Math-2A
Lesson 10-2

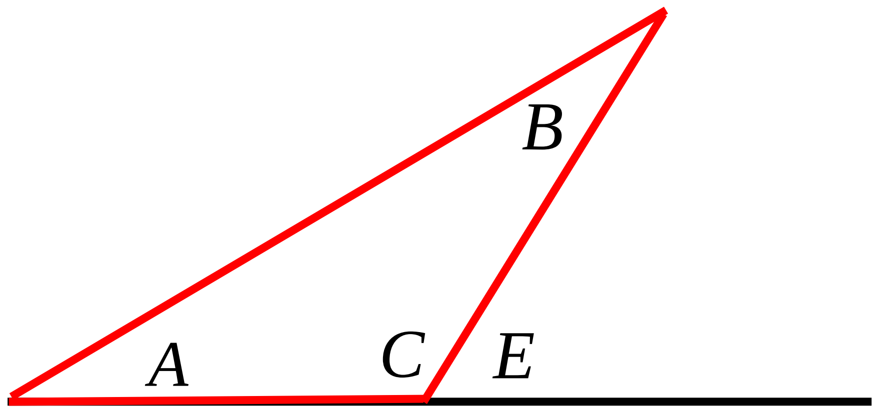
Exterior Angle Theorem,

Arcs, Central Angles, and

Inscribed Angles in Circles

Exterior angle: An angle formed by one side of a triangle and the extension of the adjacent side of the triangle.

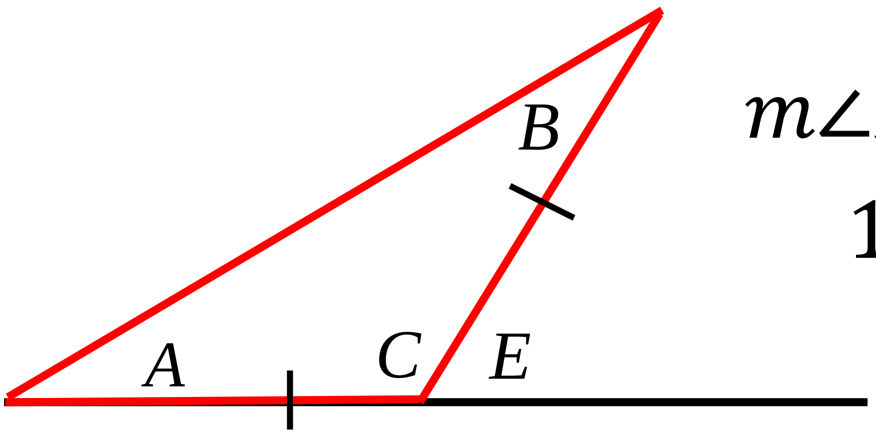
Angle “E” is an exterior angle to triangle ABC.



Remote interior angle: The two angles of a triangle that are on opposite sides of the triangle from the exterior angle.

Angles “A” and “B” are “remote interior” angles to exterior angle “E”.

Triangle ABC is Isosceles. The measure of exterior angle-E is 100. Find the measure of angle A.



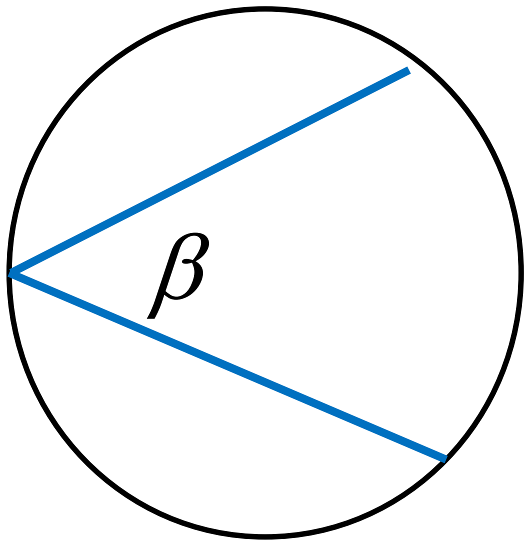
$$m\angle A = m\angle B$$

$$m\angle E = m\angle A + m\angle B$$

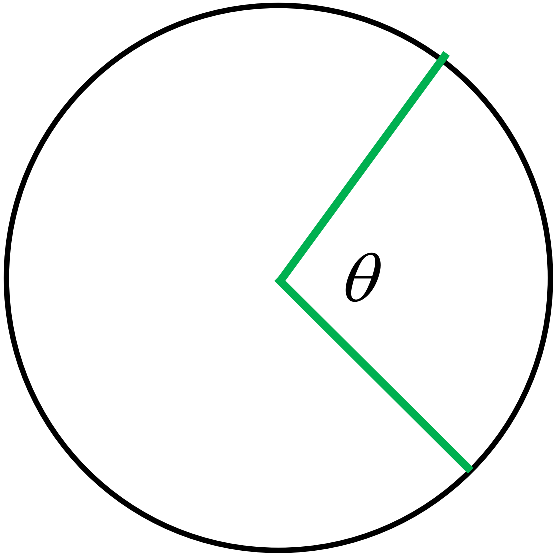
$$100 = 2 * m\angle A$$

$$50 = m\angle A$$

Inscribed angle: has its vertex on the circle.

