Math-2

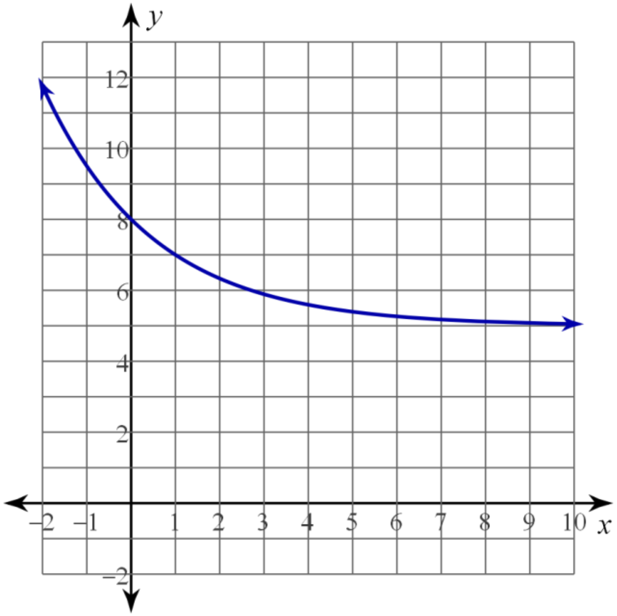
Lesson 9-5

More

Exponential Modeling

1. Money
2. Cooling

Find the equation of the graph.



1. Horizontal asymptote: y = 5

$$y = AB^x + 5$$

2. Passes through: (x, y) = (0, 8)

$$8 = AB^0 + 5 \rightarrow 8 = A + 5 \rightarrow A = 3$$

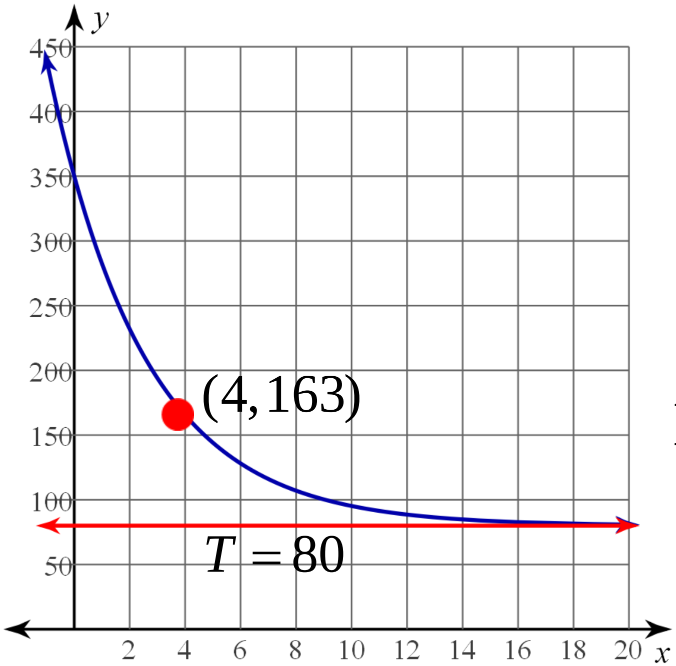
$$y = 3B^x + 5$$

3. Passes through: (x, y) = (1, 7)

$$7 = 3B^1 + 5 \rightarrow 2 = 3B \rightarrow B = \tfrac{2}{3}$$

$$\boxed{y = AB^x + k} \rightarrow \boxed{y = 3\left(\tfrac{2}{3}\right)^x + 5}$$

Find the equation of the graph.



$$T(t) = AB^t + m$$

1. Horizontal asymptote: y = 80

$$T(t) = AB^t + 80$$

2. Passes through: (t, T) = (0, 350)

$$350 = AB^0 + 80 \rightarrow 270 = A$$

$$T(t) = 270B^t + 80$$

3. Passes through: (t, T) = (4, 163)

$$163 = 270B^4 + 80 \rightarrow \frac{163-80}{270} = B^4 = 0.3074$$

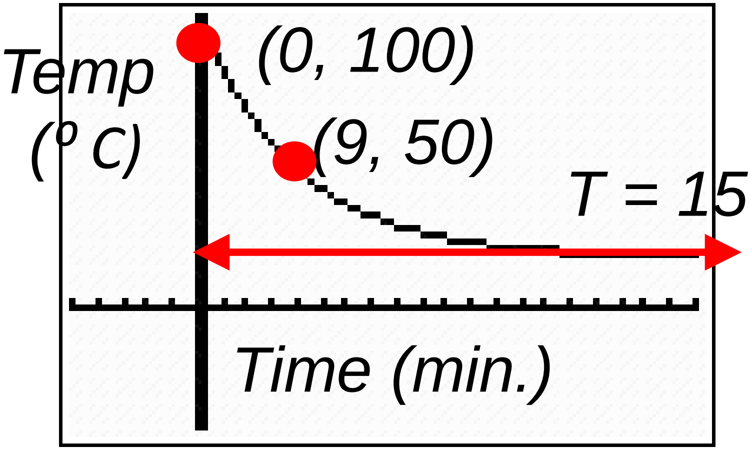
$$\rightarrow B = \sqrt[4]{0.3074} = 0.7446$$

$$\rightarrow T(t) = 270(0.7446)^t + 80$$

2. What will be the temperature in 10 minutes?

$$T(10) = 270(0.7446)^{10} + 80$$

$$T(10) = 94.1\ F$$



Boiling water (100º C) is taken off the stove to cool in a room at 15º C. After 9 minutes, the water's temperature is 50 C.

