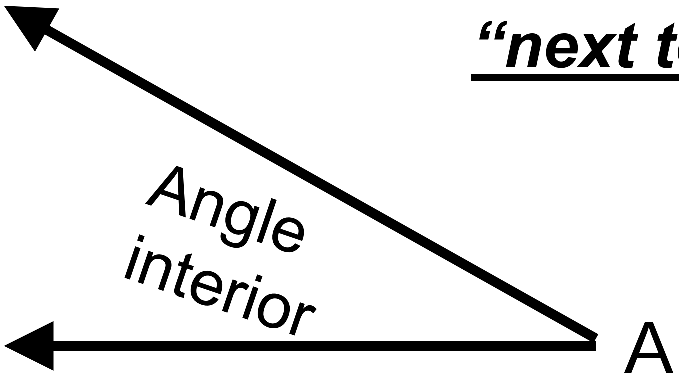
Math-2
Lesson 7-6

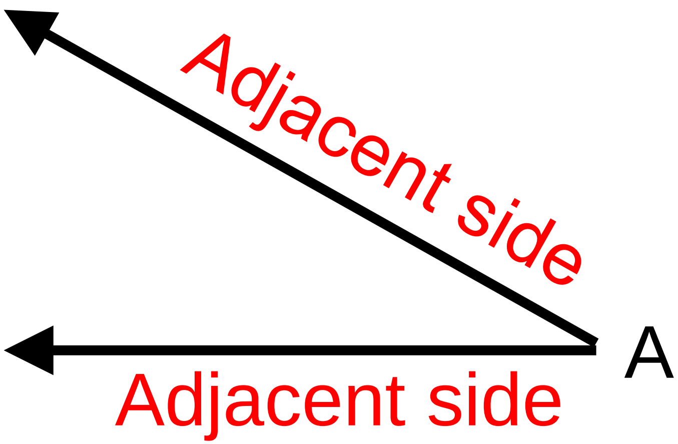
Solving Right Triangles Using Trigonometric Ratios

Angle: two rays with a common end point.

We can say the sides of the angle are ***"next to"*** the interior of the angle.

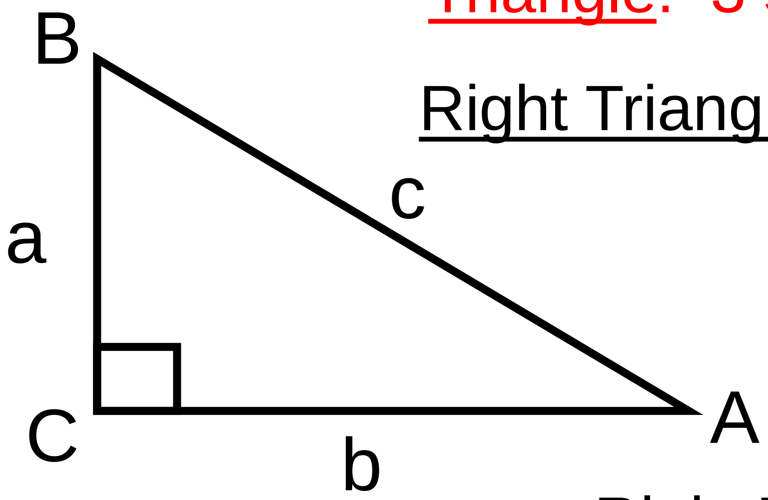


Another way to say ***"next to"*** is "ADJACENT".

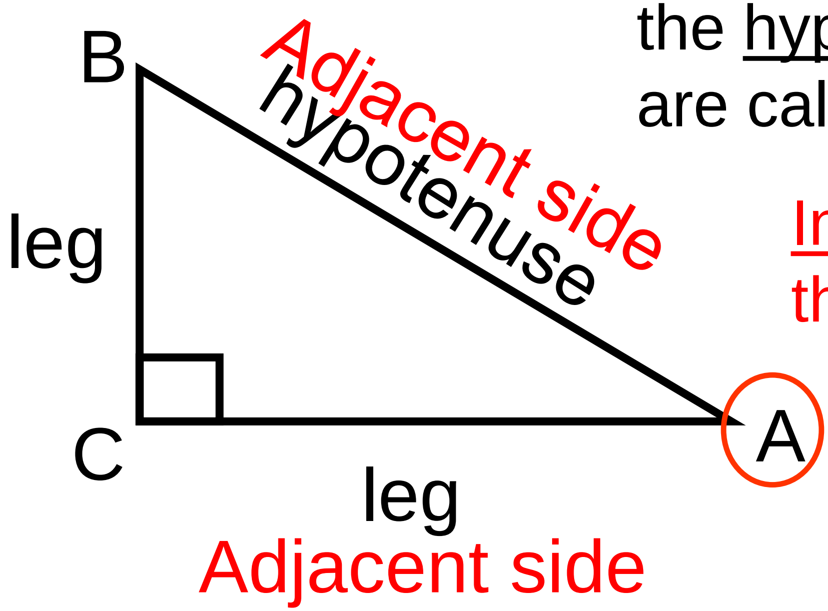


Triangle: 3 sides & 3 angles

Right Triangle: one angle measures 90º



Right Triangle: the longest side is called the hypotenuse and the other two sides are called legs.