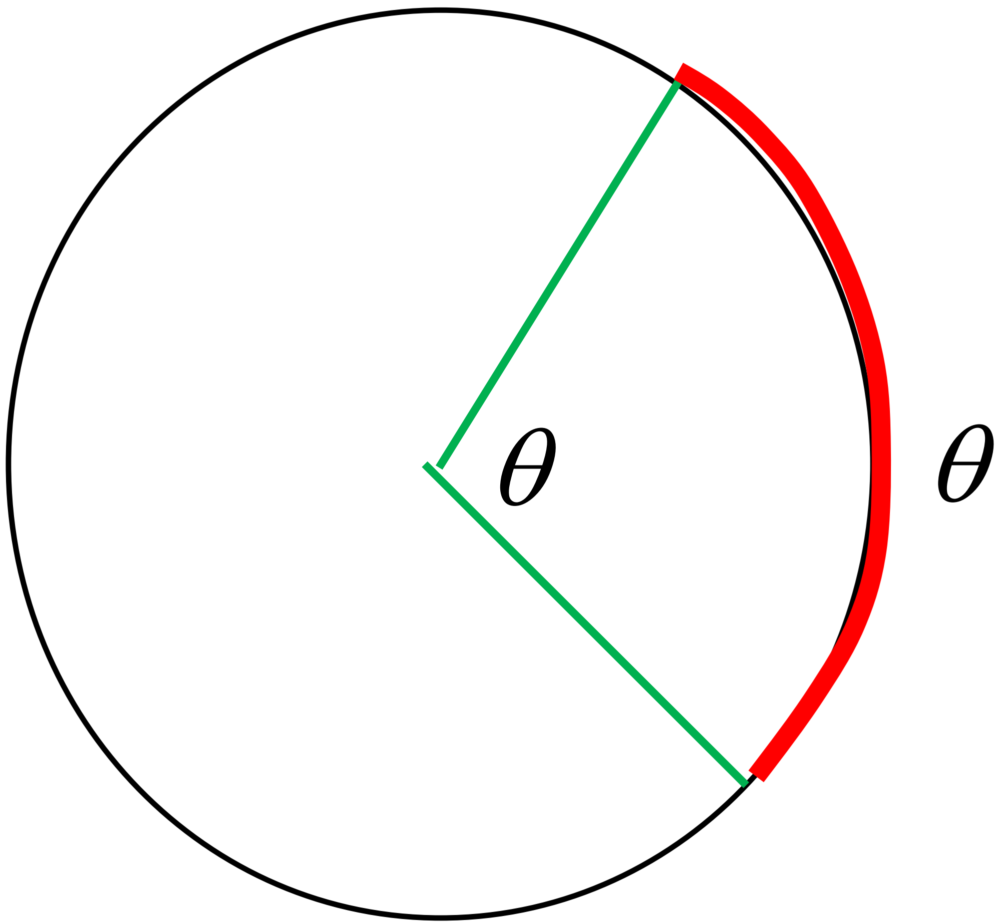


Central angle: has its vertex at the center of the circle.



