Math-2A
Lesson 2-1

Number Systems

Lesson 1-1 Vocabulary

Natural Numbers

Whole Numbers

Integers

Rational Numbers

Irrational Numbers

Real Numbers

Imaginary Numbers

Complex Numbers

Closure

Why do we need numbers?

Lebombo Plain (Africa)

Lebombo counting sticks appeared about 35,000 years ago!

How can you write the number “zero” using a counting stick?

How can you write a negative number using a counting stick?

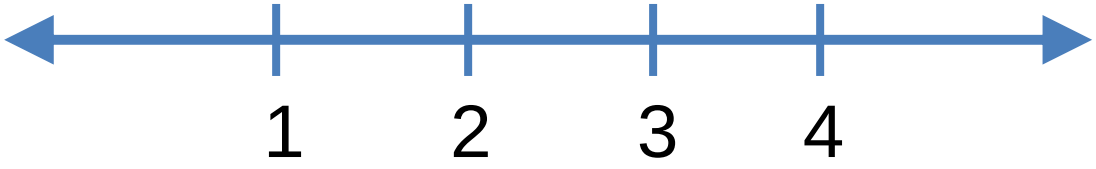
How do you count…?

1	I	6	𝍸 I
2	II	7	𝍸 II
3	III	8	𝍸 III
4	IIII	9	𝍸 IIII
5	𝍸	10	𝍸 𝍸

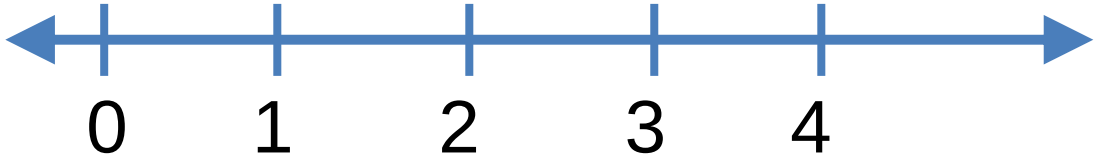
A few horses?

Vocabulary

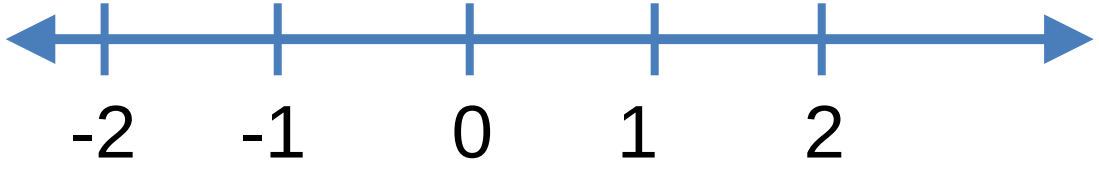
Natural numbers: the positive “counting” numbers that are usually shown on a number line.



Whole numbers: the natural numbers and the number zero.



Integers: the whole numbers and the negative “counting” numbers.



Can anyone interpret what the following means?

$$\text{Rational numbers} = \left\{ R : R = \frac{a}{b}; \text{a, b} \in \text{integers} \right\}$$

Vocabulary

Rational numbers: can be written as a ratio of integers: ½, -⅔, etc.

When converting a rational number into its decimal form (using division) the decimal with either “terminate” (1/2 = 0.5) or repeat (2/3 = 0.66666…).

