Math-3A
Lesson 5-1
Solve Rational Equations

Solving Rational Equations

Method #1: eliminate the denominators one at a time.

Method #2: Obtain common denominators for each term.

What does solve a single variable equation mean?

$$3x + 2 = 11$$

Find the value of the variable that makes the equation “true.”

What is a factor?

A number that is being multiplied by another number.

What is a least common multiple?

The smallest number that both factors divide (evenly).

Method #1: eliminate the denominators one at a time.

$4 = \frac{x}{5}$ Multiply both sides by '5' $5 * 4 = \frac{x}{5} * \frac{5}{1}$ x = 20

$4 = \frac{x}{5} + \frac{3}{2}$ Multiply both sides by '5' $5 * 4 = 5 * \left(\frac{x}{5} + \frac{3}{2}\right)$

Distributive Property: multiplication of terms being added.

$20 = \frac{5x}{5} + \frac{5 * 3}{2}$ $20 = x + \frac{15}{2}$ Multiply both sides by '2'

$2 * 20 = 2 * \left(x + \frac{15}{2}\right)$ $40 = 2x + \frac{2 * 15}{2}$

$40 = 2x + 15$ $25 = 2x$ $\boxed{x = \frac{25}{2}}$

Method #1: eliminate the denominators one at a time.

$$2 = \frac{10}{x}$$

What is the excluded value? $x \neq 0$

$$x * 2 = \frac{10}{x} * \frac{x}{1}$$

Multiply both sides by 'x'

2x = 10

x = 5

$$2 = \frac{10}{x} + \frac{3}{5}$$

Multiply both sides by 'x'

$$x * 2 = x * \left(\frac{10}{x} + \frac{3}{5}\right)$$

$$2x = \frac{x * 10}{x} + \frac{3 * x}{5}$$

$$2x = 10 + \frac{3x}{5}$$

Multiply both sides by '5'

$$5 * 2x = 5 * \left(10 + \frac{3x}{5}\right)$$

$$10x = 50 + \frac{5 * 3x}{5}$$

$$10x = 50 + \frac{5 * 3x}{5}$$

$$10x = 50 + 3x$$

$$x = \frac{50}{7}$$

$$4 = \frac{12}{x-2}$$

What is the excluded value?

How do you get the 'x – 2' out of the denominator?

Multiply both sides by '______'

$$4(x-2) = \frac{12}{(x-2)} * \frac{(x-2)}{1}$$

4(x – 2) = 12 ÷4 ÷4

x – 2 = 3 +2 +2

x = 5

The solution to the equation does not equal the excluded value.

What would happen if the solution DID equal the excluded value?

We reject the solution. It is called an *extraneous solution.*

Find the excluded values and then solve the equations.

$$5 = \frac{20}{(x+1)}$$

$$4 = \frac{30}{(x-2)} - 6$$

Variable on both sides of the equation

$$\frac{3}{x}=\frac{2}{x+1}$$

Excluded values?

$$x\neq -1,0$$

Eliminate denominators one at a time.

$$\frac{x}{1}*\frac{3}{x}=\frac{2}{x+1}*\frac{x}{1}$$

Multiply both sides by 'x'

$$3=\frac{2x}{x+1}$$

Multiply both sides by (x+1)

$$(x+1)*3=\frac{2x}{(x+1)}*\frac{(x+1)}{1}$$

$$3x+3=2x$$
$$-3 \quad -3$$

$$3x=2x-3$$
$$-2x \quad -2x$$

$$\boxed{x=-3}$$

Find the excluded values then solve the equations:

$$3 = \frac{2x}{x+1}$$

$x \neq -1$

$x = -3$

$$\frac{3}{x+1} = \frac{9}{4x+5}$$

$x \neq -1,\ -\frac{5}{4}$

$x = -2$

Eliminate one Denomiator at a time.

$$(4x + 5) * \frac{3}{x+1} = \frac{9}{4x+5} * (4x + 5)$$

$$(x + 1) * \frac{3(4x+5)}{x+1} = 9 * (x + 1)$$

$$3(4x+5) = 9(x+1)$$

Distributive property

$$12x + 15 = 9x + 9$$

$$3x + 15 = 9$$

$$3x = -6$$

$$x = -2$$

$$3(4x+5) = 9(x+1)$$

Divide both sides by 3

$$4x + 5 = 3(x+1)$$

$$4x + 5 = 3x + 3$$

$$x + 5 = 3$$

$$\boxed{x = -2}$$

Method 2: Obtain a common denominator

$$\frac{(4x+5)*}{(4x+5)*}\ \frac{3}{x+1} = \frac{9}{4x+5}\ \frac{*(x+1)}{*(x+1)}$$

