Math-3A Lesson 5-1 Solve Rational Equations

Solving Rational Equations

Method #1: eliminate the denominators one at a time.

Method #2: Obtain common denominators for each term.

What does <u>solve</u> a single variable equation mean? 3x + 2 = 11

Find the value of the variable that makes the equation "true." What is a <u>factor?</u>

A number that is being multiplied by another number.

What is a least common multiple?

The smallest number that both factors divide (evenly).

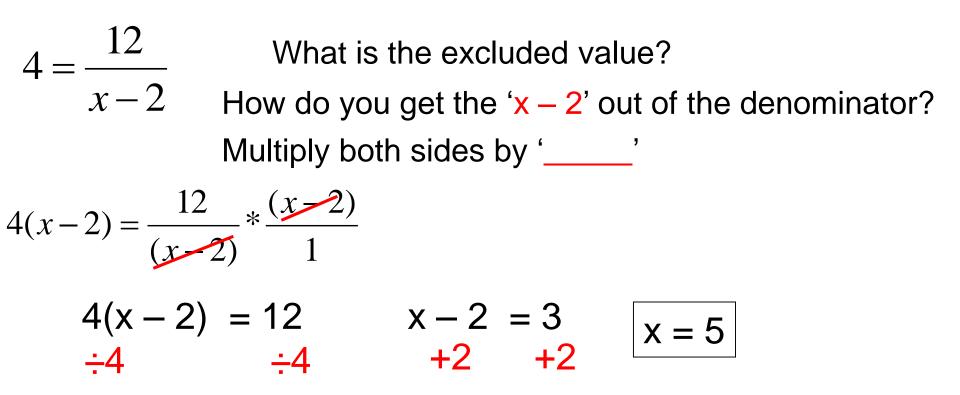
<u>Method #1:</u> eliminate the denominators one at a time.

$$4 = \frac{x}{5}$$
 Multiply both sides by '5'
$$5*4 = \frac{x}{5}*\frac{5}{1}$$
 $x = 20$
$$4 = \frac{x}{5} + \frac{3}{2}$$
 Multiply both sides by '5'
$$5*4 = 5*\left(\frac{x}{5} + \frac{3}{2}\right)$$

<u>Distributive Property</u>: multiplication of terms being added.

$$20 = \frac{5x}{5} + \frac{5*3}{2} \qquad 20 = x + \frac{15}{2} \qquad \text{Multiply both sides by '2'}$$
$$2 * 20 = 2 * \left(x + \frac{15}{2}\right) \qquad 40 = 2x + \frac{2*15}{2}$$
$$40 = 2x + 15 \qquad 25 = 2x \qquad x = \frac{25}{2}$$

Method #1: eliminate the denominators one at a time. What is the excluded value? $x \neq 0$ \boldsymbol{X} $x * 2 = \frac{10}{x} * \frac{x}{1}$ Multiply both sides by 'x' 2x = 10X = 5 $5 * 2x = 5 * \left(\frac{10 + \frac{3x}{5}}{5} \right)$ $10x = 50 + \frac{5 * 3x}{5}$ $2 = \frac{10}{x} + \frac{3}{5}$ Multiply both sides by 'x' $\boldsymbol{x} \ast 2 = \boldsymbol{x} \ast \left(\frac{10}{x} + \frac{3}{5}\right)$ $10x = 50 + \frac{5 \cdot 3x}{5}$ 10x = 50 + 3x $2x = \frac{x * 10}{x} + \frac{3 * x}{5}$ $2x = 10 + \frac{3x}{5}$ Multiply both sides by '5' $\left|x=\frac{50}{7}\right|$



The solution to the equation does not equal the excluded value.

What would happen if the solution <u>DID equal</u> the excluded value?

We reject the solution. It is called an extraneous solution.