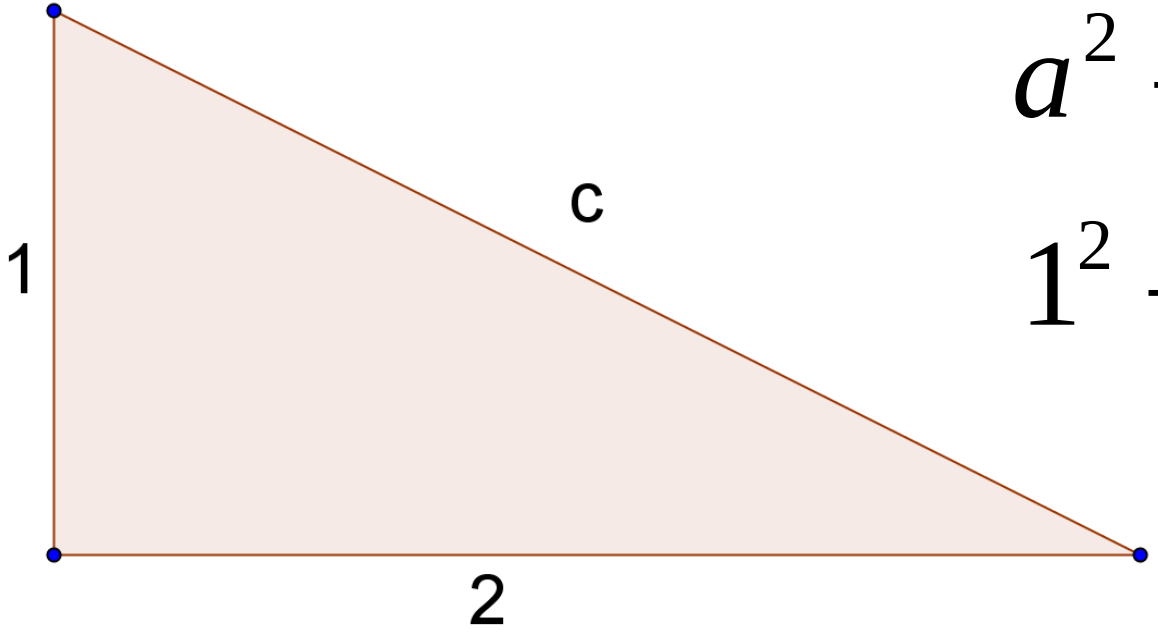
Write the integer -3 as a rational number.

Are these all the same thing? $\frac{-3}{1}, \frac{3}{-1}, -\frac{3}{1}, \frac{-3}{-1}$

Why is -3 not equal to $\frac{-3}{-1}$?

If the triangle below is a right triangle, how can we find length ‘c’ (the hypotenuse)?

Pythagorean Theorem: If it’s a right triangle, then side lengths can be related by:



$$a^2 + b^2 = c^2$$

$$1^2 + 2^2 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$

What numbers system does this number belong to?

Property of Equality: if the same operation is applied to both sides of an equal sign, then the resulting equation is still true (has the same solution).

$$x = \sqrt{4}$$

$$(x)^2 = \left(\sqrt{4}\right)^2$$ Square both sides of the equation

$$x^2 = 4$$ The same value of x makes both the 1st and last equation true (x = 2) .

$$x = 2$$

Vocabulary

Irrational numbers: cannot be written as a ratio of integers: ½, -⅔, etc.

The decimal version of an irrational number never terminates and never repeats. (0 = 5.13257306…).

If we see the radical symbol, the number is usually irrational (unless it is a "perfect square). $\sqrt{3}$

$$\sqrt{4} = 2\,(\text{rational } \#)$$

Identifying the type of number.

(1) $\frac{2}{3}$ (2) $\sqrt{7}$ (3) 5.25 (4) 26 (5) π

Natural
Whole
Integer
Rational
Irrational

Exact vs. Approximate:

Exact: $\pm\sqrt{17}$

Approximate: $\approx \pm 4.1231056\ldots.$

$\approx \pm 4.123106\ldots.$

$\approx \pm 4.12311\ldots.$

$\approx \pm 4.1231\ldots.$

$\approx \pm 4.123\ldots.$

$\approx \pm 4.12\ldots.$

$\approx \pm 4.1\ldots.$

Converting an irrational number into a decimal requires you to round off the decimal somewhere.

Irrational Numbers

The square root of 2 $x = \sqrt{2}$

really means,

"what number squared equals 2". $x^2 = 2$

Why do these both refer to the same number (that makes both equations true)?

Because of the Property of Equality.

$$\sqrt{-1}$$

The square root of -1: $x = \sqrt{-1}$

really means, "what number squared equals -1".

$$x^2 = -1$$

What real number when squared becomes a negative number ?

It doesn't exist so it must be an "imaginary number"

