## Math-3A <br> Lesson 6-12

Modeling with Quadratic Equations (Projectile Motion Problems)

## Real World

## What path does the cannon ball take?



If you drop a rock off of a cliff, what happens to the rock? Does it remain stationary?

Which direction does it fall?
As it falls, does it stay the same speed?
How fast is the rock falling after one second?
How fast is the rock falling after two seconds?

Mathematical Modeling: representing a real-world phenomenon or quantity with an equation or inequality.
"Projectile Motion" A vertical time-distance problem in two dimensions.

Height as a function of time.
$1 / 2$ the Vertical acceleration (of Gravity) multiplied by time squared gives the change in height due to gravity (English units) $\rightarrow$ (feet, seconds)


Initial Height
(height at time $=0$ )

The Initial Vertical velocity (speed at time $=0$ ) multiplied by time gives the change in vertical height from its starting point.

Mathematical Modeling: representing a real-world phenomenon or quantity with an equation or inequality.
"Projectile Motion" A vertical time-distance problem in two dimensions.

Height as a
function of time.

Initial Height
(height at time $=0$ )

$$
h(t)=-4.9 t^{2}+V_{0} t+h_{0}
$$

$1 / 2$ the Vertical acceleration (of Gravity) multiplied by time squared gives the change in height due to gravity (Standard International units) $\rightarrow$ (meters, sec.)

The Initial Vertical velocity (speed at time $=0$ ) multiplied by time gives the change in vertical height from its starting point.

An object is launched vertically upward from the ground at an initial velocity of 250 ft per second.

$$
h(t)=-16 t^{2}+V_{0} t+h_{0}
$$

a. When will the object be at its maximum height?
b. What is the object's maximum height?

$$
h(t)=-16 t^{2}+250 t
$$

Find the vertex using "technology".
a) 976.6 ft
b) 7.8 sec


An object is launched vertically upward from the ground at an initial velocity of 250 ft per second.

$$
h(t)=-16 t^{2}+V_{0} t+h_{0}
$$

When will the object fall back to the ground?

$$
h(t)=-16 t^{2}+250 t
$$

Find the x-intercept (use technology").

$$
\rightarrow 15.6 \mathrm{sec}
$$



An object is launched vertically upward from the top of a 20 foot building at an initial velocity of 310 ft . per second.

$$
\begin{aligned}
h(t) & =-16 t^{2}+V_{0} t+h_{0} \\
h(t) & =-16 t^{2}+310 t+20
\end{aligned}
$$

a) Find the maximum height
b) Find the time it takes to reach maximum height
c) Find the time when it falls to the ground.
a) 1521.6 ft
b) 9.7 sec
c) 19.4 sec


An object is launched vertically upward from the ground at an initial velocity of 250 ft per second.

$$
h(t)=-16 t^{2}+V_{0} t+h_{0}
$$

When will the object reach 500 feet?

$$
\begin{aligned}
& h(t)=-16 t^{2}+250 t \\
& h(t)=500
\end{aligned}
$$

Find the (time, height) pairs
$\rightarrow$ points of intersection.
$(t, h)=\left(t_{1}, 500\right),\left(t_{2}, 500\right)$
$(3.35,500)$ and $(13.27,500)$


An object is launched vertically upward from the ground at an initial velocity of 200 ft per second.

$$
h(t)=-16 t^{2}+V_{0} t+h_{0}
$$

During what period of time with the object be above 500 feet?

$$
\begin{aligned}
& h(t)=-16 t^{2}+200 t \\
& h(t)=500
\end{aligned}
$$

Find the (time, height) pairs
$\rightarrow$ points of intersection.
$(t, h)$
$(3.5,500)$ and $(9.1,500)$
Time (sec) $=(3.5,9.1)$


An object is launched vertically upward from the ground at an initial velocity of 450 ft per second.

$$
h(t)=-16 t^{2}+V_{0} t+h_{0}
$$

For what periods of time is the object below 2500 feet? $h(t)=-16 t^{2}+450 t$

$$
h(t)=2500
$$

Find the (time, height) pairs
$\rightarrow$ points of intersection.
 $(7.6,2500)$ and $(20.1,2500)$
Find the time that it hits the ground.
$(28.1,0)$
Time $(\mathrm{sec})=[0,7.6) \cup(20.1,28.1)$

