Math-2a

Lesson 11-6:
Counting, Permutations, and Combinations

How many ways can you arrange the letters A, B, and C in order?

SIX ways

Any one of the following 3 could be the 1st letter.

A B C

Given the first letter above, the second letter could be:

B or *C* | *A* or *C* | *A* or *B*

The only option for the 3rd letter in each case is:

ABC ACB BAC BCA CBA CAB

The "multiplication principle" of counting: When arranging things in order (letters A, B, and C), the total number arrangements is the product of the number of possibilities for each step.

"Each step" means:
1st step → pick the first item
2nd step → pick the second item, etc.

$\underline{3} * \underline{2} * \underline{1}$

How many ways are there to arrange the letters A, B, C, and D in order?

$\underline{\ 4\ } * \underline{\ 3\ } * \underline{\ 2\ } * \underline{\ 1\ }$

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

24 Ways

Use the Multiplication rule of counting to count total number of ways Abby, Ben, and Cassie could stand in a line at the grocery store.

There are 3 people to choose from for the 1st position in line

3 possibilities

For the 2nd position in line, one person is "used up" (she cannot be in both the 1st AND 2nd positions in the line).

2 possibilities

Since you "use up a person", each subsequent position has 1 less possibility than the previous position.

For the 3rd position in line, there is only one person left to choose from.

1 possibility

$$\underline{\;3\;} * \underline{\;2\;} * \underline{\;1\;}$$

ABC ACB BAC BCA CBA CAB

How many ways are there to arrange the 8 people in a line?

$$\underline{8} * \underline{7} * \underline{6} * \underline{5} * \underline{4} * \underline{3} * \underline{2} * \underline{1}$$

How many people do we have to choose from for the "head of the line?"

8

How many people do we have to choose from for 2nd person in the line?

7

Total number of ways = 8!

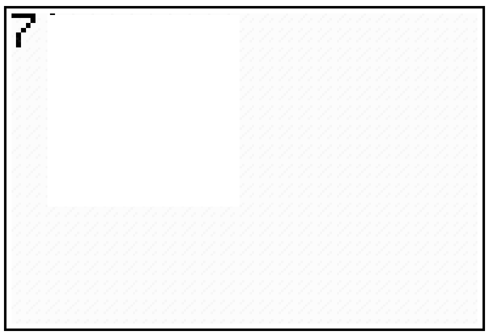
"!" means "factorial"

8 ! = 40,320

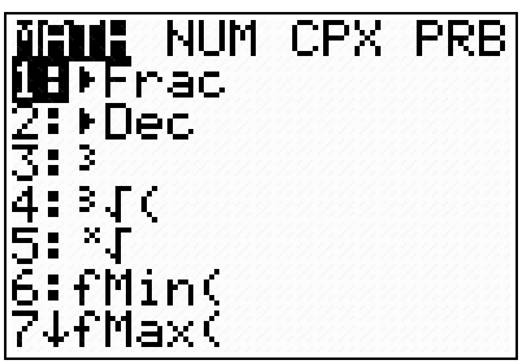
<u>Factorial</u>: Multiply a <u>*natural number*</u> by <u>*every smaller natural number.*</u>

3! = 3*2*1 3! = 6 Calculate 5! 5! = 120

Using your calculator:
Type in the number….7.

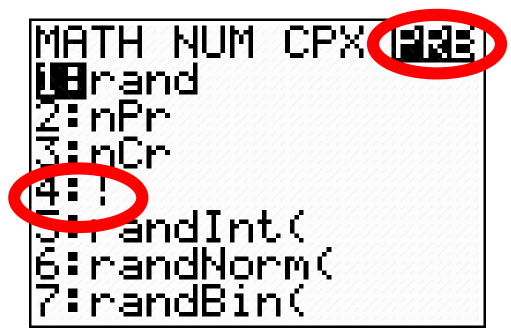


<u>Factorial</u>: Press the “math” button



Scroll over to “PRB” (probability)

Use option “4”



Hit “enter”

A car dealership has a large showroom. It has room for 12 cars in a row. How many different ways can you arrange the 12 cars?

12 ! 479,001,600

Taking the “factorial” of a number can result in HUGE numbers!

Use your calculator <u>factorial feature</u> to calculate: 0 !

0 ! = 1

Math with factorial.

$$\frac{7!}{3!} = \frac{7*6*5*4*\cancel{3}*\cancel{2}*\cancel{1}}{\cancel{3}**\cancel{2}*\cancel{1}}$$

$$= 7*6*5*4 = 840$$

Or….

$$\frac{7!}{3!} = \frac{7*6*5*4*\cancel{3!}}{\cancel{3!}}$$

$$= 7*6*5*4 = 840$$

Permutation: The number of ways a group of items can be arranged ***in order*** without re-using items.

ways to arrange letters, people, numbers, in order; were all permutations.

Another version of a permutation. Arranging fewer than the total number of items in the group.

For example: Sean's band has 10 original songs. The recording company will only accept 6 songs on a demo CD. How many different ways can you pick 6 of the 10 and then arrange them on the demo disk?