$$\sqrt{-3} = \sqrt{(-1)*3} = \sqrt{(-1)} * \sqrt{3}$$

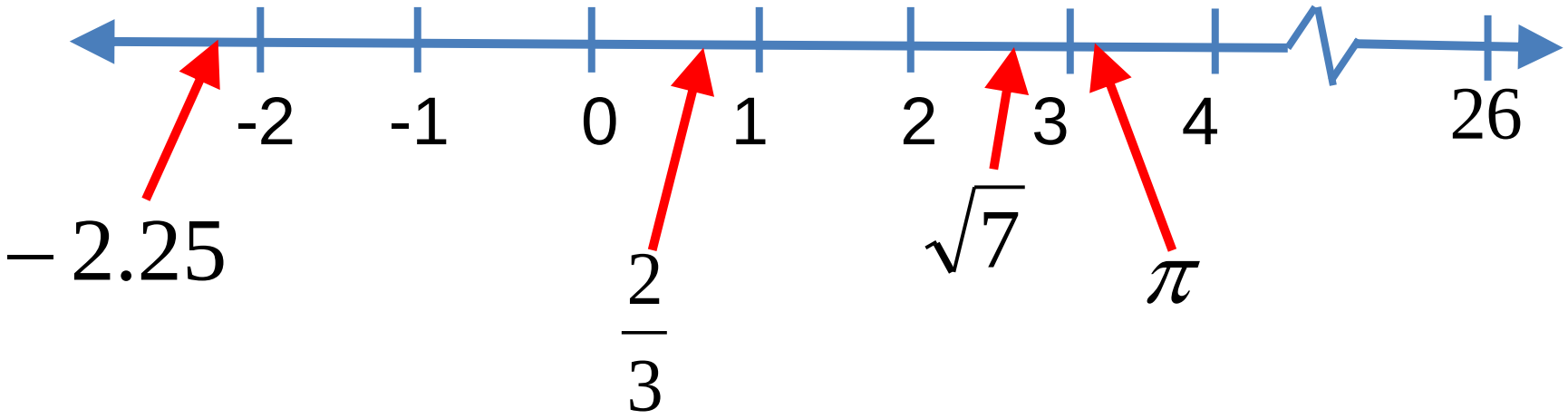
$$= i\sqrt{3}$$

Vocabulary

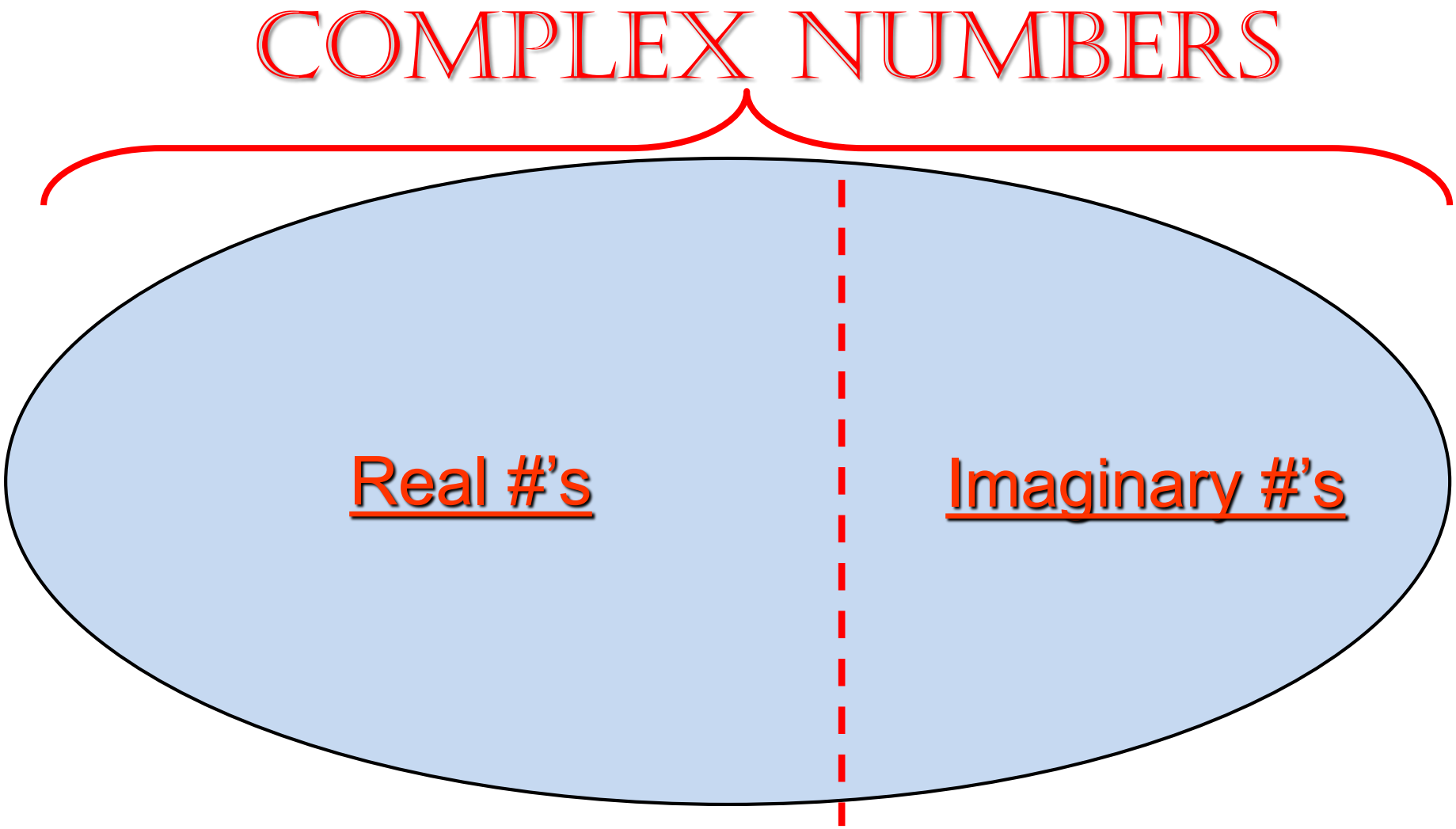
imaginary numbers: a number that includes the square root of a negative number.