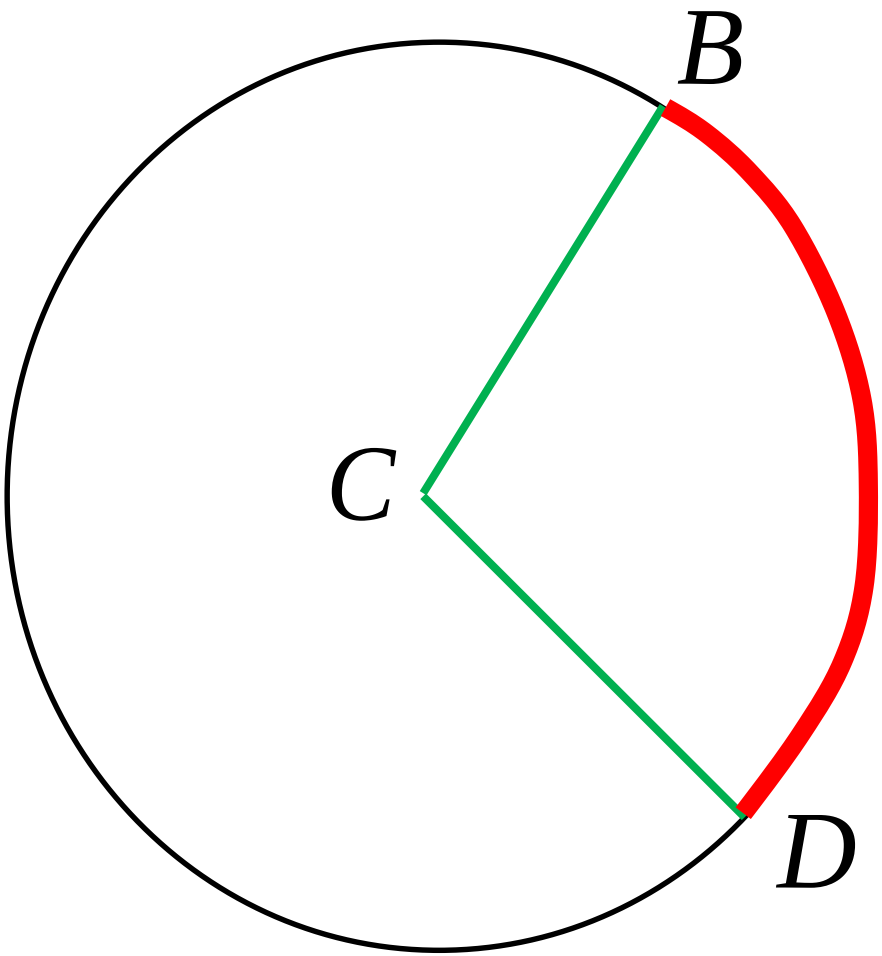
"intercepted" Arc

<u>Intercepted arc</u>: the arc of the circle that is in the interior of the angle. It has the <u>same measure as the central angle.</u>



<u>Naming Arcs</u>

The arc subtended by Center Angle C is $\overset{\frown}{BD}$



<u>Spoken</u>: "arc BD"

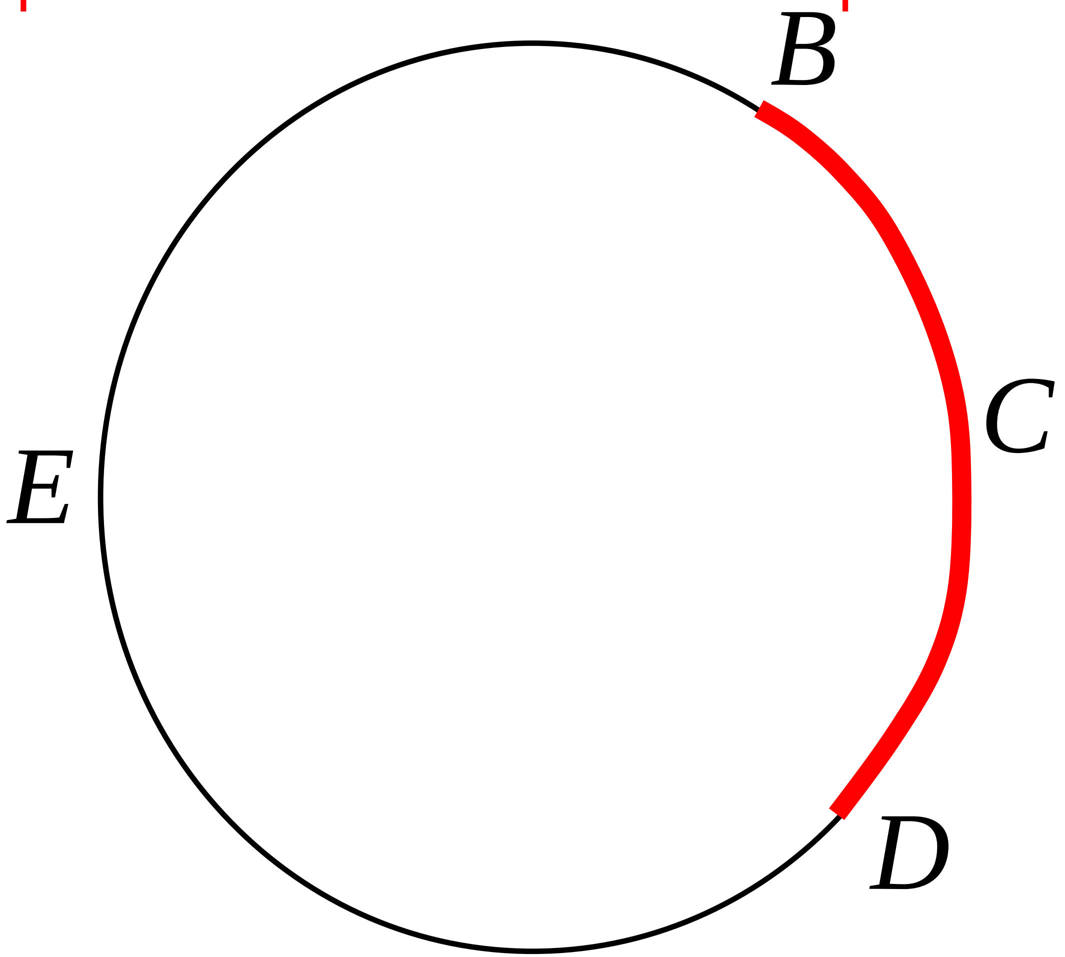
$m\overset{\frown}{BD}$

<u>Spoken</u>: "the measure of arc BD"

Minor Arcs: arcs that are less than half the circle.

Major Arcs: arcs that are more than half the circle.