Find the excluded values and then solve the equations.

$$5 = \frac{20}{(x+1)}$$

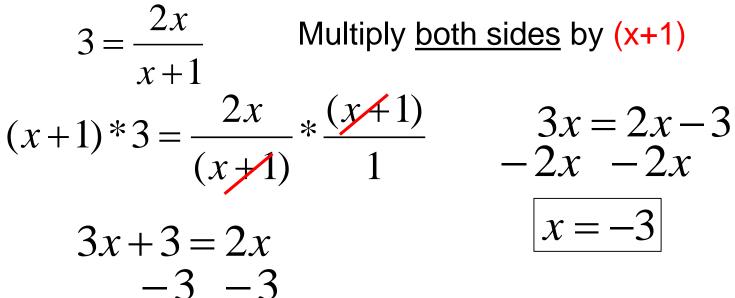
$$4 = \frac{30}{(x-2)} - 6$$

Variable on both sides of the equation

$$\frac{3}{x} = \frac{2}{x+1}$$
 Excluded values? $x \neq -1,0$

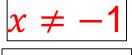
Eliminate denominators one at a time.



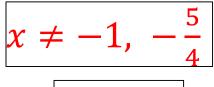


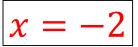
Find the excluded values then solve the equations:

$$3 = \frac{2x}{x+1}$$



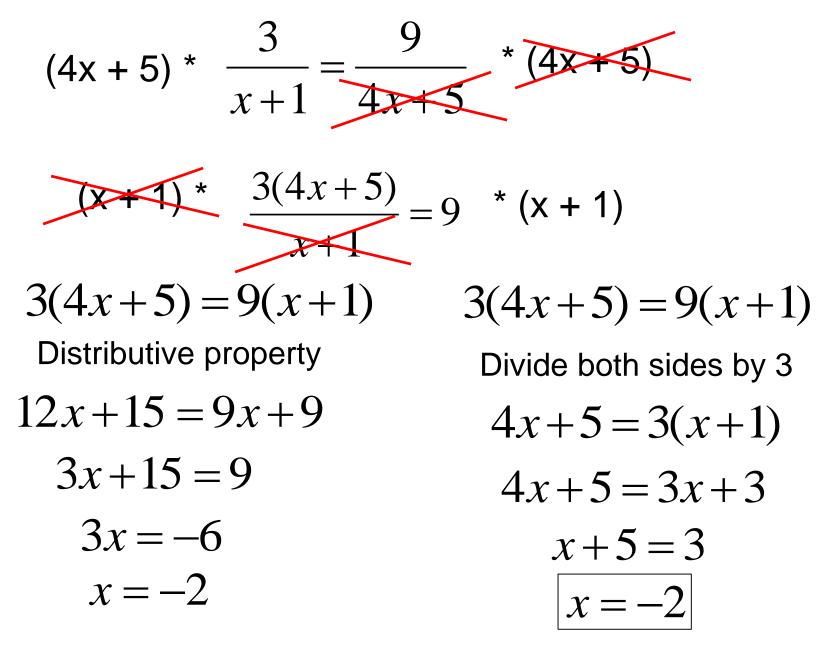






 $\frac{3}{x+1} = \frac{9}{4x+5}$

Eliminate one Denomiator at a time.



Method 2: Obtain a common denominator

$$\begin{array}{c} (4x+5)^{*} & \frac{3}{x+1} = \frac{9}{4x+5} & ^{*}(x+1) \\ \end{array} \\ \end{array}$$

Multiply both sides by the common denominator

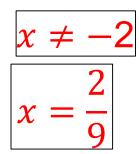
$$(4x+5)(x+1) \quad 3(4x+5) = \frac{9(x+1)(4x+5)(x+1)}{(4x+5)(x+1)} = \frac{9(x+1)(4x+5)(x+1)}{(4x+5)(x+1)}$$

Solve:
$$3(4x+5) = 9(x+1)$$

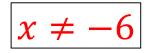
$$x = -2$$

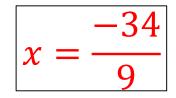
Identify the excluded value then solve.

$$\frac{9}{5} = \frac{4}{x+2}$$



$\frac{9}{5} = \frac{4}{x+6}$





$$\frac{4}{x} + x = 5 \qquad \mathbf{x}^* \left(\frac{4}{x} + x\right) = 5^* \mathbf{x}$$
$$4 + x^2 = 5x \qquad \text{Now what?} \qquad \text{It's a que$$

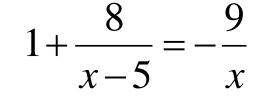
It's a quadratic, solve the quadratic!

put into standard form !!!

$$x^{2} - 5x + 4 = 0$$

(x-4)(x-1) = 0
x = 4 x = 1

Rational equations with 2 solutions.