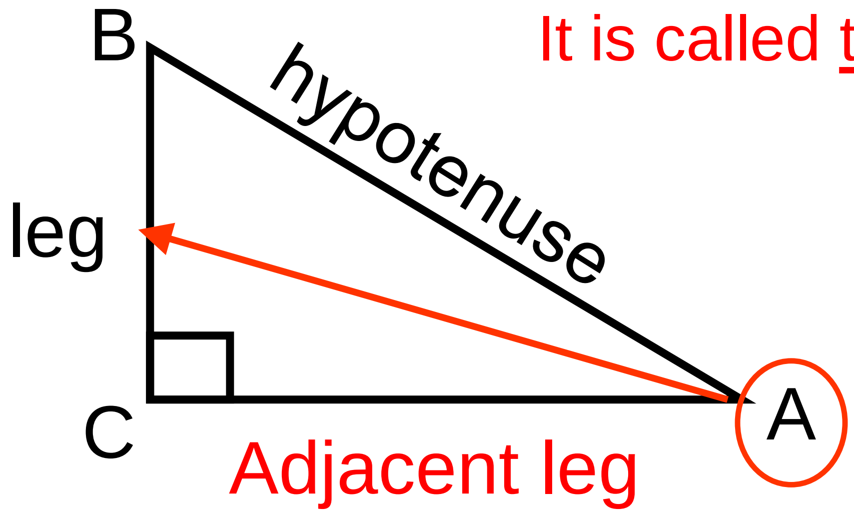
In a right triangle we do not refer to the hypotenuse as an adjacent side.

One adjacent side of Angle A will be the hypotenuse and the other side will be a leg.

Using Angle A as the "***reference***" angle, we need a name for the other leg of the right triangle.

It is called the ***opposite leg.***

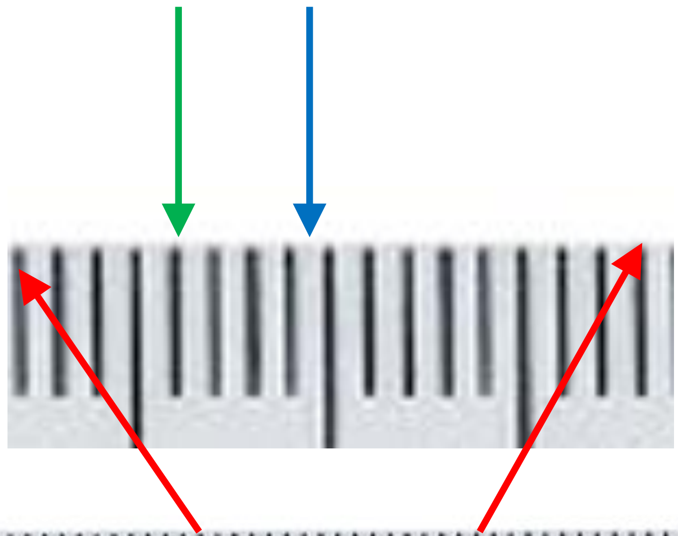
opposite leg.



Accuracy of measurements.

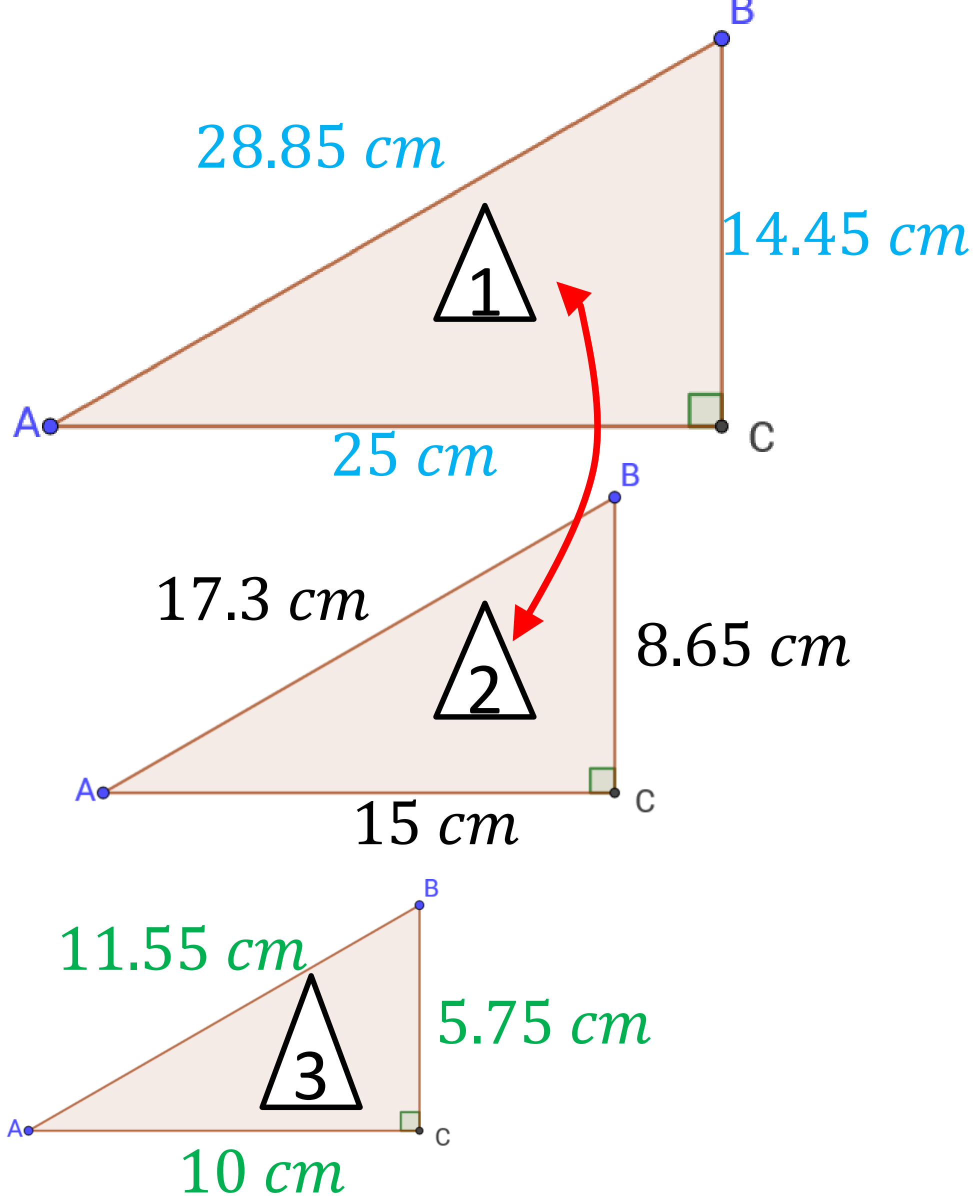
The smallest distance between each “tic mark” on the ruler is 0.1 cm (or 1 mm).