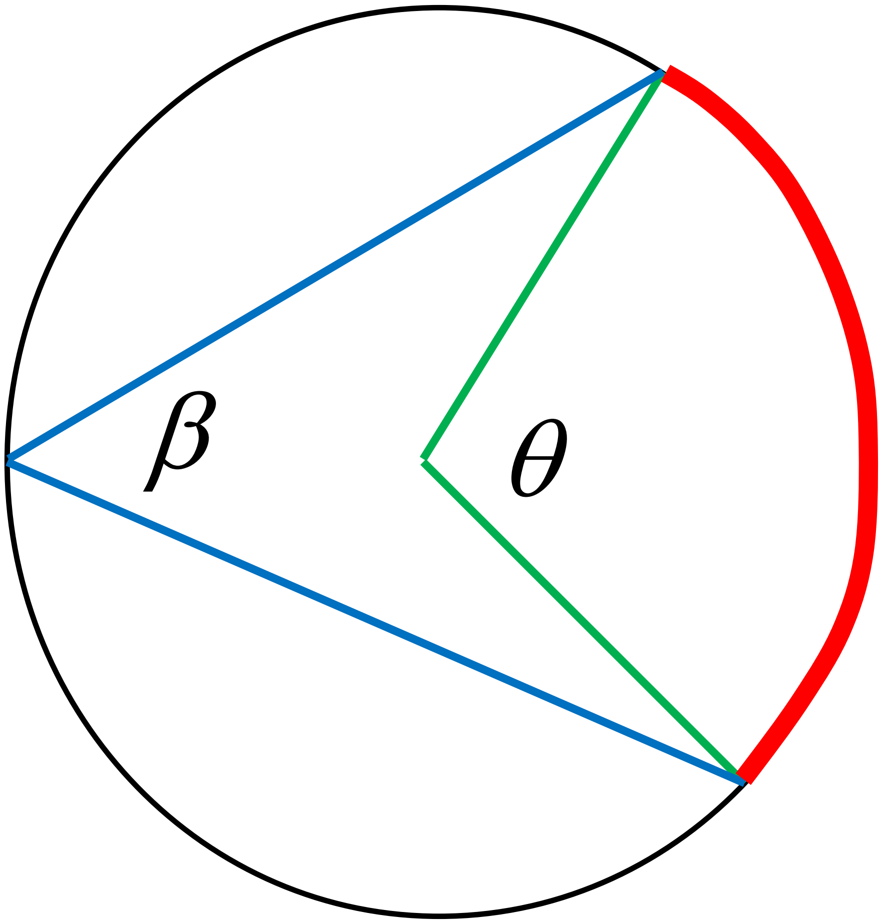
To distinguish between minor arc BD and major arc BD, we could add a letter between 'B' and 'D' to indicate a point in between that the arc passes through.



$\overset{\frown}{BCD}$ (minor arc)

$\overset{\frown}{BED}$ (major arc)

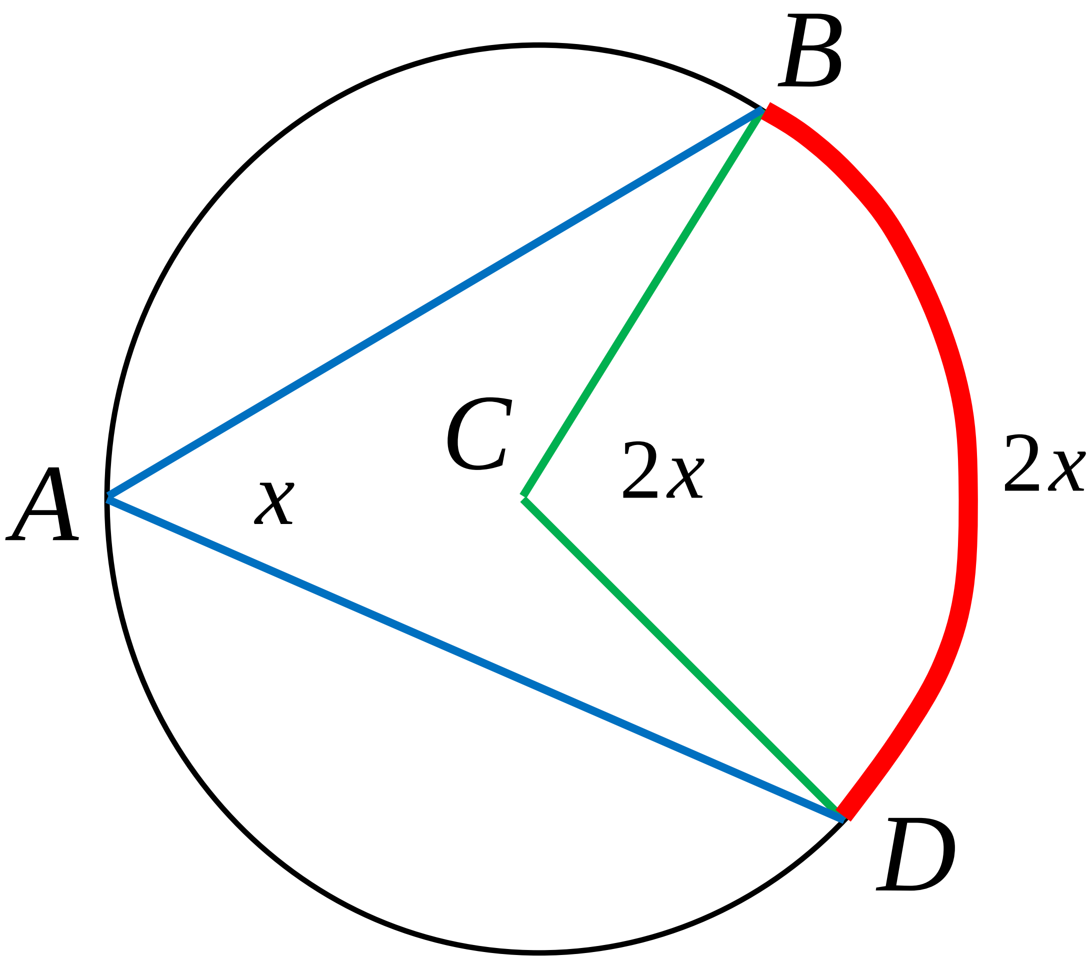
The “Central” Angle and the “inscribed” angle intercept (“cut”) the same arc.