 $1 + \frac{8}{x-5} = -\frac{9}{x}$ Eliminate denominators one at at time.

$$\frac{(x-5)}{1}\left(1+\frac{8}{x-5}\right) = \left(-\frac{9}{x}\right)\frac{(x-5)}{1}$$
 Multiply left/right by $(x-5)$
$$\left(\frac{(x-5)}{1}*\frac{1}{1}\right) + \left(\frac{(x-5)}{1}*\frac{8}{(x-5)}\right) = \left(-\frac{9}{x}\right)\frac{(x-5)}{1}$$
$$\frac{(x-5)}{1} + \left(\frac{8}{1}\right) = \frac{-9(x-5)}{x}$$

(continued)

 $x+3 = \frac{-9(x-5)}{x}$

$$\frac{(x-5)}{1} + \left(\frac{8}{1}\right) = \frac{-9(x-5)}{x} \qquad \text{simplify}$$

$$1 + \frac{8}{x-5} = -\frac{9}{x}$$

$$x(x+3) = \frac{-9(x-5)}{x} * \frac{x}{1}$$

$$x^2 + 3x = -9(x-5)$$
 simplify

$$x^2 + 3x = -9x + 45$$

 $x^2 + 12x - 45 = 0$ factor

$$(x+15)(x-3) = 0$$

 $x = -15$ $x = 3$

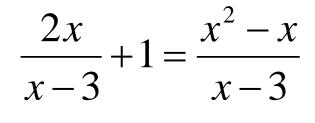
Neither solution is an excluded value! Extraneous Solution: a solution obtained algebraically that is not in the domain of the original equation.

$$\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$$
 What are the excluded values
 $x \neq 0,3$

Eliminate denominators one at at time. Careful: distributive property

$$\left(\frac{x}{1}\right)\left(\frac{2}{x-3} + \frac{1}{x}\right) = \left(\frac{x-1}{x-3}\right)\left(\frac{x}{1}\right)$$

$$\frac{2x}{x-3} + 1 = \frac{x^2 - x}{x-3}$$



 $\frac{2x}{x-3} + 1 = \frac{x^2 - x}{x-3}$ Eliminate denominators one at at time. Careful: distributive property

$$\left(\frac{x-3}{1}\right)\left(\frac{2x}{x-3}+1\right) = \left(\frac{x^2-x}{x-3}\right)\left(\frac{x-3}{1}\right)$$

$$2x + (x-3) = x^2 - x$$

$$3x - 3 = x^2 - x$$

 $x^2 - 4x + 3 = 0$

Vocabulary

Extranious Solution: a solution obtained algebraically that is an excluded value of the original equation.

$$\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3} \qquad \qquad \frac{2}{1-3} + \frac{1}{1} = \frac{1-1}{1-3}$$
$$x^2 - 4x + 3 = 0 \qquad \qquad \frac{2}{-2} + \frac{1}{1} = \frac{0}{-2}$$
$$(x-3)(x-1) = 0 \qquad \qquad -1+1=0$$

Are <u>both</u> x = 1, 3 in the domain of the original equation?

<u>Only</u> x = 1 is in the domain.

Solve. Check to see if the solution is extraneous.

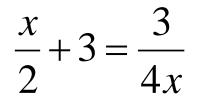
$$\frac{3x+6}{x^2-4} = \frac{x+1}{x-2}$$

x = 2, -2

both are excluded values: "no solution"

$$x = -3 \pm \sqrt{21}$$

neither are excluded values



$$\frac{1}{2x} = \frac{1}{6} + \frac{x^2 - x - 12}{6x^2}$$

$$x = 1 \pm \sqrt{7}$$

<u>neither</u> are excluded values

