

Math-2  
Lesson 9-2

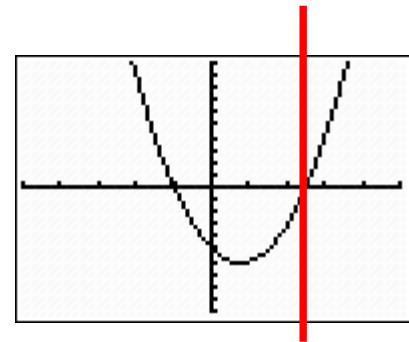
Function Composition

1. Is the following relation a function?

$(-2, 5)$ ,  $(5, 6)$ ,  $(-2, 6)$ ,  $(7, 6)$

No. Input value -2 has two output values.

2. Is the following relation a function?



Does the graph of the relation pass the “vertical line test” ?

Yes. Each input value has exactly one output value.

# Function Notation

$$y = f(x) \quad \text{“y is a function of x”}$$

‘y’ equals ‘f’ of ‘x’

A function is a rule that matches input values to out put values.

$$f(x) = 2x + 1$$

(Input)	(rule)	(output)	
x	$2x + 1$	y	
2	$2(2) + 1$	5	$f(2) = 5$
3	$2(3) + 1$	7	$f(3) = 7$

## Compositions of Functions

$$f(x) = 2x \quad \rightarrow \quad f(3) = ?$$

Means: wherever you see an 'x' in the function, replace it with a 3.

1. Replace the 'x' with a set of parentheses.

$$f(3) = 2( \quad )$$

2. Put the input value '3' into the parentheses.

$$f(3) = 2(3)$$

3. Find the output value.

$$f(3) = 6$$

## Compositions of Functions

$$f(x) = x^2 - 3x + 2 \quad \rightarrow f(2) = ?$$

Means: wherever you see an 'x' in the function, replace it with a '2'.

1. Replace the 'x' with a set of parentheses.

$$f(x) = ( )^2 - 3( ) + 2$$

2. Put the input value '2' into the parentheses.

$$f(x) = (2)^2 - 3(2) + 2$$

3. Find the output value.

$$f(2) = 0$$

**Cool, we found a zero of the function.**

$$f(x) = x^3 - 1 \quad f(-2) = ?$$

$$f(-2) = (-2)^3 - 1 \quad f(-2) = -9$$

$$f(x) = 2x^{1/2} \quad f(9) = ?$$

$$f(9) = 2(9)^{1/2} \quad f(9) = 6$$

$$f(x) = \frac{2(x-4)}{x^2 + x - 20} \quad f(-2) = ?$$

$$f(-2) = \frac{2(-2-4)}{((-2)^2 + (-2) - 20)} \quad f(-2) = \frac{2}{3}$$

# Function Notation

$f(x) = 2x + 1$	(Input) $x$	(rule) $2x + 1$	(output) $f(x)$
	2	$2(2) + 1$	5 $f(2) = 5$
	3	$2(3) + 1$	7 $f(3) = 7$
$f(x - 1) = 2x - 1$	$x - 1$	$2(x - 1) + 1$	$2x - 1$
$f(3x) = 6x + 1$	$3x$	$2(3x) + 1$	$6x + 1$

If your input is an expression instead of a number:  
replace 'x' with parentheses and "plug in" the expression  
→ parentheses, substitute, simplify

$$f(x) = 3x - 1$$

(Input) $x$	(rule) $3x - 1$	(output) $f(x)$
2	$3(2) - 1$	5 $f(2) = 5$
$x^2$	$3( ) - 1$	? $f(x^2) = 3x^2 - 1$
$x + 2$	$3( ) - 1$	? $f(x + 2) = 3x + 5$
$3 - 2x$	$3( ) - 1$	? $f(3 - 2x) = 8 - 6x$



Your turn:  $f(x) = x^2 + 1$   
input the expressions

$$f(2) = ? = 5$$

$$f(x^3) = ? = x^6 + 1$$

$$f(x+2) = ? = (x+2)^2 + 1 = x^2 + 4x + 5$$

$$f(-2x+3) = ? = (-2x+3)^2 + 1 \\ = 4x^2 - 12x + 10$$

## Compositions of Functions

$$f(x) = 2x$$

and

$$g(x) = x^2$$

Let's use  $f(x)$  as the input to  $g(x)$

$$g(f(x)) = ?$$

$$g(..) = (..) ^2$$

$$g(2x) = (2x) ^2$$

$$g(f(x)) = 4x^2$$

1. Replace the 'x' with a set of parentheses.
2. Put the input value "2x" into the parentheses.
3. Find the output value.