A length either falls
on a tic mark, or between a tic mark.



The limit of our ability to measure accurately is halfway between each tic mark or 0.05 cm (0.5 mm).

When proving Triangle Similarity, we compared the lengths of corresponding sides between two different triangles.



$$\frac{BC_1}{BC_2} = \frac{14.45}{8.65} = 1.67$$

$$\frac{AC_1}{AC_2} = \frac{25}{15} = 1.67$$

$$\frac{AB_1}{AB_2} = \frac{28.85}{17.3} = 1.67$$

$$Scale\ Factor_{\Delta 2 \rightarrow \Delta 1} = 1.67$$

$\Delta 1 \sim \Delta 2$

By SSS

Now we “shift gears” to learn a completely new concept which is
GIGANTICALY important in mathematics.

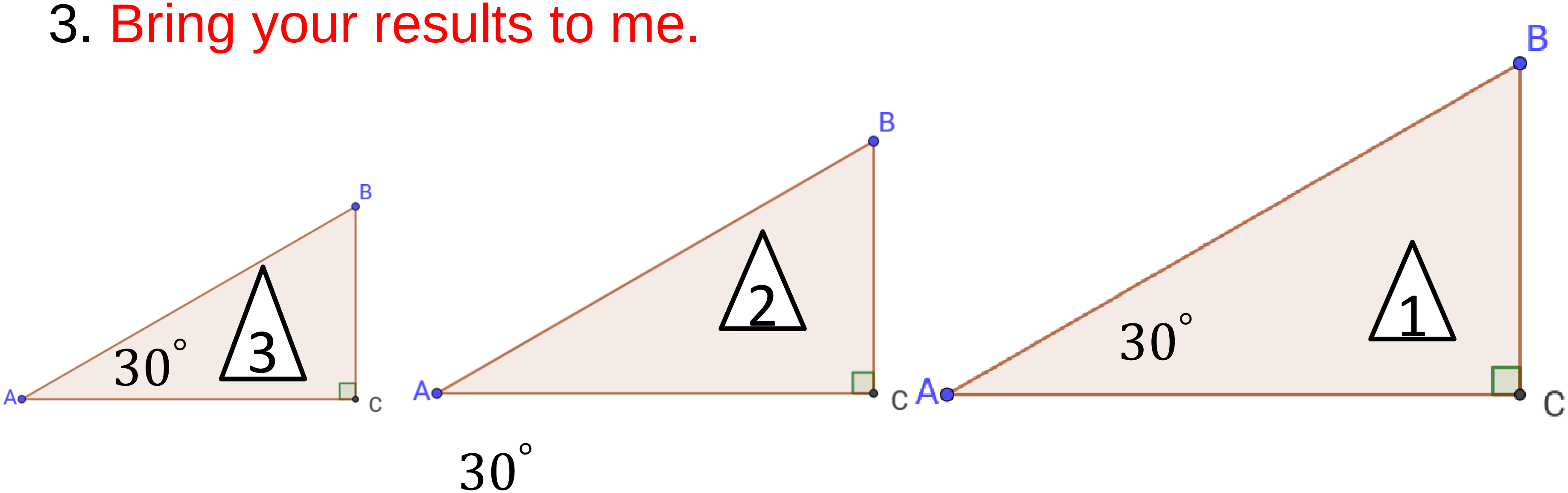
The entire basis of this idea comes from the measures of sides and angles of similar right triangles.

Group Activity: Each of you has a 30-60-90 triangle. There are three different sizes of triangles. All three triangles are similar by the AA Similarity Theorem.