$$\sqrt{-1} \qquad i\sqrt{3} \qquad \sqrt{-3}$$

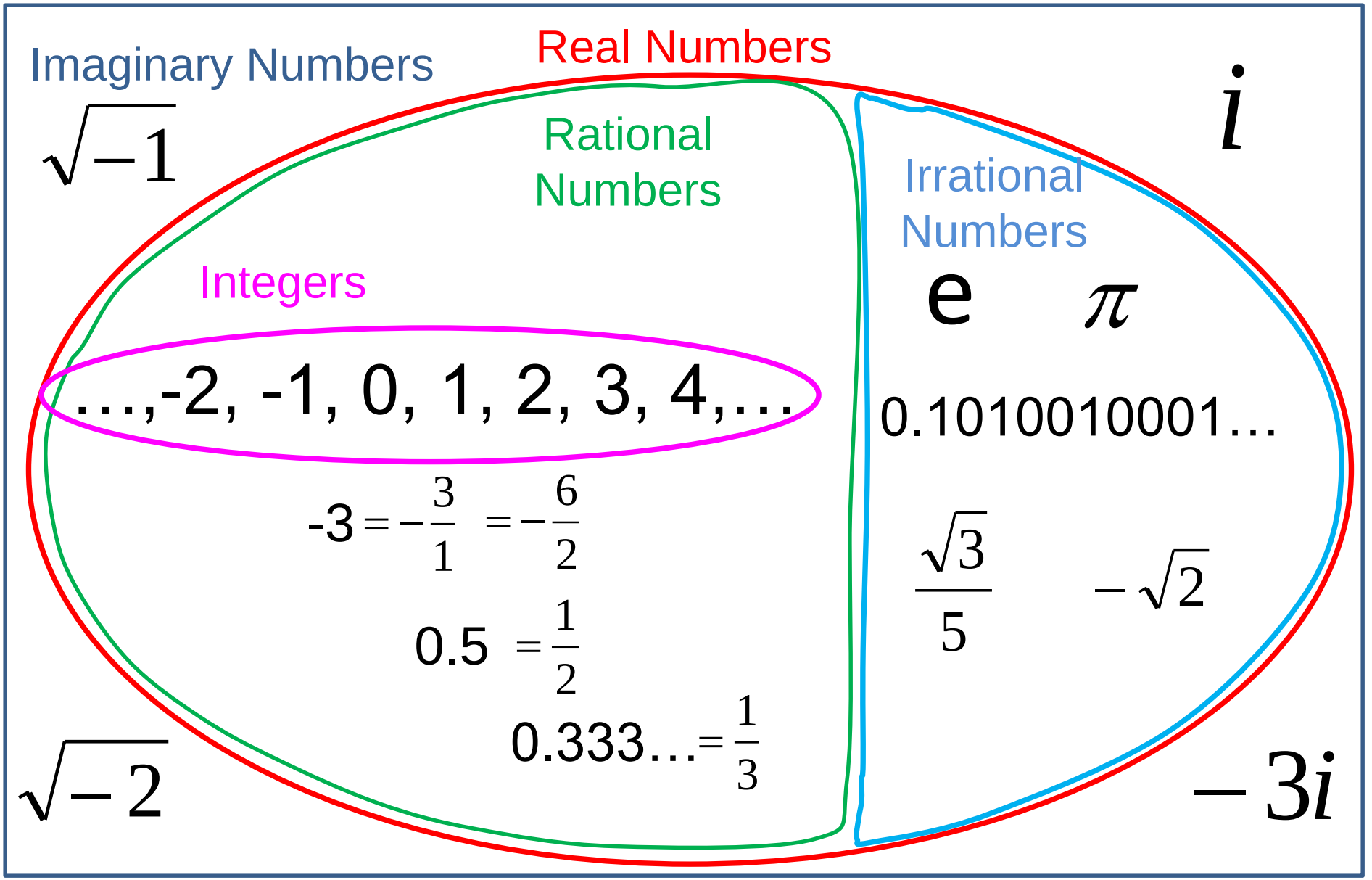
real numbers: a number that can be found on the number line.