## Compositions of Functions

$$f(x) = 2x + 3 \quad \text{and}$$

$$g(x) = x^2$$

$$f(g(x)) = ?$$

1. The input value to  $f(x)$  is  $g(x)$ .

$$f(..) = 2(..) + 3$$

2. Replace the 'x' in  $f(x)$  with a set of parentheses.

$$f(x^2) = 2(x^2) + 3$$

3. Put the input value ( $g(x)$ ) into the parentheses.

$$f(g(x)) = 2x^2 + 3$$

4. Find the output value.

Function “composition”

$$f(x) = x^2 + 1 \quad g(x) = x^2$$

$$f(2) = ? \quad \text{What does this mean?}$$

**“Substitute ‘2’ in for ‘x’ in the function f(x).”**

$$f(g(x)) = ? \quad \text{What does this mean?}$$

**“Substitute ‘g(x)’ in for ‘x’ in the function f(x).”**

$$f(g(x)) = (g(x))^2 + 1$$

**“Which means the same as...”**

$$f(x^2) = (x^2)^2 + 1 = x^4 + 1$$

## Composition of Functions

$$f(x) = 2x + 1 \quad g(x) = 3x + 2 \quad h(x) = x + 5$$

$$f(g(x)) = ? = 2(\quad) + 1 = 2(3x + 2) + 1$$

$$h(g(x)) = ? = (\quad) + 5 = (3x + 2) + 5$$

$$h(f(x)) = ? = (\quad) + 5 = (2x + 1) + 5$$

$$g(h(x)) = ? = 3(\quad) + 2 = 3(x + 5) + 2$$

$$f(f(x)) = ? = 2(\quad) + 1 = 2(2x + 1) + 1$$

# New Notation for the Composition of Functions

$$(f \circ g)(x) = f(g(x)) \quad \text{"g" plugged into rule "f"}$$

$$f(x) = 4x - 1 \quad g(x) = -5x + 3$$

$$(f \circ g)(x) = ? = 4(\quad) - 1 = 4(-5x + 3) - 1$$

$$\text{"g" plugged into rule "f"} \quad (f \circ g)(x) = -20x + 11$$

$$(g \circ f)(x) = ? = -5(\quad) + 3 = -5(4x - 1) + 3$$

$$\text{"f" plugged into rule "g"} \quad (g \circ f)(x) = -20x + 8$$

$$(f \circ f)(x) = ? = 4(\quad) - 1 = 4(4x - 1) - 1$$

$$\text{"f" plugged into rule "f"} \quad (f \circ f)(x) = 16x - 5$$

$$(g \circ g)(x) = ? = -5(\quad) + 3 = -5(-5x + 3) + 3$$

$$\text{"g" plugged into rule "g"} \quad (g \circ g)(x) = 25x - 12$$

One more layer!

$$f(x) = 3x \quad g(x) = x^2$$

$$f(g(4))$$

4

$$g(\quad) = (\quad)^2$$

$$g(4) = (4)^2$$

16

$$f(\quad) = 3(\quad)$$

$$f(16) = 3(16)$$

48

$$f(g(x)) = 3(g(x)) = 3x^2$$

$$f(g(4)) = 3(g(4)) = 3(4)^2 = 48$$

One more layer.  $g(x) = x^2$   $f(x) = 3x$

$$(g \circ f)(-1) = ?$$

Rewrite in “old” notation

$$g(f(-1)) = ?$$

The input to  $f(x)$  is -1.

$$f(-1) = 3(-1)$$

$$f(-1) = -3$$

The output of  $f(-1)$  is -3.

The input to  $g(x)$  is -3.

$$g(-3) = 9$$

$$g(f(-1)) = 9$$