Which angle has the larger measure?

Inscribed/Center Angle/Inscribed Arc Theorem

If an inscribed angle and a central angle subtend the same arc, then the measure of the central angle equals twice the measure of the inscribed angle.



If a central angle subtends an arc, then the measure of the arc equals twice the measure of the inscribed angle.

Find the measure of the angle.

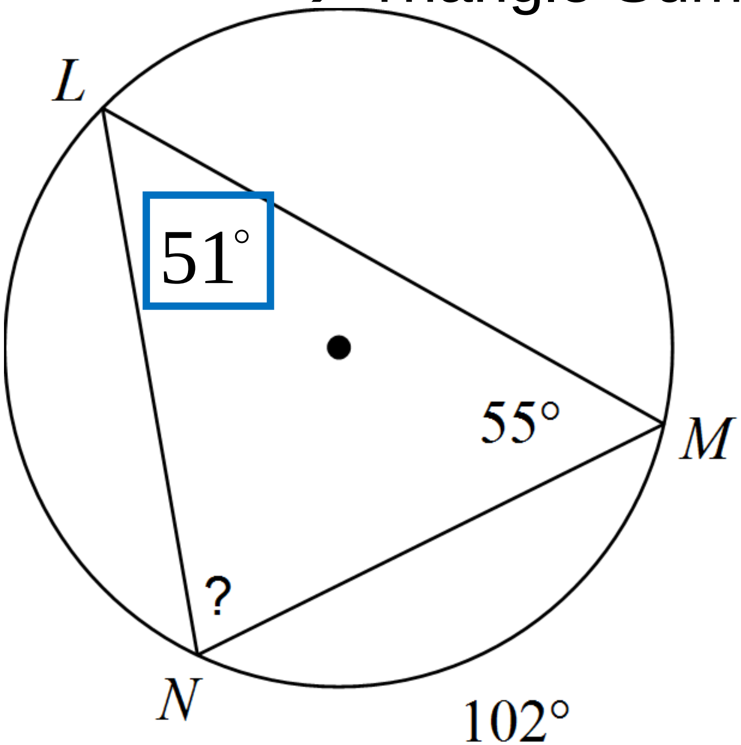
To solve for an unknown value, you need an equation.