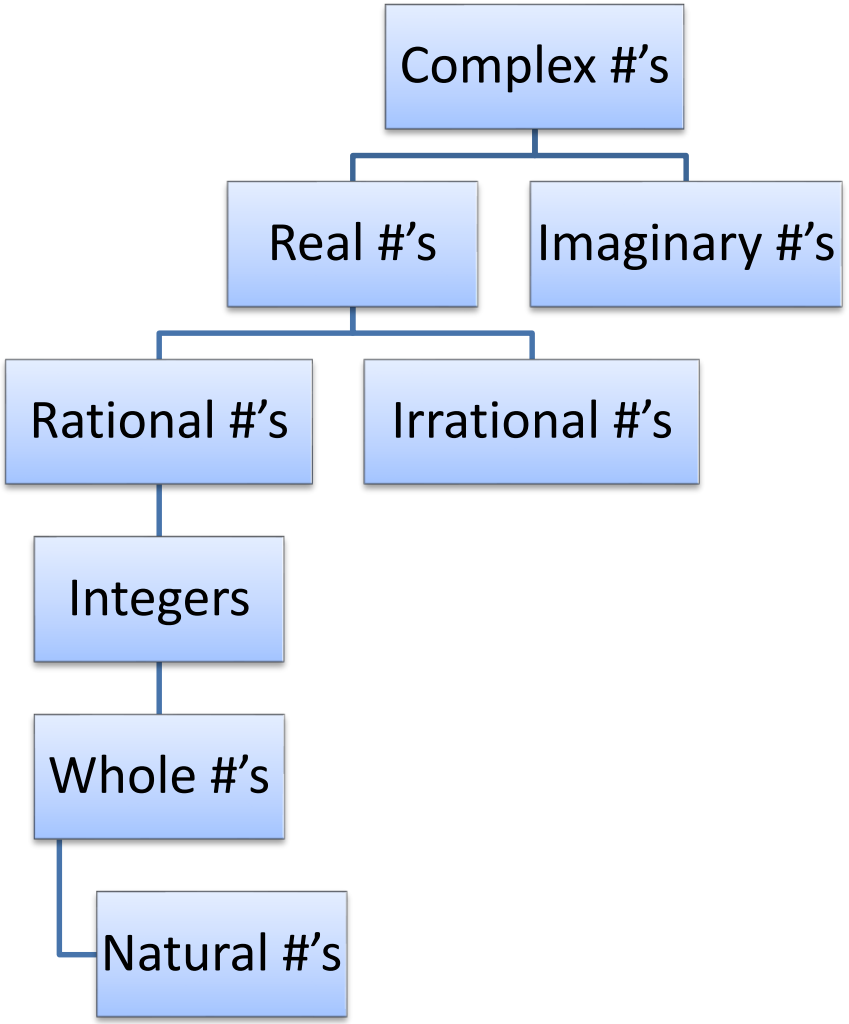


Think of the complex numbers as the “universe of numbers”.



Venn Diagram





$$1+2=3$$

$$\text{natural}+\text{natural}=\text{natural}$$

Is this always true?

$$3*2=6$$

$$\text{natural}*\text{natural}=\text{natural}$$

Is this always true?

$$-2+1=-1$$

$$\text{integer}+\text{integer}=\text{integer}$$

Is this always true?

$$4-3=1$$

$$\text{natural}-\text{natural}=\text{natural}$$

Is this always true?

Vocabulary

Closure: a number system is “closed” for a particular operation (add, subtract, multiply, divide, etc.) when two numbers have an operation performed on them and the resulting number is still in the number system.

We say that whole numbers and natural numbers are not closed “under” subtraction.

Is there another operation for which the whole numbers or the natural numbers are not closed?

$$\frac{-7}{0} = ? \qquad \frac{1}{2} = ?$$