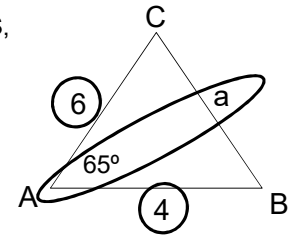
Math-3A
Lesson 10-6

Area of Triangles

Quiz:

Solve ΔABC given that $b = 6, c = 4$, and $\angle A = 65°$.

What triangle configuration is this (SSS, AAS, SSA, SAS, ASA)?



a = ? B = ? C = ?

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 6^2 + 4^2 - 2(6)(4)\cos 65$$

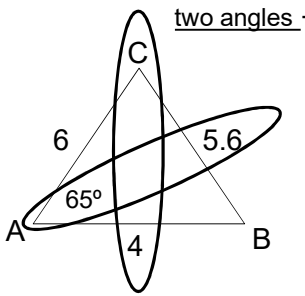
$$a^2 = 36 + 16 - 48\cos 65$$

$$a^2 = 52 - 48\cos 65$$

$$a^2 = 31.7$$

$$a = 5.6$$

Use Law of Sines to find the smaller of the two angles → Angle C.



$$\frac{\sin C}{4} = \frac{\sin 65}{5.6}$$

$$\sin C = \frac{4\sin 65}{5.6}$$

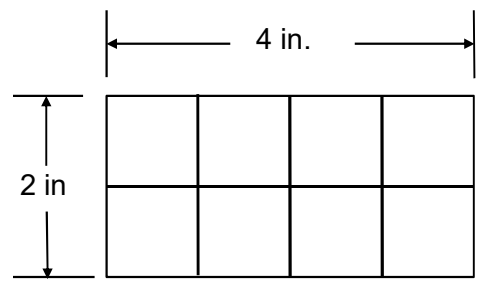
$$C = \sin^{-1}\left(\frac{4\sin 65}{5.6}\right)$$

$$C = 40.3^\circ$$

$$B = 180^\circ - 65^\circ - 40.3^\circ$$

$$B = 74.7^\circ$$

The area of this rectangle is….?

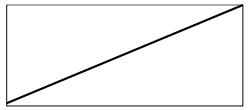


$$\text{area}_{\text{rectangle}} = \text{length} * \text{width}$$

Rectangle area formula. $A_{\text{rectangle}} = L * W$

W = width

L = length

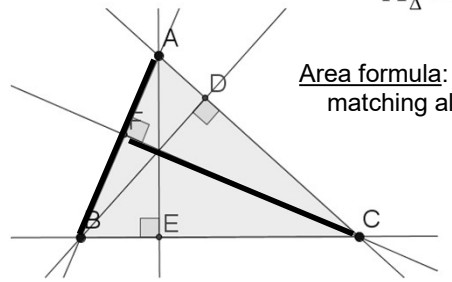


Triangle area formula.

$$A_{\text{triangle}} = \frac{1}{2} * A_{\text{rectangle}} = \frac{1}{2} * L * W$$

$$A_{\text{triangle}} = \frac{1}{2} * B * h$$

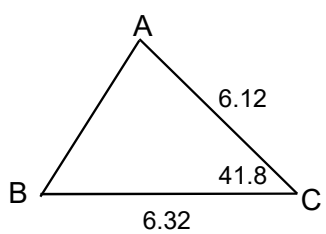
$$A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$$

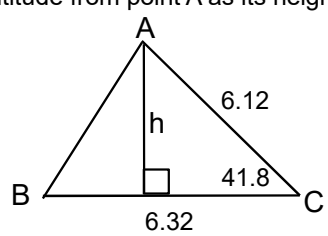


Area formula: requires the use of matching altitudes and sides.

Using segment AB as the base, requires the use of segment FC as the height.

Find the triangle area. (Use the altitude from point A as its height.)





$$A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$$

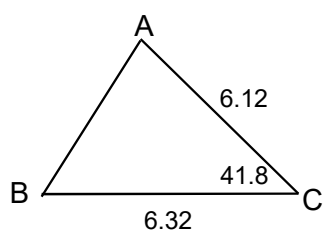
$$\frac{h}{6.12} = \sin 41.8^\circ$$

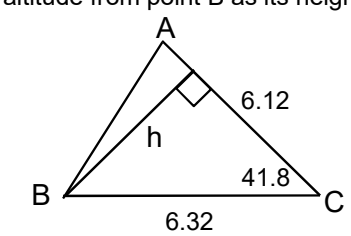
$$h = 6.12 \sin 41.8^\circ$$

$$A_{\Delta} = \frac{1}{2} * (6.32) * (6.12 sin(41.8))$$

$$A_{\Delta} = 12.89 \text{ units}^2$$

Find the triangle area. (Use the altitude from point B as its height.)





$$A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$$

$$\frac{h}{6.32} = \sin 41.8^\circ$$

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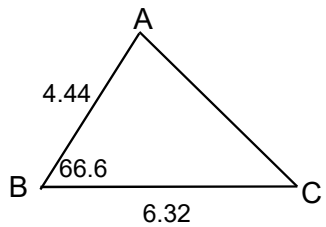
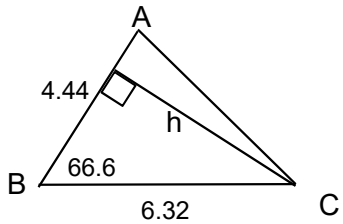
$$A_{\Delta} = \frac{1}{2} * (6.12) * (6.32 sin(41.8))$$

$$A_{\Delta} = 12.89 \text{ units}^2$$

Find the triangle area. (Use the altitude from point c as its height.)