Write the modeling equation as a base 'b' exponential.

$$T(t) = a(b)^t + k$$

1) Horizontal Asymptote

$$T(t) = a(b)^t + 15$$

2) y-intercept

$$100 = a(b)^0 + 15$$

$$a = 85$$

3) "nice point"

$$50 = 85(b)^9 + 15$$

$$\left(\frac{50-15}{85}\right) = (b)^9$$

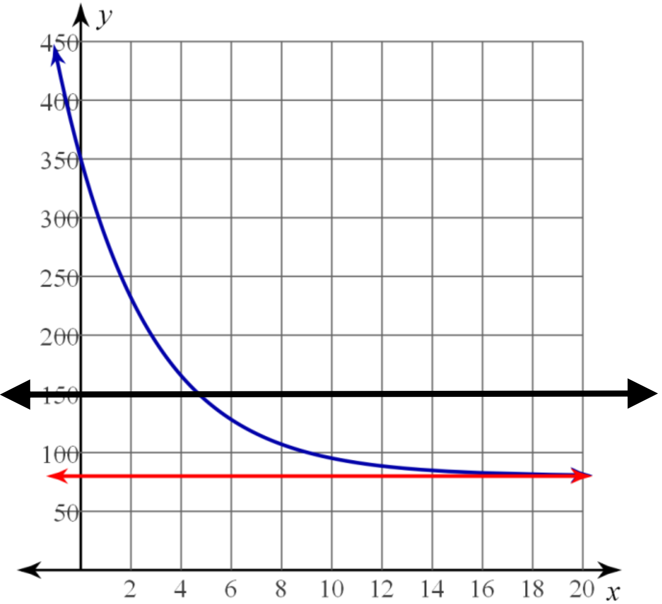
$$\left(\frac{50-15}{85}\right)^{1/9} = b$$

$$b = 0.906$$

4) Final equation

$$T(t) = 85(0.906)^t + 15$$

Find the equation of the graph.



1. Find 't' to reach 150 F

$$T(t) = 270(0.7446)^t + 80$$

$$150 = 270(0.7446)^t + 80$$

$$\frac{150 - 80}{270} = (0.7446)^t$$

$$\rightarrow 0.2593 = (0.7446)^t$$

How do we solve for 't'?

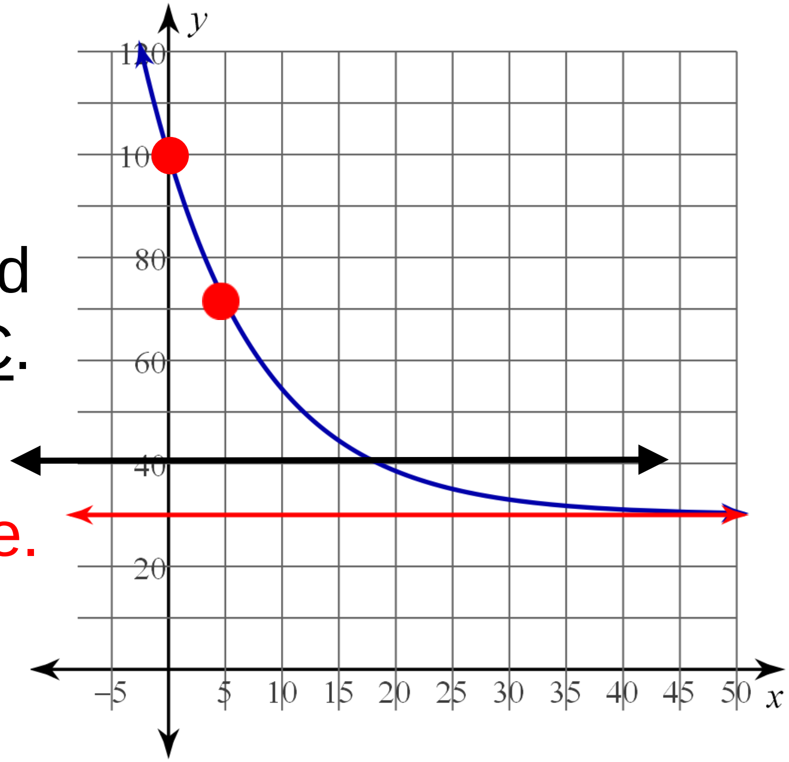
1. "Guess and check" → build a table and try some values for 't'

t	4	4.2	4.4	4.5	4.6
T	0.307	0.290	0.273	0.265	0.258

2. Solve by graphing: $y_1 = 270(0.7446)^t + 80$

$$y_2 = 150$$

A cup of hot water is taken out of the microwave oven. Its initial temperature is 100 C. It is placed on the counter in a room whose temperature is 30 C. In 5 minutes it has cooled to 72 C. When will it reach 40 C.



