Math-3A
Lesson 12-8
Solve Rational Equations

What does <u>solve</u> a single variable equation mean?

$$3x + 2 = 11$$

Find the value of the variable that makes the equation "true."

What is a <u>factor?</u>

A number that is being multiplied by another number.

What is a <u>least common multiple</u>?

The <u>smallest</u> number that both factors divide (evenly).

<u>Solving Rational Equations</u>

Method #1: eliminate the denominators one at a time.

Method #2: Obtain common denominators for each term and then eliminate all the denominators in one step by multiplying by the common denominator.

Method #3: Determine what the common denominator will be and multiply by that number.

<u>Method #1</u>: eliminate the denominators one at a time.

$$2 = \frac{10}{x} + \frac{3}{5} \quad \text{Multiply both sides by 'x'}$$

$$x * 2 = x * \left(\frac{10}{x} + \frac{3}{5} \right) \quad \text{Careful! (Distributive Property)}$$

$$2x = \frac{\cancel{x} * 10}{\cancel{x}} + \frac{3 * x}{5}$$

$$2x = 10 + \frac{3x}{5} \quad \text{Multiply both sides by '5'}$$

$$5 * x = \left(10 + \frac{3x}{5} \right) * 5$$

$$10x = 50 + \frac{\cancel{5} * 3x}{\cancel{5}}$$

$$10x = 50 + 3x$$

$$\boxed{x = \frac{50}{7}}$$

Method #2: Obtain a common denominator (then multiply by the common denominator.

$$2 = \frac{10}{x} + \frac{3}{5}$$ Multiply by "one in the form of…"

$$\frac{5x}{5x} * 2 = \frac{5}{5} * \frac{10}{x} + \frac{3}{5} \frac{x}{* x}$$

$$\frac{10x}{5x} = \frac{50}{5x} + \frac{3x}{5x}$$ Multiply by the common denominator

$$10x = 50 + 3x$$ Solve for 'x'

$$7x = 50 \qquad \boxed{x = \frac{50}{7}}$$

Method #3: multiply by the common denominator

$$2 = \frac{10}{x} + \frac{3}{5}$$ The common denominator would be: __5x__

$$2 * 5x = \left(\frac{10}{x} + \frac{3}{5}\right) * 5x$$ Multiply by the common denominator

$$2 * 5x = \frac{10}{x} * 5x + \frac{3}{5} * 5x$$ Careful! (Distributive Property)

$$10x = 50 + 3x$$ Solve for 'x'

$$7x = 50 \qquad \boxed{x = \frac{50}{7}}$$

Rational equations with 2 solutions.

$$1 + \frac{8}{x-5} = -\frac{9}{x}$$ Multiply by the common denominator

$$x(x-5) * \left(1 + \frac{8}{x-5}\right) = -\frac{9}{x} * x(x-5)$$

Careful! (Distributive Property)

$$x(x-5) + 8x = -9(x-5)$$ Simplify left side/right side

$$x^2 - 5x + 8x = -9x + 45$$ Non-standard quadratic

$$x^2 + 4x - 45 = 0$$ Standard form quadratic

$$(x+9)(x-5) = 0$$ Solve for 'x'

(1) factor

$$x = -9, 5$$ (2) Convert to vertex form
(3) Quadratic formula

Extraneous solution?

Extraneous Solution: a solution obtained algebraically that is not in the domain of the original equation.

$$\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$$ What are the excluded values

$$x \neq 0, 3$$

Multiply by the common denominator
Careful: distributive property

$$x(x-3) * \left(\frac{2}{x-3} + \frac{1}{x}\right) = \frac{(x-1)}{(x-3)} * x(x-3)$$

$$2x + (x-3) = x(x-1)$$ Simplify left side/right side

$$3x - 3 = x^2 - x$$ Non-standard quadratic

$$0 = x^2 - 4x + 3$$ Standard form quadratic

$$0 = (x-3)(x-1) \qquad x = 3, 1$$ Extraneous solution?

Solve $\dfrac{1}{2x} = \dfrac{1}{6} + \dfrac{x^2 - x - 12}{6x^2}$

$$6x^2 * \left(\dfrac{1}{2x}\right) = \left(\dfrac{1}{6} + \dfrac{x^2 - x - 12}{6x^2}\right) * 6x^2$$

$3x = x^2 + (x^2 - x - 12)$ Simplify left side/right side

$3x = 2x^2 - x - 12$ Non-standard quadratic

$0 = 2x^2 - 4x - 12$ Divide by 2

$0 = x^2 - 2x - 6$ Non-factorable

(1) Quadratic formula

$0 = (x - 1)^2 - 7$ (2) Convert to vertex form

$\boxed{x = 1 \pm \sqrt{7}}$ neither are excluded values