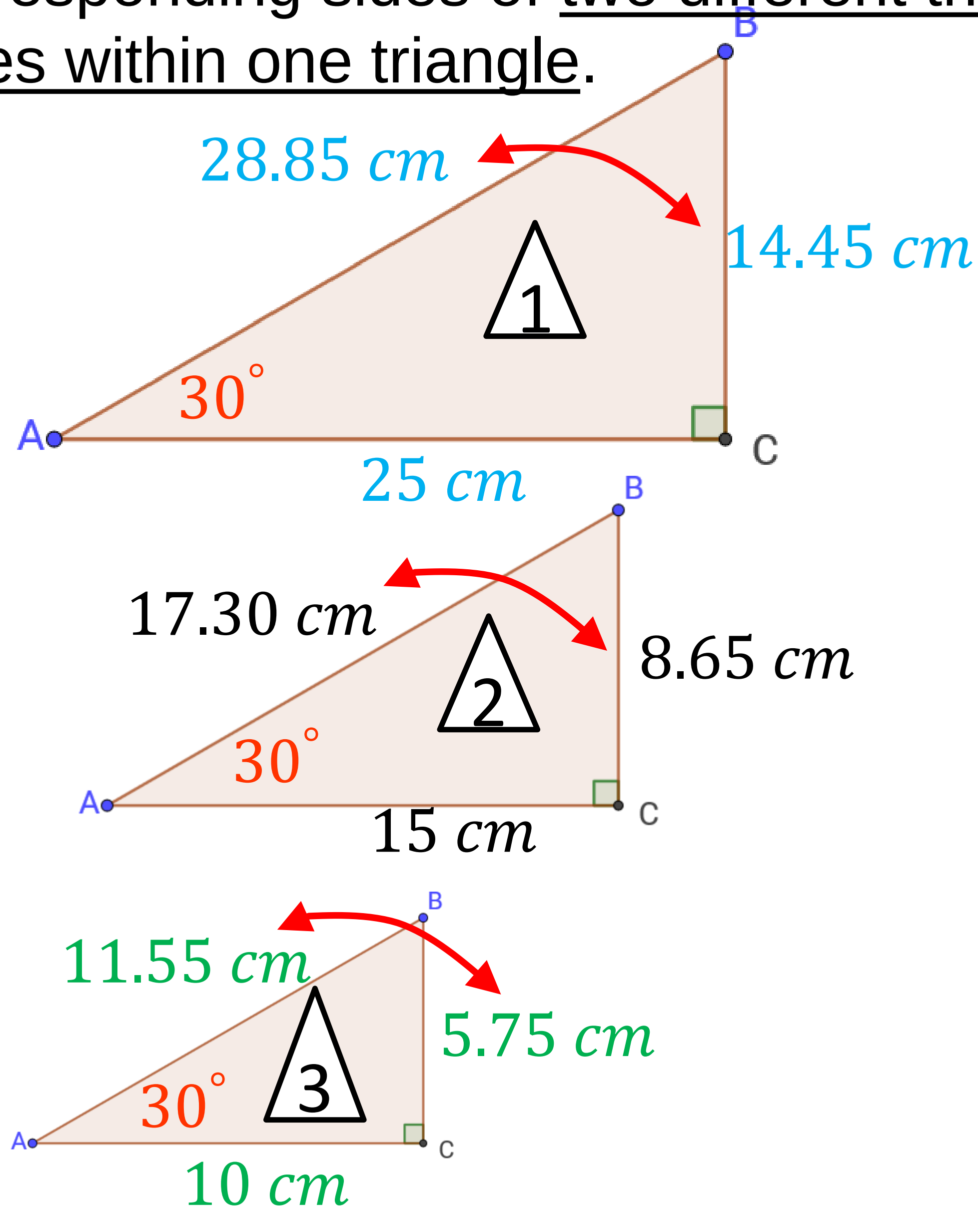
1. To the nearest 0.1 centimeter, measure lengths: BC, AC, and AB (write these lengths on your triangles.

2. Calculate and write the value of the ratios (in order), BC/AB, AC/AB, and BC/AC.

3. Bring your results to me.



Getting back to our Activity: instead of making ratios out of corresponding sides of two different triangles we made ratios of sides within one triangle.



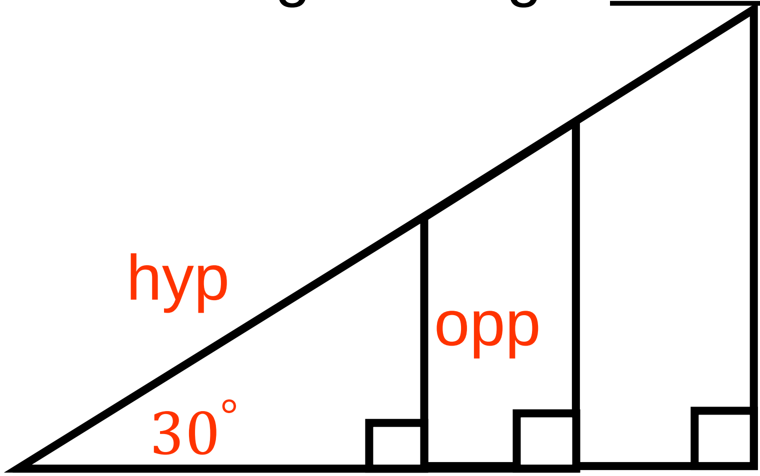
$$\frac{BC_1}{AB_1} = \frac{14.45}{28.85} = 0.50$$

We compare this to the same ratio of sides of the other triangles.

$$\frac{BC_2}{AB_2} = \frac{8.65}{17.30} = 0.50$$

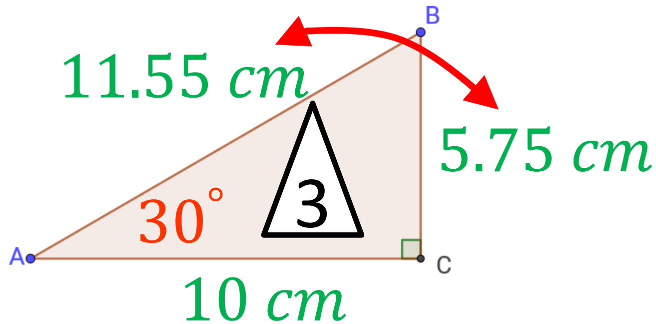
$$\frac{BC_3}{AB_3} = \frac{5.75}{11.55} = 0.50$$

Conclusion: the size of the triangle DOES NOT MATTER—the ratio of the opposite side of a 30º angle to the hypotenuse of a 30-60-90 right triangle will always be the same number.



