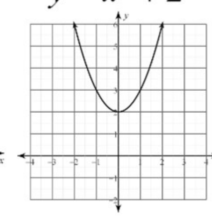
<u>Replacing</u> 'x' with 'x – 2' translates the parent function <u>right</u> by 2.

<u>Compare</u> the two parabolas.

$y = x^2$



$y = x^2 + 2$



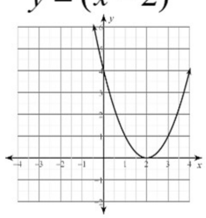
Adding 2 to the parent function moves the function up 2 units.

Subtract 2 from both sides yields: $y - 2 = x^2$

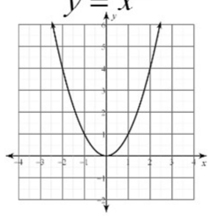
$(y-2) = x^2$

Replace "y" in the parent function with (y – 2) moves the graph up 2.

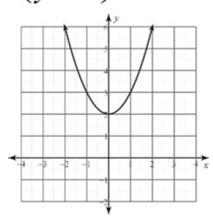
$y = (x-2)^2$



$y = x^2$



$(y-2) = x^2$



<u>Replacing</u> 'x' with (x – 2) moves the parent function <u>right</u> by 2.

Replace "y" with (y – 2) moves the parent function <u>up</u> by 2.

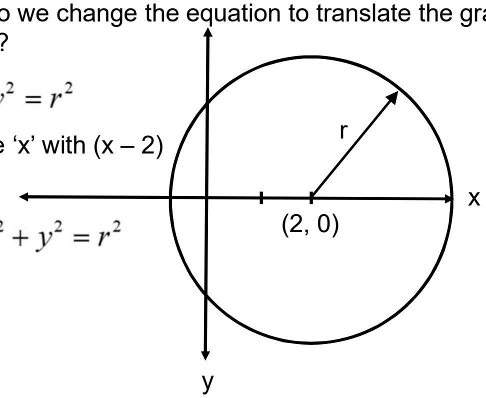
Let's move the circle 2 spaces to the right.

How do we change the equation to translate the graph right 2?

$x^2 + y^2 = r^2$

Replace 'x' with (x – 2)

$(x-2)^2 + y^2 = r^2$



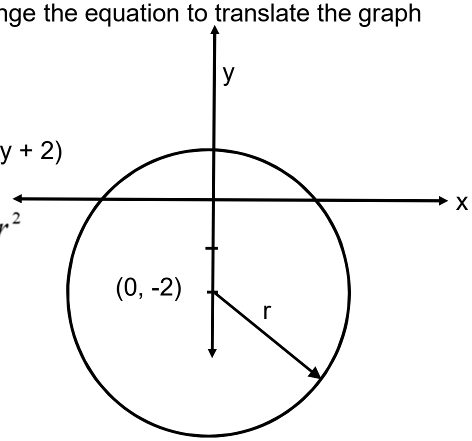
Let's move the circle 2 spaces downward.

How do we change the equation to translate the graph down 2?

$x^2 + y^2 = r^2$

Replace 'y' with (y + 2)

$x^2 + (y+2)^2 = r^2$



Prove that a point is on a circle:

The circle below is the graph of: $(x-1)^2 + (y-2)^2 = 4$

Is the point $(2, 2+\sqrt{3})$ on the circle?

Plug in x = 2, solve for 'y'.

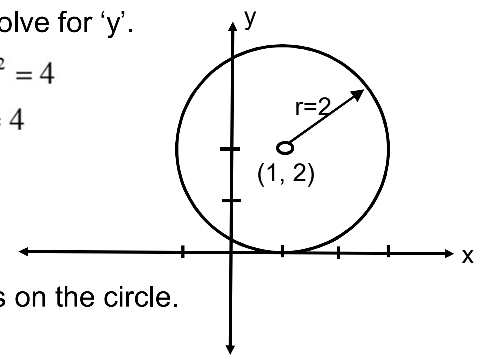
$(2-1)^2 + (y-2)^2 = 4$

$(1)^2 + (y-2)^2 = 4$

$(y-2)^2 = 3$

$y = 2 \pm \sqrt{3}$

Yes $(2, 2+\sqrt{3})$ is on the circle.



Solve the system:

$(x+3)^2 + (y-2)^2 = 9$

$(y-2)^2 = 9 - (x+3)^2$

$(y-2) = \pm\sqrt{9-(x+3)^2}$

This is <u>two</u> different equations, which are the upper and lower halves of the circle!

$(y-2) = \sqrt{9-(x+3)^2}$

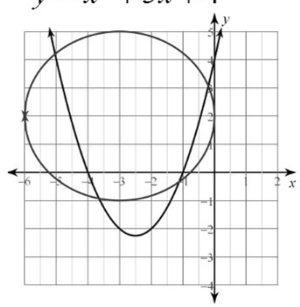
$(y-2) = -\sqrt{9-(x+3)^2}$

$y = 2 + \sqrt{9-(x+3)^2}$

$y = 2 - \sqrt{9-(x+3)^2}$

Prepare to graph: solve for 'y'

$y = x^2 + 5x + 4$



$x = -5.04,\ -3.64,\ -1.12, -0.19$

Solve the system:

$y = 2x - 3$

$y = x^2 - 4$