Multiply both sides by the common denominator

$$(4x+5)(x+1)\frac{3(4x+5)}{(4x+5)(x+1)} = \frac{9(x+1)}{(4x+5)(x+1)}(4x+5)(x+1)$$

Solve: $3(4x+5) = 9(x+1)$

$x = -2$

Identify the excluded value then solve.

$$\frac{9}{5} = \frac{4}{x+2}$$

$x \neq -2$

$x = \frac{2}{9}$

$$\frac{9}{5} = \frac{4}{x+6}$$

$x \neq -6$

$x = \frac{-34}{9}$

$$\frac{4}{x} + x = 5$$

$$x * \left(\frac{4}{x} + x\right) = 5 * x$$

$$4 + x^2 = 5x$$

Now what?

It's a quadratic,
solve the quadratic!

put into standard form !!!

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$\boxed{x = 4} \qquad x = 1$$

Rational equations with 2 solutions.

$$1+\frac{8}{x-5}=-\frac{9}{x}$$

Eliminate denominators one at at time.

$$\frac{(x-5)}{1}\left(1+\frac{8}{x-5}\right)=\left(-\frac{9}{x}\right)\frac{(x-5)}{1}$$

Multiply left/right by (x – 5)

Carefull: distributive property

$$\left(\frac{(x-5)}{1}*\frac{1}{1}\right)+\left(\frac{\cancel{(x-5)}}{1}*\frac{8}{\cancel{(x-5)}}\right)=\left(-\frac{9}{x}\right)\frac{(x-5)}{1}$$

$$\frac{(x-5)}{1}+\left(\frac{8}{1}\right)=\frac{-9(x-5)}{x}$$

(continued)

$$1+\frac{8}{x-5}=-\frac{9}{x}$$

$$\frac{(x-5)}{1}+\left(\frac{8}{1}\right)=\frac{-9(x-5)}{x}$$ simplify

$$x+3=\frac{-9(x-5)}{x}$$

Multiply left/right by x

Careful: distributive property

$$x(x+3)=\frac{-9(x-5)}{x}*\frac{x}{1}$$

$$x^2+3x=-9(x-5)$$ simplify

$$x^2+3x=-9x+45$$

$$x^2+12x-45=0$$ factor

$$(x+15)(x-3)=0$$

$$x=-15 \qquad x=3$$

Neither solution is an excluded value!

Extraneous Solution: a solution obtained algebraically that is not in the domain of the original equation.

$$\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$$

What are the excluded values

$$x \neq 0,3$$

Eliminate denominators one at at time.

Careful: distributive property

$$\left(\frac{x}{1}\right)\left(\frac{2}{x-3}+\frac{1}{x}\right)=\left(\frac{x-1}{x-3}\right)\left(\frac{x}{1}\right)$$

$$\frac{2x}{x-3}+1=\frac{x^2-x}{x-3}$$

$$\frac{2x}{x-3}+1=\frac{x^2-x}{x-3}$$

Eliminate denominators one at at time.

<u>Careful</u>: distributive property

$$\left(\frac{\cancel{x-3}}{1}\right)\left(\frac{2x}{\cancel{x-3}}+1\right)=\left(\frac{x^2-x}{\cancel{x-3}}\right)\left(\frac{\cancel{x-3}}{1}\right)$$

$$2x+(x-3)=x^2-x$$

$$3x-3=x^2-x$$

$$x^2-4x+3=0$$

Vocabulary

Extranious Solution: a solution obtained algebraically that is an excluded value of the original equation.

$$\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$$

$$\frac{2}{1-3}+\frac{1}{1}=\frac{1-1}{1-3}$$

$$x^2-4x+3=0$$

$$\frac{2}{-2}+\frac{1}{1}=\frac{0}{-2}$$

$$(x-3)(x-1)=0$$

$$-1+1=0$$

$$x=1,3$$

Are both x = 1, 3 in the domain of the original equation?

Only x = 1 is in the domain.

Solve. Check to see if the solution is extraneous.

$$\frac{3x+6}{x^2-4}=\frac{x+1}{x-2}$$

$x = 2, -2$

both are excluded values: "no solution"

$$\frac{x}{2}+3=\frac{3}{4x}$$

$x = -3 \pm \sqrt{21}$

neither are excluded values

$$\frac{1}{2x} = \frac{1}{6} + \frac{x^2 - x - 12}{6x^2}$$

$$\boxed{x = 1 \pm \sqrt{7}}$$

neither are excluded values

$$\frac{1}{x^2} = \frac{1}{4x^2} - \frac{x+3}{4x^2}$$

$$\boxed{x = -6}$$

Solution is not an excluded value