The $\dfrac{opposite}{hypotenuse}$ ratio is called the sine ratio.

$$\frac{BC_3}{AB_3} = \frac{opp_{30}}{hyp_{30}} = \frac{5.75}{11.55} = 0.50$$



Another way to say $\dfrac{opp_{30}}{hyp_{30}}$

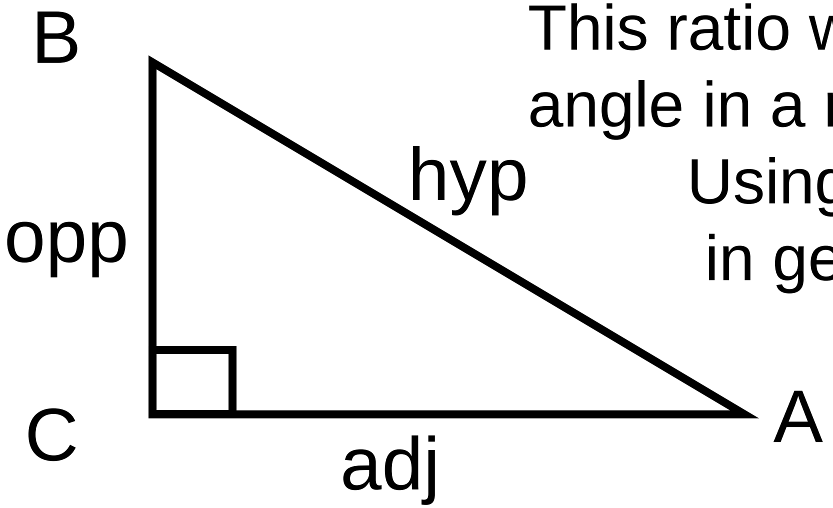
Is $\sin 30 = \dfrac{opp_{30}}{hyp_{30}}$

“Ratios” are decimal form (not in fraction form).

These “Ratios” are unique numbers for each angle; they are ***Properties of the angle.***

Angle	$\frac{opp}{hyp}$
10°	0.1736
20°	0.3420
30°	0.5
43.9°	0.6934
60°	0.8660

Radian	Degree	Sine	Cosine	Tan
0.000	0	0.000	1.000	0.0
0.017	1	0.017	1.000	0.0
0.035	2	0.035	0.999	0.0
0.052	3	0.052	0.999	0.0
0.070	4	0.070	0.998	0.0
0.087	5	0.087	0.996	0.0
0.105	6	0.105	0.995	0.1
0.122	7	0.122	0.993	0.1
0.140	8	0.139	0.990	0.1
0.157	9	0.156	0.988	0.1
0.175	10	0.174	0.985	0.1
0.192	11	0.191	0.982	0.1
0.209	12	0.208	0.978	0.2



This ratio will work for any measure of acute angle in a right triangle!

Using Angle A as our reference angle, in general we say:

$$\sin A = \frac{opp}{hyp}$$

This is an ***EQUATION.***

"The sine ratio (for reference angle A) is the length of the leg opposite of angle A divided by the length of the hypotenuse".

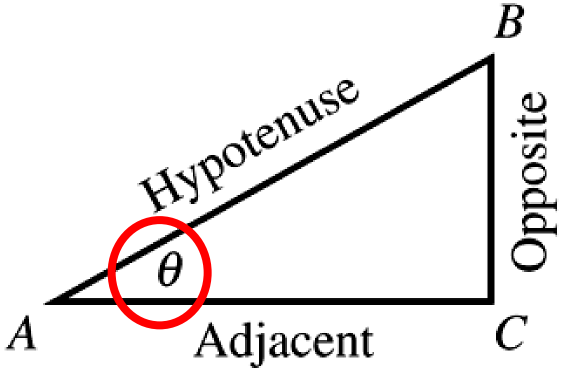
$$\cos A = \frac{adj}{hyp}$$

"The cosine ratio (for reference angle A) is the length of the leg adjacent to angle A divided by the length of the hypotenuse."

$$\tan A = \frac{opp}{adj}$$

"The tangent ratio (for reference angle A) is the length of the leg opposite to angle A divided by the length of the leg adjacent to angle A."

The ratio is a property of the angle. We <u>must know which of the two acute angles we are referring</u> to in order to find the correct ratio.



$$sin = \frac{opp}{hyp}$$

NO!!!

$$\sin\theta = \frac{opp}{hyp}$$

YES

SOH-CAH-TOA

$$\sin\theta = \frac{o}{h}$$

$$cos\,\theta = \frac{a}{h}$$

$$tan\,\theta = \frac{o}{a}$$

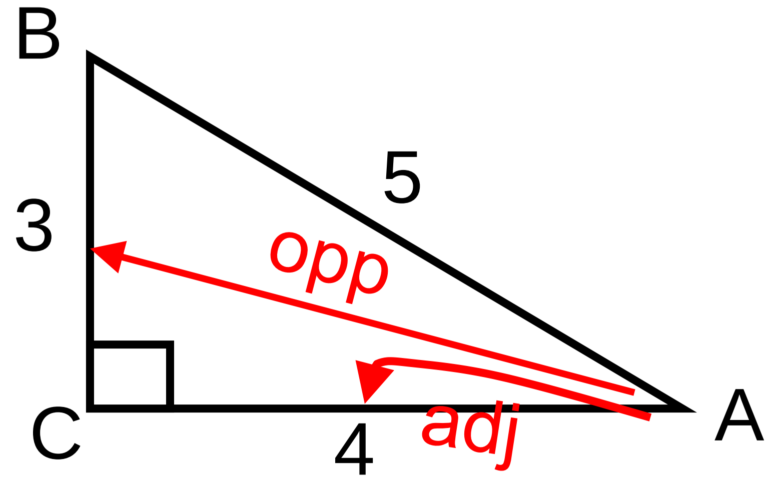
The <u>easy way to remember</u> what sides of the triangle to use in ratios.