1. Inscribed Angle. → Inscribed/Central Angle/Inscribed Arc Theorem

$m\angle L = ?$ $2m\angle L = m\widehat{NM}$ $m\angle L = 0.5 * 102^{\circ}$

$m\angle L = 51$

1. Triangle → Triangle Sum Theorem



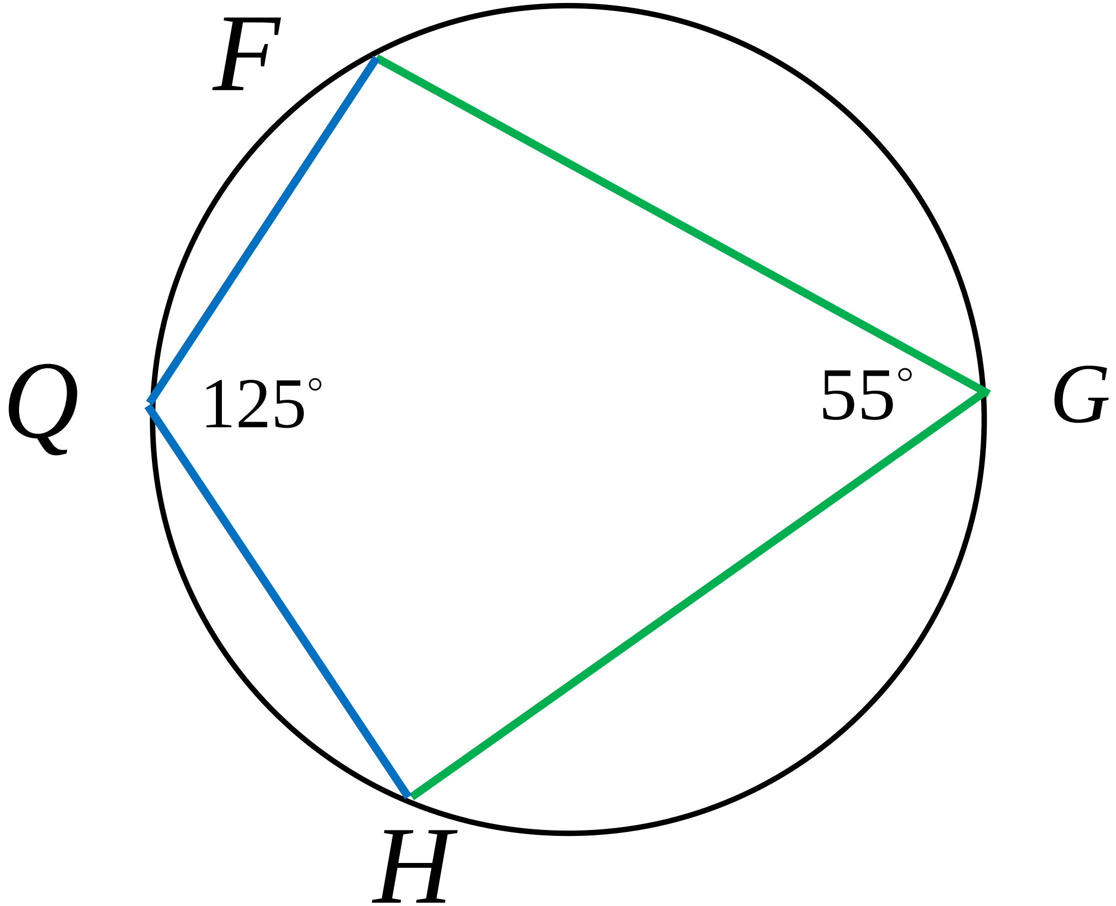
$m\angle N = ?$

$m\angle N = 180 - 55 - 51$

$m\angle N = 74$

An interesting (and useful) result

Inscribed angles that "cut opposite arcs are supplementary (add up to 180).



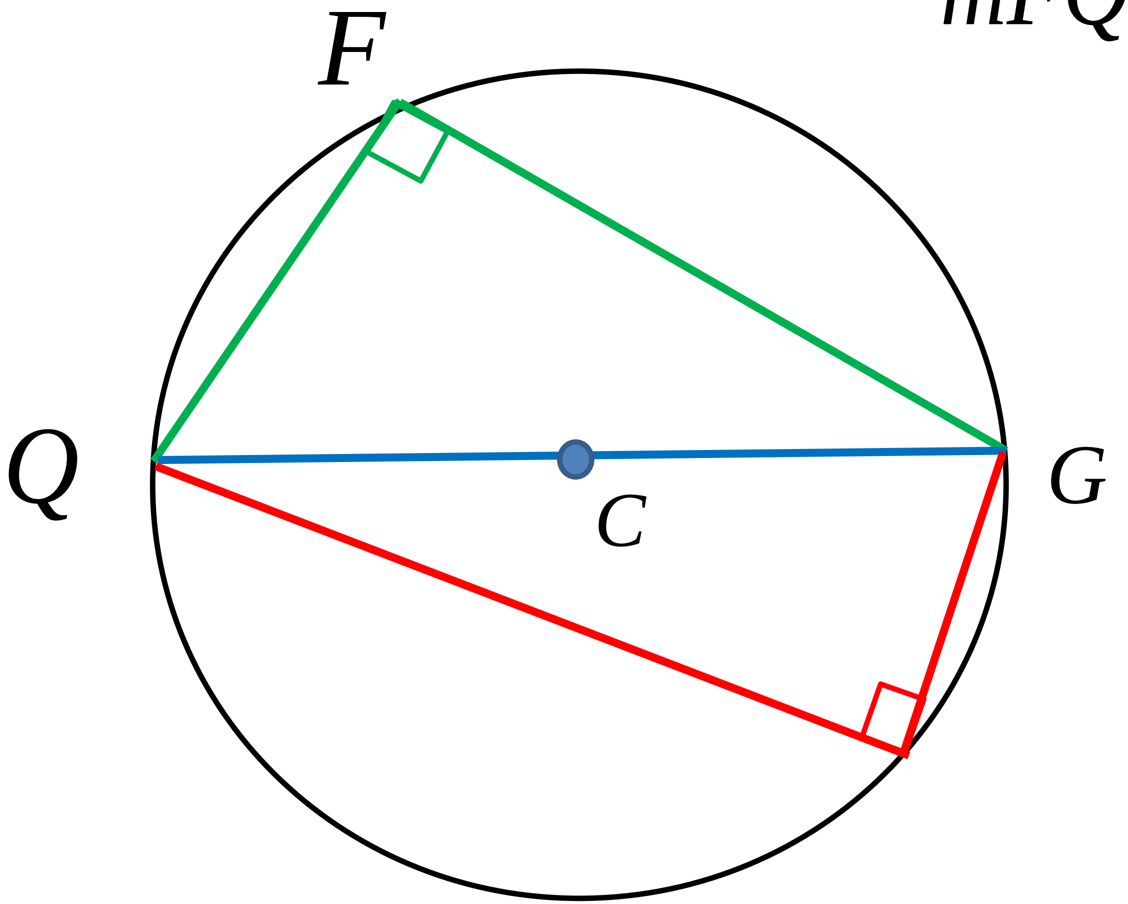
An interesting (and useful) result:

Segment QG is a diameter of circle C.

$$m\angle F = ?$$

$$m\overset{\frown}{FQH} = ? = 180^\circ$$

$$= 90^\circ$$



An inscribed angle that "cuts a diameter" always has a measure of 90.