We call this a permutation of 'n' items taken 'r' at a time.

$_{n}P_{r}$ For Sean's CD: $_{10}P_{6}$ "10 taken 6 at a time"

"Pick from 10 items, put then in 6 spots"

10 songs taken 6 at a time

10 ___ 9 ___ 8 ___ 7 ___ 6 ___ 5 ___

What about the remaining spaces?

How many options *left* for the second space?

How many options for the first space?

<u>The "multiplication principle."</u>

When arranging things in order (letters A, B, and C), the <u>total</u> number of possible ways to arrange things is the <u>product</u> of the number of possibilities for each step.

$$10 * 9 * 8 * 7 * 6 * 5 = 151{,}200$$

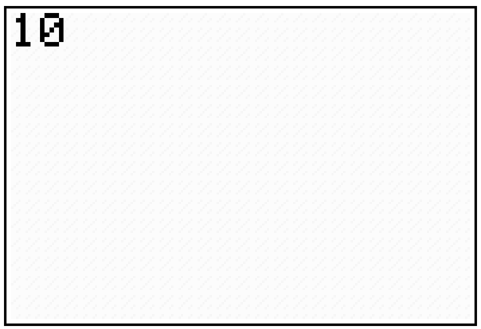
$$_nP_r = \frac{n!}{(n-r)!} \qquad {}_{10}P_6 = \frac{10!}{(10-6)!}$$

Permutations using your calculator

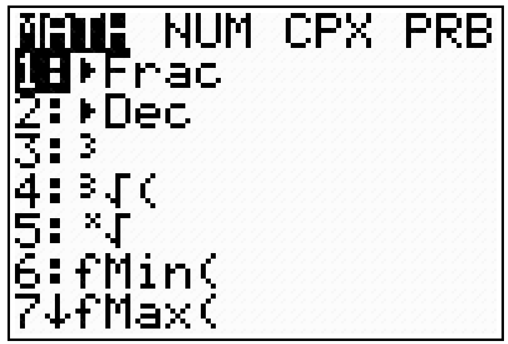
$$_{n}P_{r} = \frac{n!}{(n-r)!} \qquad _{10}P_{6} = \frac{10!}{(10-6)!}$$

“10 permutate 6”

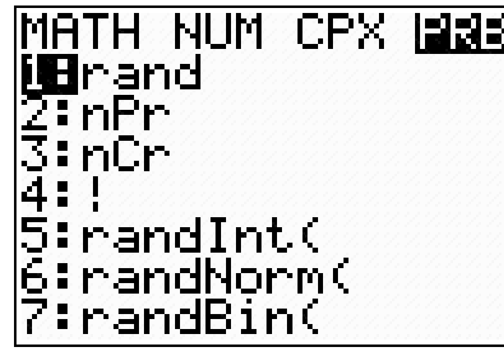
Clear your screen then enter “10”



“Math” button

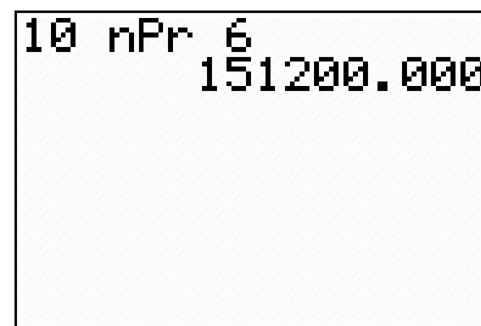


Scroll to “PRB”



Select option “2”

Hit “6” then “enter”



There are 10 candidates. The one with the highest number of votes will be president, the 2nd highest will be vice president and the 3rd highest will be secretary.

How many ways are there to arrange 3 candidates chosen from a group of 10 in the positions of president, vice president and secretary?

$${}_{10}P_3 = \frac{10!}{7!} = 720$$

Permutations.

If we were making a permutation using the letters 'D', 'A', 'W', and 'G' DAWG and WADG

ORDER MATTERS!! (with permutations) → a different order of members is a different group all together!!

I have 4 bills in my wallet: $1, $2, $5, $10 $\quad {}_4P_3 = 24$

How many different <u>sequences</u> of bills can I take out of my wallet, if I only take 3 out?

24 ways

1, 2, 5	10, 2, 1	1, 10, 5	10, 2, 5
1, 5, 2	10, 1, 2	1, 5, 10	10, 5, 2
2, 1, 5	2, 10, 1	10, 1, 5	2, 10, 5
2, 5, 1	2, 1, 10	10, 5, 1	2, 5, 10
5, 1, 2	1, 10, 2	5, 1, 10	5, 10, 2
5, 2, 1	1, 2, 10	5, 10, 1	5, 2, 10

Each of these groups is just a permutation of the # of ways to arrange 3 different bills.