These ratios only work for <u>right triangles</u>!!!

Sine Ratio

What is the sine ratio of angle A?

$$\sin A = \frac{opp}{hyp}$$



Sine ratio of angle A is $\frac{3}{5}$

$$\sin A = \frac{3}{5}$$

Cosine Ratio

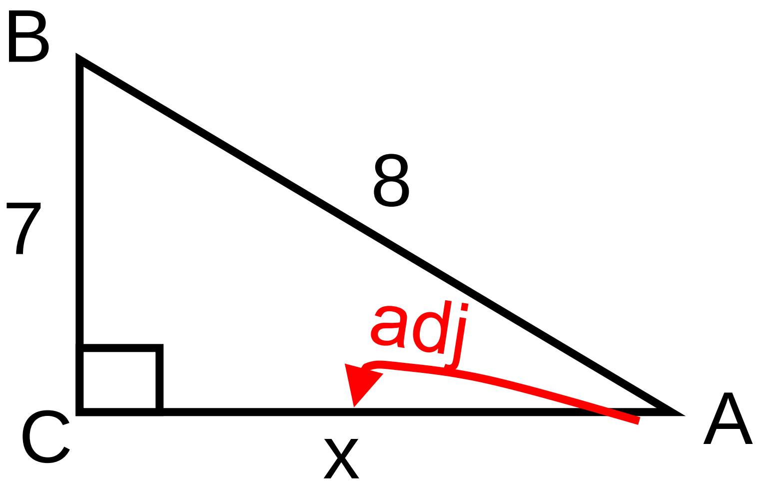
What is the cosine ratio for angle A?

$$\cos A = \frac{adj}{hyp}$$

Cosine ratio for angle A is $\frac{4}{5}$

$$\cos A = \frac{4}{5}$$

What is the cosine ratio of angle A?



$$\cos A = \frac{Adj.}{hyp.}$$

$$\cos A = \frac{x}{8}$$

How do we find the value represented by 'x'?

IF (right triangle) THEN

$$a^2 + b^2 = c^2$$

$$7^2 + x^2 = 8^2$$

$$x^2 = 8^2 - 7^2$$

$$x = \sqrt{64 - 49}$$

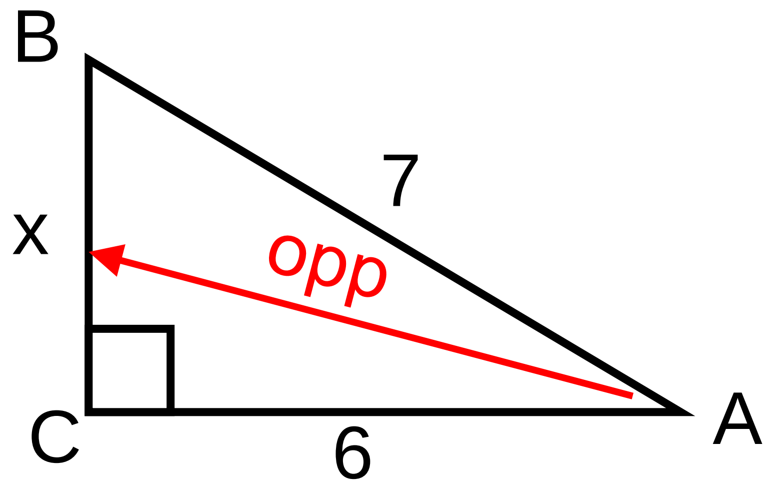
$$x = \sqrt{15}$$

$$\cos A = \frac{\sqrt{15}}{8}$$

What is the sine ratio of angle A?

$$\sin A = \frac{opp}{hyp}$$

$$sin\,A = \frac{x}{7}$$



How do we find the value represented by ‘x’?

<u>IF</u> (right triangle) <u>THEN</u>

$$a^2 + b^2 = c^2$$

$$x^2 + 6^2 = 7^2$$

$$x^2 = 7^2 - 6^2$$

$$x = \sqrt{49 - 36}$$

$$x = \sqrt{13}$$

$$\sin A = \frac{\sqrt{13}}{7}$$

Solve a triangle: to find the measure of the unknown angles and side lengths.

To find an unknown value you need an equation!

There are five equations that relate to right triangles.