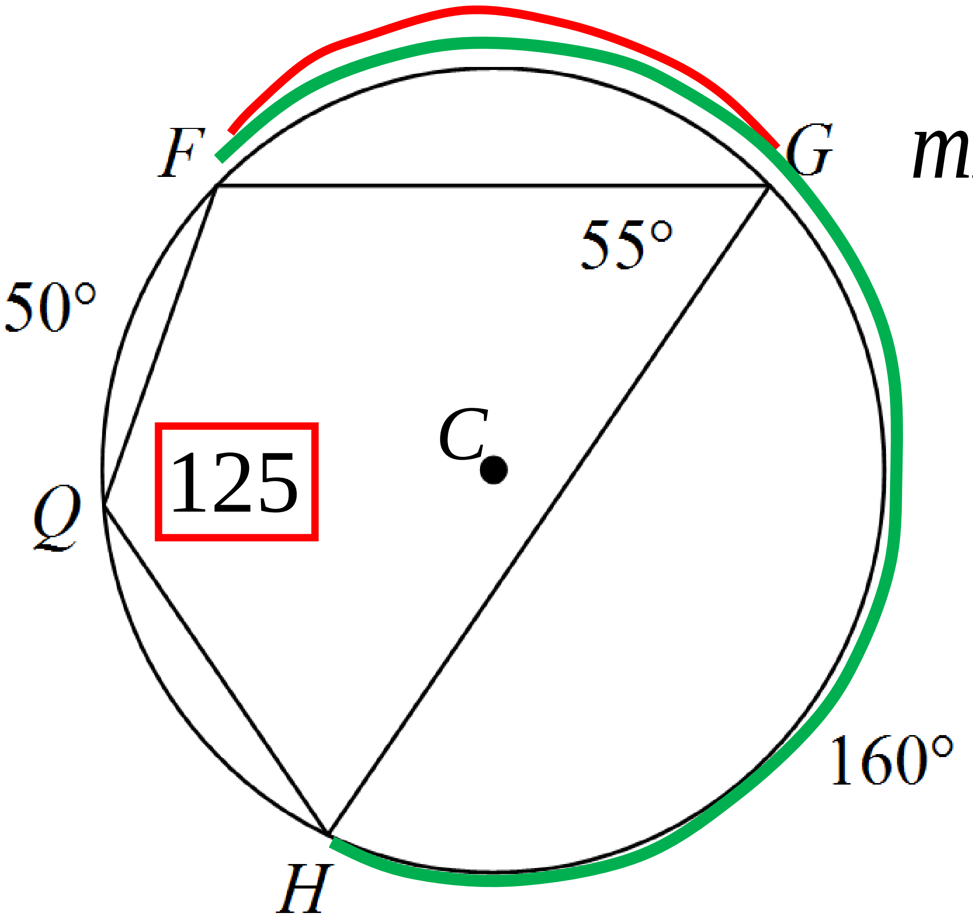
Find the measure of the angle.