I have 4 bills in my wallet: $1, $2, $5, $10

How many different <u>sums</u> of money can I take out of my wallet, if I only take 3 bills out? 4 ways

1, 2, 5	10, 2, 1	1, 10, 5	10, 2, 5
1, 5, 2	10, 1, 2	1, 5, 10	10, 5, 2
2, 1, 5	2, 10, 1	10, 1, 5	2, 10, 5
2, 5, 1	2, 1, 10	10, 5, 1	2, 5, 10
5, 1, 2	1, 10, 2	5, 1, 10	5, 10, 2
5, 2, 1	1, 2, 10	5, 10, 1	5, 2, 10
= $8	= $13	= $16	= $17

ORDER Doesn't MATTER!! → a different order of pulling the same 3 bills out doesn't make a different sum.

If order doesn't matter, then we have "<u>double counted</u>" the number of sums by the number of ways to arrange 3 different bills in order.

We call this new method of counting a "*combination*".

$$\frac{{}_4P_3}{3!} = \frac{24}{6} = 4$$

$$\frac{{}_nP_r}{r!} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

1, 2, 5	10, 2, 1	1, 10, 5	10, 2, 5
1, 5, 2	10, 1, 2	1, 5, 10	10, 5, 2
2, 1, 5	2, 10, 1	10, 1, 5	2, 10, 5
2, 5, 1	2, 1, 10	10, 5, 1	2, 5, 10
5, 1, 2	1, 10, 2	5, 1, 10	5, 10, 2
5, 2, 1	1, 2, 10	5, 10, 1	5, 2, 10
= $8	= $13	= $16	= $17

Using the multiplication principle of counting we must divide out the number of ways we have "double counted".

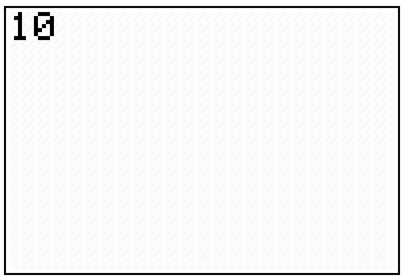
Combinations using your calculator

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$

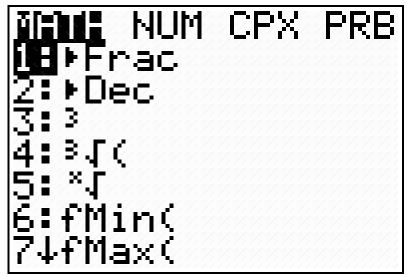
$$ {}_{10}C_{5} = \frac{10!}{5!(10-5)!} $$

“10 choose 5”

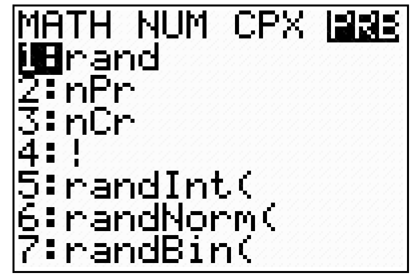
Clear your screen then enter “10”



“Math” button

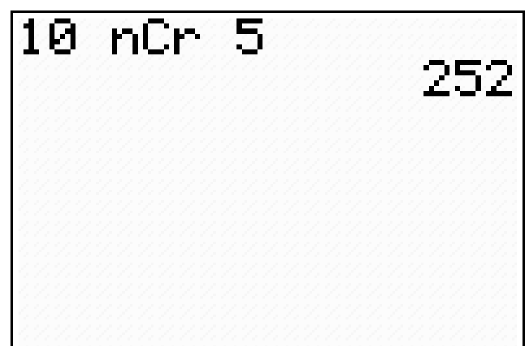


Scroll to “PRB”



Select option “3” then hit “5”

Now “enter”



“Permutation” “Combination”

You are tasked to count the number of ways the following items could occur. Decide if you will use a permutation or a combination (write “P” or “C”) for each of the following:

3 people chosen out of a group of 10 to be the president, vice president and secretary of a club.

3 people chosen out of a group of 10 to members of a committee.

The top 3 finishers of a race involving 20 runners.

The 1^{st}, 2^{nd}, and 3^{rd} place finishers of a race involving 20 runners.

“Order Matters” vs. “Order Doesn’t Matter”

Permutation Different order of the same items counted as a separate arrangement

→ Different ways to line up people/things in order

→ If you see the words “…in order” in the question (“golf” and “flog” are different words using the same 4 letters).

→ Different presidencies

→ Different prizes based upon order of finish in a race

“Order Matters” vs. “Order Doesn’t Matter”

Combination Different order of the same items
→ can not be counted as separate arrangement

→ Different total scores (summing the roll of two dice, etc.)

→ Different total amounts of money

→ Different “hands” of cards dealt in a game of cards (in games where you can rearrange the cards in your hand once they are dealt)

→ Different committees of people

How many different committees with 5 members can be formed when choosing from 25 candidates?

You are dealt 5 cards in a card game where you are allowed to rearrange the cards in your hand. How many different “5 card hands” are possible? (you may rearrange the cards after they have been dealt).

The number of ways 700 people can line up while in the lunch line.

1. The multiplication rule for counting ways things can be arranged in order.

2. The difference between a permutation and a combination when counting the ways to arrange things in order.

3. How to use a calculator to find the number of ways to arrange things in order (permutation or combination).