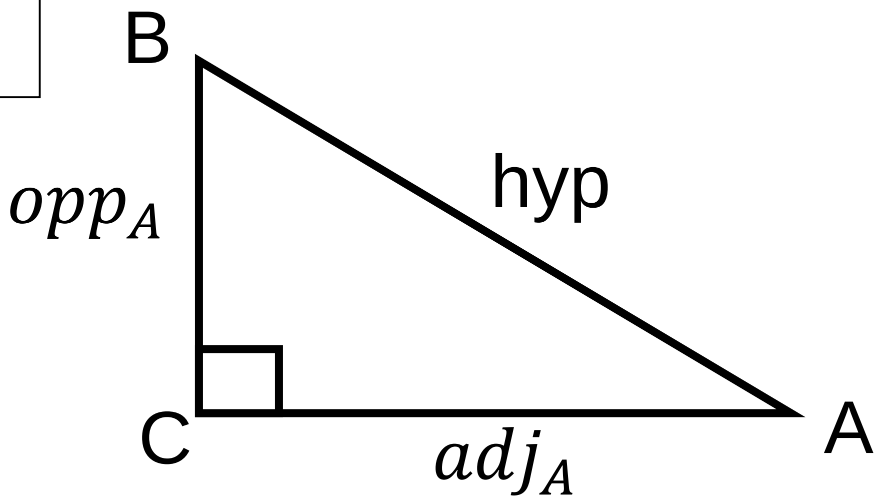
IF (right triangle) THEN

1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$

3) $\cos A = \frac{a}{h}$

4) $\tan A = \frac{o}{a}$



and IF (any triangle) THEN

5) $m\angle A + m\angle B + M\angle C = 180^0$

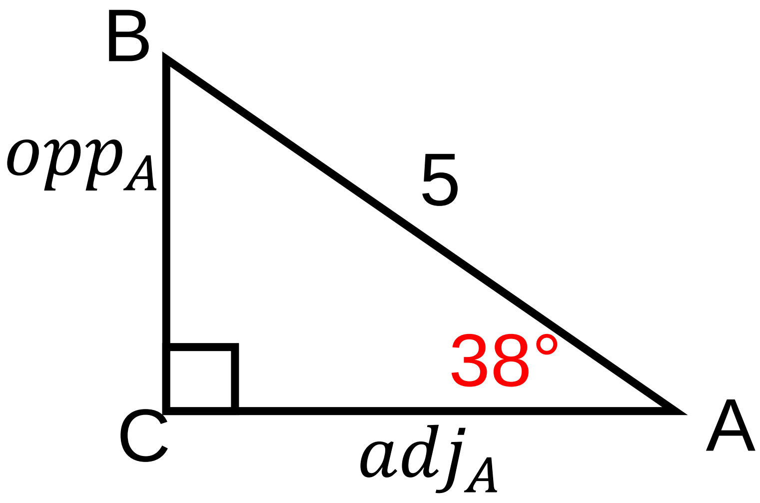
"Solve the triangle" (find the missing side and angle measures)

1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$

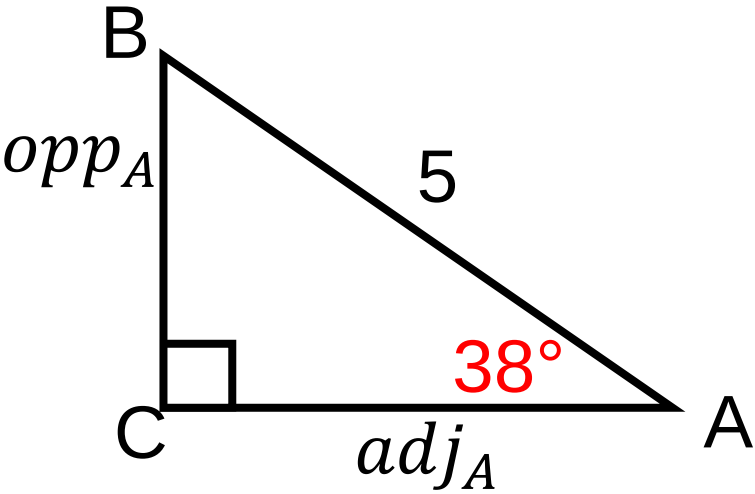
3) $\cos A = \frac{a}{h}$

4) $\tan A = \frac{o}{a}$



5) $m\angle A + m\angle B + M\angle C = 180^0$ → $m\angle B = 52^0$

<u>Solve the triangle</u>.



1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$ → $\sin 38^\circ = \frac{opp}{5}$

$0.616 = \frac{opp}{5}$

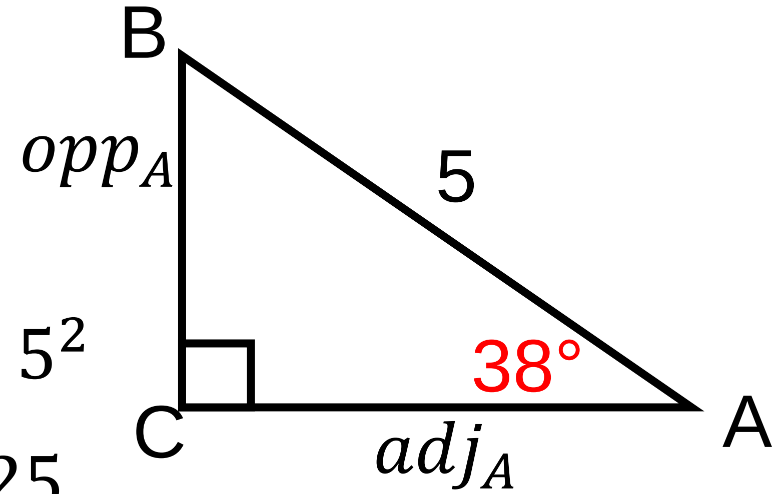
3) $\cos A = \frac{a}{h}$

$5(0.616) = \text{opp} = 3.08$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$

Solve the triangle.



1) $a^2 + b^2 = c^2$ →

$(adj)^2 + (3.08)^2 = 5^2$

$(adj)^2 + 9.49 = 25$

$(adj)^2 = 25 - 9.49$

$(adj)^2 = 15.51$

$adj = \sqrt{15.51} = 3.94$

2) $\sin A = \frac{o}{h}$

3) $\cos A = \frac{a}{h}$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$

Solve the triangle.