$m\overset{\frown}{FG} = ?$

$m\angle Q = ?$ $\boxed{125}$

$m\overset{\frown}{FGH} = ?$

$= 2(125^\circ) = 250^\circ$



$m\overset{\frown}{FG} = m\overset{\frown}{FGH} - m\overset{\frown}{GH}$

$= 250^\circ - 160^\circ$

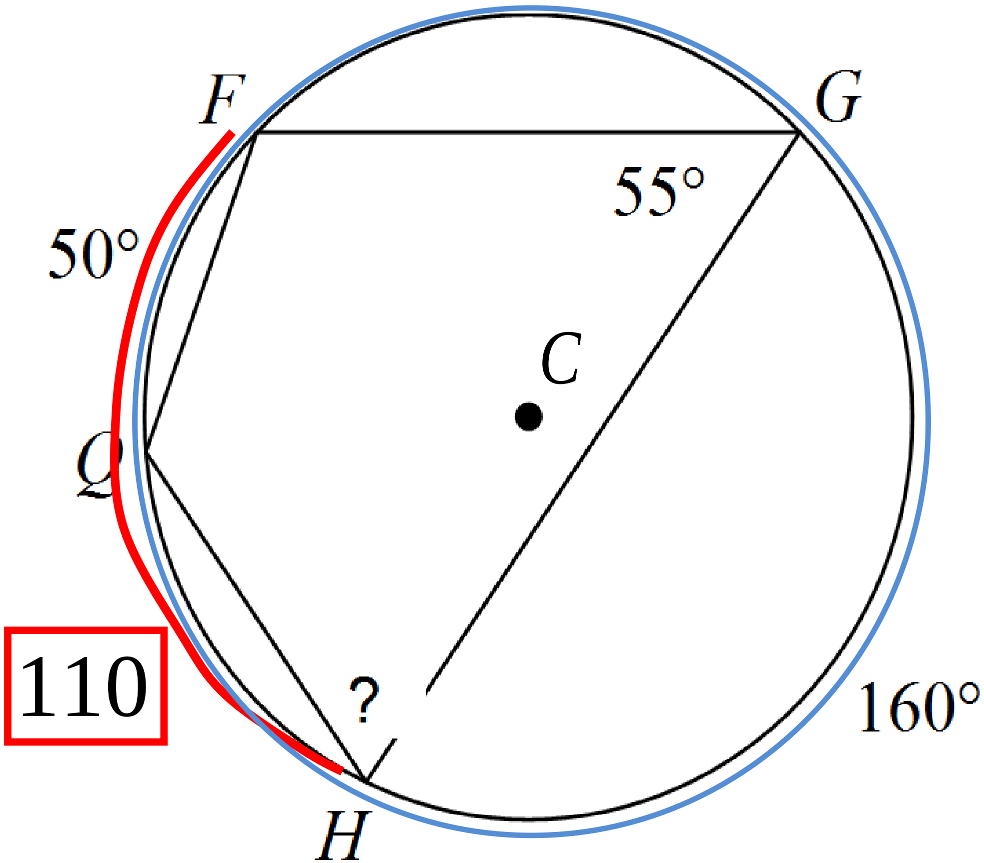
$= 90^\circ$

Find the measure of the angle.

To solve for an unknown value, you need an ____________.

1. Inscribed Angle. → Inscribed/Central Angle/Inscribed Arc Theorem

$m\angle Q = ?$



$$m\overset{\frown}{FQH} = ? = 2(55^\circ) = 110^\circ$$

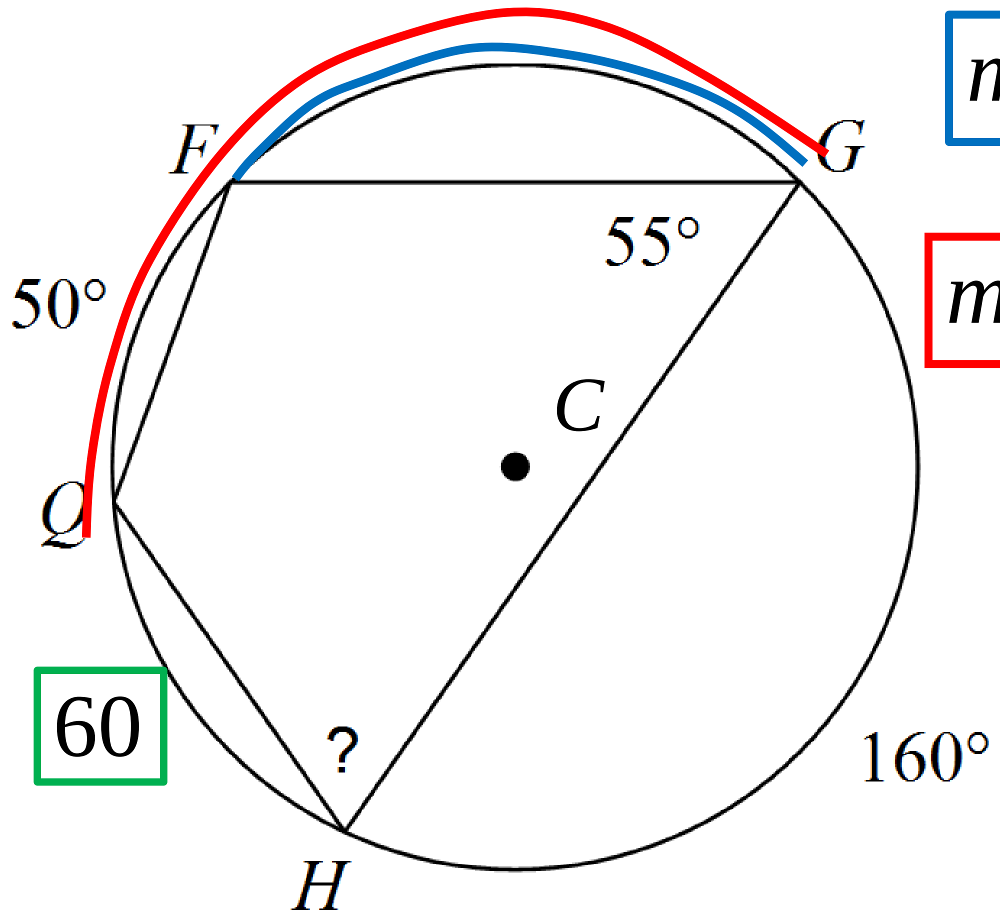
$$\boxed{m(Circle) = 360}$$

$$m\overset{\frown}{FGH} = ? = 360 - 110$$
$$= 250$$

$$m\angle Q = \frac{1}{2} * 250$$

$$m\angle Q = 125$$

$$m(arc\ FG) = 360 - 60 - 50 - 160$$



$$m(arc\ FG) = 90$$

$$m(arc\ QFG) = 50 + 90$$

$$m(arc\ QFG) = 140$$

$$m\angle QHG = 70$$