$$A_{\Delta} = \frac{1}{2} * \text{base} * \text{height}$$

$$\frac{h}{6.32} = sin66.6°$$

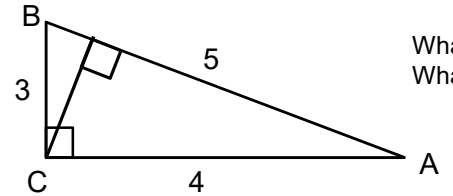
$$h = 6.32 \sin 66.6$$

$$A_{\Delta} = \frac{1}{2} * (4.44) * (6.32sin(66.6))$$

$$A_{\Delta} = 12.89 \text{ units}^2$$

Area of a Triangle

$$Area = \frac{1}{2} * \text{base} * \text{height}$$

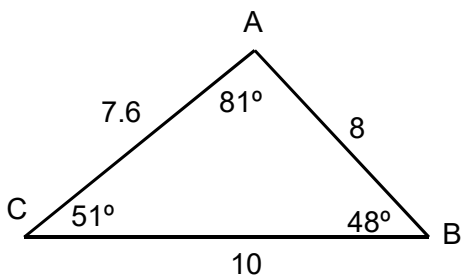


What is the base?
What is the height?

For <u>right triangles</u> one of the legs is the base, the other leg is the height (or vice-versa).

There is no point in finding the height from Angle C to side AB.

Find the Area of the Triangle



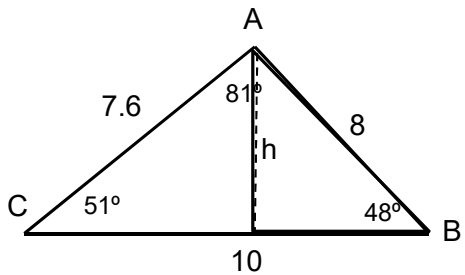
There are <u>two</u> right triangles that can be used to solve for 'h'.

$$\sin A = \frac{opp}{hyp}$$

$$\sin(48) = \frac{h}{8}$$

h = 8*Sin(48º)

h = 5.9



$$Area = 0.5 * \text{base} * \text{height}$$

Area = ½(10)(5.9)

Area = 29.5 square units

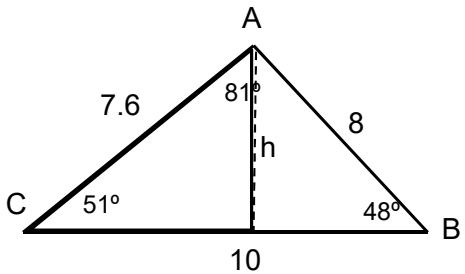
The "height" of a triangle is the distance from a vertex to the opposite side of the vertex.

$$\sin A = \frac{opp}{hyp}$$

$$\sin(52.2) = \frac{h}{7.6}$$

h = 7.6*Sin(51º)

h = 5.9



$$Area = 0.5 * \text{base} * \text{height}$$

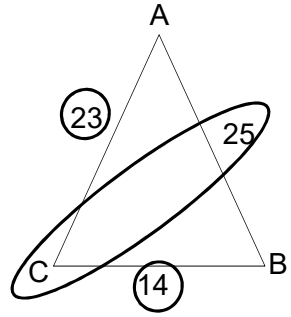
Area = ½(10)(5.9)

Area = 29.5 square units

What is the area of ΔABC given that $c = 25$, $b = 23$, and $a = 14$.

1. Find an angle!

SSS → find largest angle 1st.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

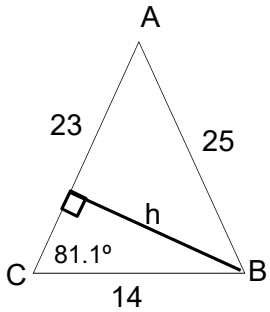
$$25^2 = 23^2 + 14^2 - 2(23)(14)\cos A$$

$$C = \cos^{-1}\left(\frac{(25^2 - 23^2 - 14^2)}{(-2*23*14)}\right)$$

$$C = 81.1^\circ$$

What is the area of ΔABC given that $c = 25$, $b = 23$, and a $= 14$.

2. Using angle C, find the appropriate height..



$$\sin 81.1 = h/14$$

$$h = 14\sin 81.1$$

$$h = 13.8$$

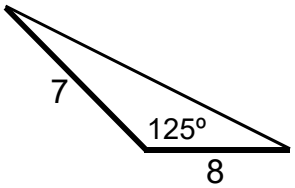
$$Area = 0.5 * \text{base} * \text{height}$$

$$Area = 0.5(23)(13.8)$$

$$Area = 158.7 \text{ units}^2$$

Find the area of triangle ABC

Area = ½*a*b*sin C



Area = ½(7)(8)sin(25)

Area = 11.8

SIDE ANGLE SIDE