1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$

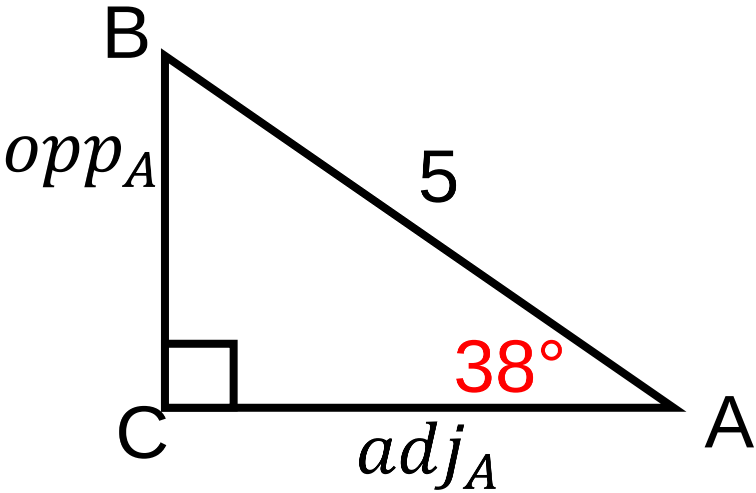
3) $\cos A = \frac{a}{h}$ → $\cos 38^\circ = \frac{adj}{5}$

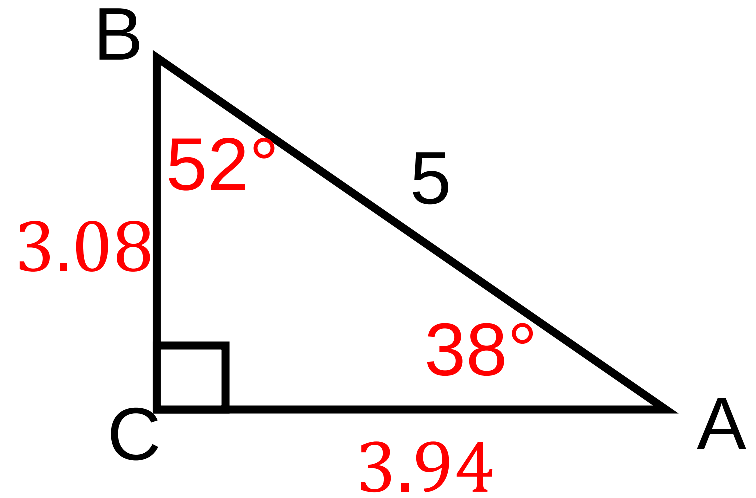
$0.788 = \frac{adj}{5}$

$5(0.788) = \text{adj}$ $= 3.94$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$





Usually, the problem only asks you to solve for 'x'

1) $a^2 + b^2 = c^2$

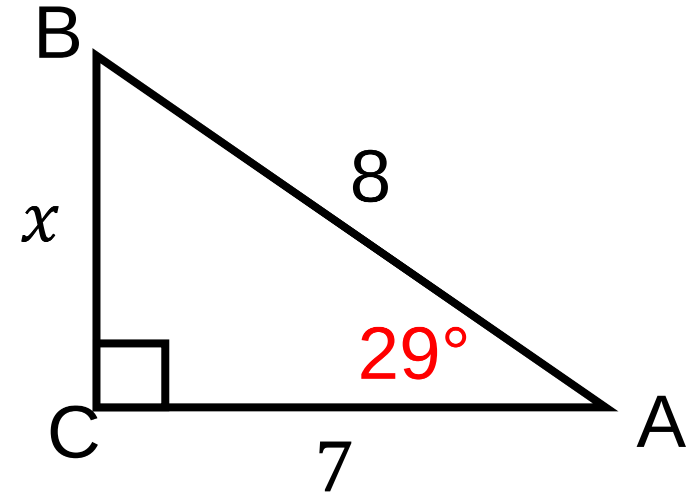
2) $\sin A = \frac{o}{h}$ → $\sin 29^{\circ} = \frac{x}{8}$

3) $\cos A = \frac{a}{h}$ $\quad 0.485 = \frac{x}{8}$

$8(0.485) = \text{opp} \quad = 3.9$

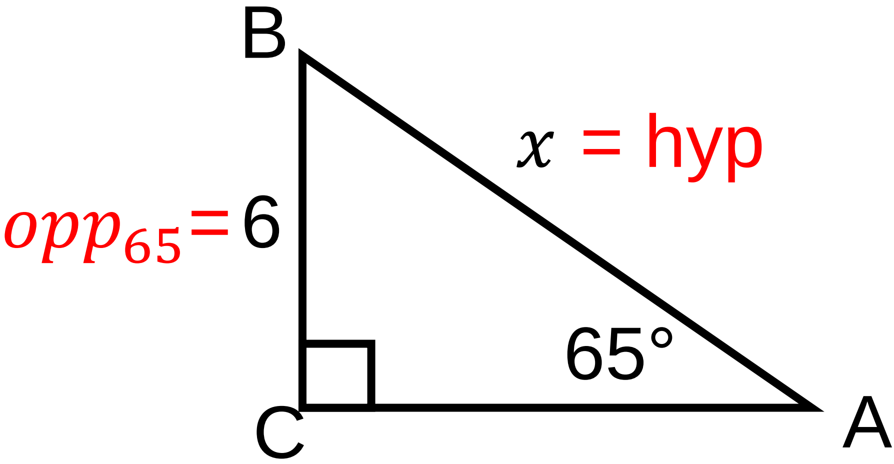
4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$



The Hardest problem

$\sin A = \frac{o}{h}$ $\sin 65^{\circ} = \frac{6}{x}$



$$0.906 = \frac{6}{x}$$

'x' is in the denominator!

$$x(0.906) = 6$$

"*undo*" division by 'x'

$$x = \frac{6}{0.906} = 6.6$$