Math-3A
Lesson 11-11
Find Zeroes Using "Long Division", Synthetic Division, and Box Division

Our goal is to find the x-intercepts of polynomials.

We've learned how to factor:

1) Quadratic form $y = x^4 + 4x^2 + 3$

2) 3rd degree polynomials with a common factor of 'x'

$$y = x^3 + 4x^2 + 3x$$

3) 3rd degree polynomials that have a "nice pattern"

$$y = x^3 + 2x^2 + 3x + 6$$

4) Sum and Difference of 2 "perfect cubes"

$y = x^3 + 8$ $\quad$ $y = x^3 - 27$

Now we learn how to factor Polynomials that ***don't*** have a "nice pattern".

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\frac{x^3 + 3x^2 + 14x - 18}{(x-1)} = ax^2 + bx + c$$

Divide Evenly: A divisor divides evenly if there is a zero for the remainder.

Polynomial Long Division

$$\begin{array}{r} x^2 \\ x - 1 \overline{) \, x^3 + 3x^2 + 14x - 18} \end{array}$$

1) Look at left-most numbers

2) What # times "left" = "left"?

$$x^3/x = ? = x^2$$

3) Multiply

$$x^2(x-1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$\begin{array}{r} x^2 \\ x-1 \overline{) x^3+3x^2+14x-18} \\ -(x^3-x^2) \\ \hline 4x^2 \ +14x-18 \end{array}$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$\begin{array}{r} x^2 \ \textcircled{+4x} \\ \textcircled{x}-1 \overline{) x^3+3x^2+14x-18} \\ -(x^3+x^2) \\ \hline \textcircled{4x^2}+14x-18 \\ -(4x^2-4x) \\ \hline 18x \end{array}$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$\frac{4x^2}{x} = ? \quad = 4x$

3) Multiply

$4x(x-1) \ = 4x^2 - 4x$

4) Subtract

$-(4x^2-4x)$

Polynomial Long Division

$$\begin{array}{r} x^2 \ +4x \\ x-1 \overline{) x^3+3x^2+14x-18} \\ -(x^3+x^2) \\ \hline 4x^2+14x-18 \\ -(4x^2-4x) \\ \hline 18x \ -18 \end{array}$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$\begin{array}{r} x^2 \ +4x \ \textcircled{+18} \\ \textcircled{x}-1 \overline{) x^3+3x^2+14x-18} \\ -(x^3+x^2) \\ \hline 4x^2+14x-18 \\ -(4x^2-4x) \\ \hline \textcircled{18x}-18 \\ -(18x-18) \\ \hline 0 \end{array}$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$\frac{18x}{x} = \ 18$

3) Multiply

$18(x-1) \ = 18x - 18$

4) Subtract

$-(18x-18)$

$$x^2 + 4x - 18$$
$$x-1 \overline{)\, x^3 + 3x^2 + 14x - 18}$$
$$-(x^3 + x^2)$$
$$4x^2 + 14x - 18$$
$$-(4x^2 - 4x)$$
$$18x - 18$$
$$-(-18x + 18)$$
$$0$$

$$x^3 + 3x^2 + 14x - 18 = (x-1)(x^2 + 4x - 18)$$

How do we find the zeroes of the unfactorable quadratic factor?

Convert to vertex form and take square roots.

$$x - 8 \overline{)\, x^3 + 2k^2 - 90x + 76}$$

Problem #3 from homework

Synthetic Division

$$x-1 \overline{)\, x^3 - 4x^2 - 15x + 18}$$

1)	1	-4	-15	18
	↓			
	1			

1st step: Write the polynomial with only its coefficients.

2nd step: Write the "zero" of the linear divisor.

3rd step: Bring down the lead coefficient

$$x-1 \overline{)\, x^3 - 4x^2 - 15x + 18}$$

1)	1	-4	-15	18
		1		
	1	-3		

4th step: Multiply the "zero" by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

$$x-1\overline{\big)\ x^3-4x^2-15x+18}$$

1	1	-4	-15	18
		1	-3	
	1	-3	-18	

7th step: Multiply the "zero" by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

$$x-1\overline{\big)\ x^3-4x^2-15x+18}$$

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

10th step: Multiply the "zero" by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$x-1\overline{\big)\ x^3-4x^2-15x+18} = x^2-3x-18$$

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

This last number is the <u>remainder</u> when you divide:

$$x^3-4x^2-15x+18$$

by

$$x-1$$

Because the <u>remainder = 0</u>, then (x – 1) is a factor <u>AND</u> x = 1 is a zero of the original polynomial!

$$x-3\overline{\big)\ 10x^3-35x^2+17x-7}$$

Problem #7 from homework

Division of Polynomials — Box Method

$$(2x^3 + 15x^2 + 27x + 5) \div (2x + 5)$$

Only the upper left and bottom right boxes are known.

	x^2	$5x$	1
$2x$	$2x^3$	$10x^2$	$2x$
5	$5x^2$	$25x$	5

$15x^2$ $27x$

Diagonals have "like terms"

$$(2x^3 + 15x^2 + 27x + 5) \div (2x + 5) = x^2 + 5x + 1$$

Division of Polynomials — Box Method

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1)$$

Only the upper left and bottom right boxes are known.

Diagonals have "like terms"

	x^3	$4x^2$	$4x$	2
$3x^2$	$3x^5$	$12x^4$	$12x^3$	$6x^2$
$0x$	$0x^4$	$0x^3$	$0x^2$	$0x$
-1	$-x^3$	$-4x^2$	$-4x$	-2

$12x^4$ $11x^3$ $2x^2$ $-4x$ -2

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1) = x^3 + 4x^2 + 4x + 2$$

Division with remainders

$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$

Only the upper left and bottom right boxes are known.

	$-x^3$	$-2x^2$	$4x$	-8
x	$-x^4$	$-2x^3$	$4x^2$	$-8x$
3	$-3x^3$	$-6x^2$	$12x$	-24

Remainder 25

$-x^4$ $-5x^3$ $-2x^2$ $+4x$ $+1$

$\frac{25}{x+3}$

Diagonals have "like terms"

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

$$= (-x^3 - 2x^2 + 4x - 8 + \frac{25}{x+3})$$

Divide.

$$k^3 + 12k^2 + 19k - 72 \quad \div k + 9$$

Problem #9 from homework