1. Draw a graph that shows temperature as a function of time.

2. What is the equation of the graph? (use the following equation). $T(t) = AB^t + k$

3. Draw a horizontal line for T = 40

4. Solve by graphing

$$y_1 = 70(0.903)^t + 30$$
$$y_2 = 40$$

A cake taken out of the oven at temperature of 450º F. It is placed on in a room with an ambient temperature of 75ºF to cool. 10 minutes later the temperature of the cake is 180ºF. When will the cake be cool enough to put the frosting on (90ºF)? (t=?, 90ºF)

Start with either:

$$T(t) = AB^t + k$$

$$T(t) = 375(0.8805)^t + 75$$

$$T(t) = 90$$

Solve by graphing

A hard-boiled egg at temperature 212^{o} F is placed in 60^{o} F water to cool. 5 minutes later the temperature of the egg is 95^{o} F. When will the egg be 75^{o}C?

A cake taken out of the oven at temperature of 350^{o} F. It is placed on in a room with an ambient temperature of 70^{o}F to cool. Ten minutes later the temperature of the cake is 150^{o}F. When will the cake be cool enough to put the frosting on (90^{o}F) ?

You deposit $100 money into an account that pays 3.5% interest per year. The interest is "compounded" monthly. How much money will be in the account at the end of the 1st year?

$$A(1) = \$100 + \$100(0.035)$$

Original amount ($100)will still be in the account.

There will be a small amount of growth(3% of $100)

Factor out the common factor $100

$$A(1) = \$100(1 + 0.035) = \$100(1.035)$$

$$A(2) = \$100(1.035)^2$$

$$A(3) = \$100(1.035)^3$$

$$A(t) = \$100(1.035)^t$$

$$A(t) = A_0(1 + r)^t$$

A bank pays 3% interest per year, and they pay you each month, what is the monthly interest rate?

$$\frac{0.03}{\cancel{year}} * \frac{\cancel{year}}{12\, months} \rightarrow \frac{0.03}{12\, month} \rightarrow \frac{0.03}{12}\ per\ month$$

$$\rightarrow 0.0025\ per\ month$$

A bank pays 5% interest per year, and they pay you each month, what is the monthly interest rate?

$$0.05\ per\ year \rightarrow \frac{0.05}{12}\ per\ month \rightarrow 0.0042\ per\ month$$

The exponential growth equation for money in a bank for account where the bank pays you more frequently than at the end of the year is:

Amount of $$ in the account as a function of time

Initial value

Annual interest rate

Years after the deposit

$$A(t) = A_0(1 + r/k)^{k*t}$$

# of times the bank pays you each year

"Compounding period" → the number of times the bank pays you each year.

Values of "k"	
Words to look for	K
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365

"A bank pays 3% per year compounded monthly."

$$A(t) = A_0(1 + 0.03/12)^{12*t}$$

You deposit $100 money into an account that pays 3.5% interest per year. The interest is “compounded” monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{k*t}$$

$$A(t) = A_0(1+r)^t$$

$$A(5) = 100\left(1 + \frac{0.035}{12}\right)^{12*5}$$

$$A(5) = 100(1+0.035)^{(5)}$$

$$A(5) = \$119.09$$

$$A(5) = \$118.77$$

Interest paid at the end of each month

Interest paid at the end of each year

You deposit $200 money into an account that pays 5.5% interest per year. How much money will be in the account at the end of the 20th year?

$$A(t) = A_0(1+r)^t$$

$$A(20) = \$200(1+0.055)^{(20)}$$

$$A(20) = \$583.55$$

You buy a car for $18,500. It depreciates at 15% per year. What is the value of the car (what you could sell it for) after 7 years?

$$V(t) = V_0(1-r)^t$$

$$V(t) = 18{,}500(1-0.15)^{(t)}$$

$$V(t) = 18{,}500(0.85)^{(t)}$$

What is the growth factor?

Is it "growth" or "decay"?

$$V(7) = 18{,}500(0.85)^{(7)}$$

$$V(7) = \$5930.68$$

You deposit $100 money into an account that pays 3.5% interest per year. The interest is "compounded" monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{kt} \qquad A(5) = 100(1 + 0.035/12)^{12(5)}$$

$$A(5) = \$119.09$$

What is the doubling time for this account?

$$200 = 100(1 + 0.035/12)^{12t}$$

$$2 = (1.0029)^{12t}$$

$$y_1 = (1.0029)^t$$
$$y_2 = 2$$

Solve by graphing

You deposit $200 money into an account that pays 5.5% interest per year. The interest is “compounded” quarterly. How long will it take for your money to triple?

$$A(t) = A_0(1 + r/k)^{kt} \qquad 600 = 200(1 + 0.055/4)^{4(t)}$$

$$3 = (1.0138)^{4t}$$

$$y_1 = (1.0138)^t$$
$$y_2 = 3$$

Solve by graphing