Math-2
8-5

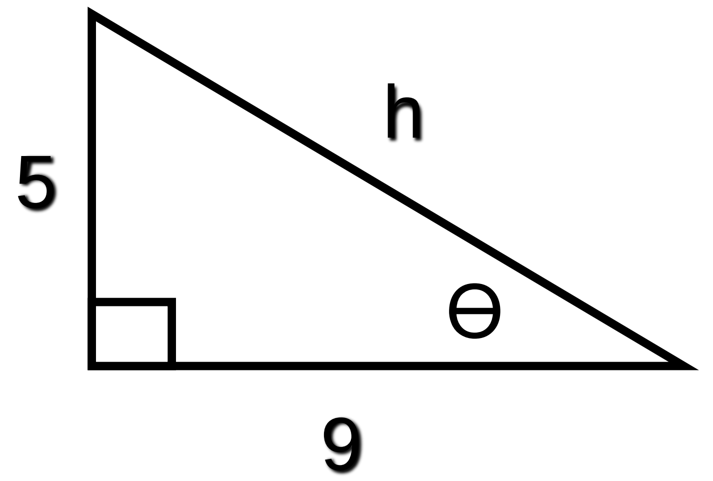
Radian Measure

and

the Measures of Arcs

$$\tan\theta = 5/9$$

$$\cos\theta = ?$$



Tangent ratio gives 2 sides of a right triangle.

$$h = \sqrt{5^2 + 9^2}$$

$$h = \sqrt{25 + 81}$$

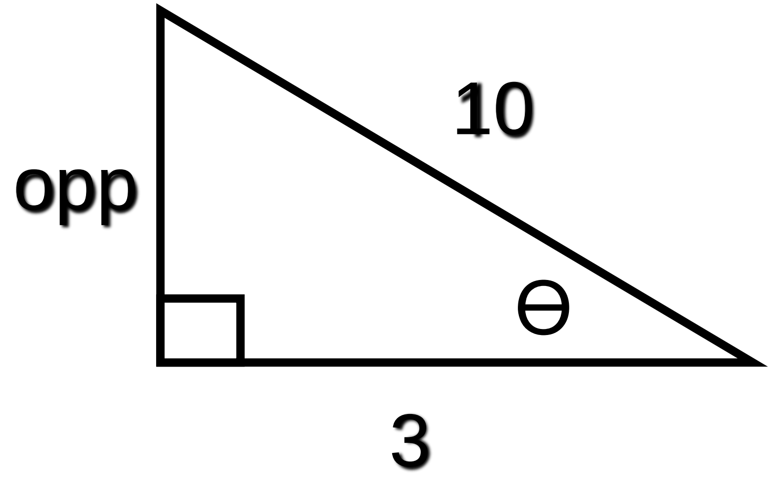
$$h = \sqrt{106}$$

$$\cos\theta = \frac{9}{\sqrt{106}}$$

$$\cos\theta = \frac{9\sqrt{106}}{106}$$

$$\cos\theta = {}^{3}\!/_{10}$$

$$\sin\theta = ?$$



cosine ratio gives 2 sides of a right triangle.

$$h = \sqrt{10^2 - 3^2}$$

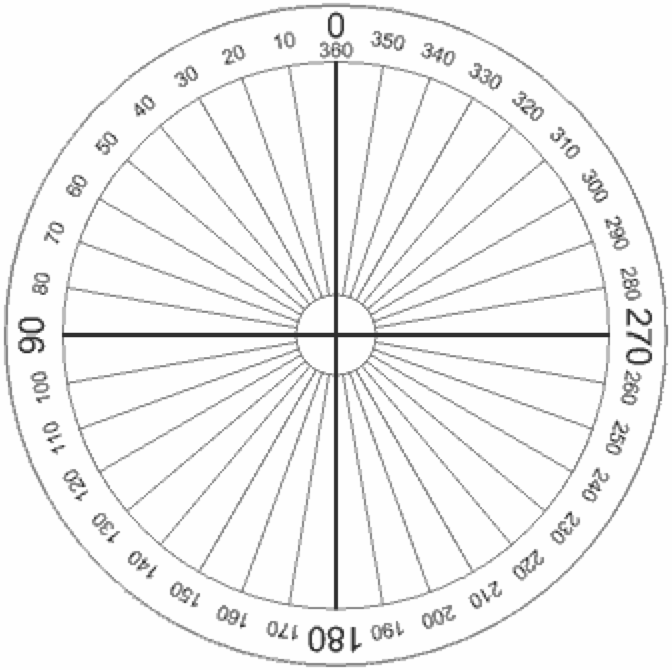
$$h = \sqrt{100 - 9}$$

$$h = \sqrt{91}$$

$$\sin\theta = \frac{\sqrt{91}}{10}$$

Why 360º?

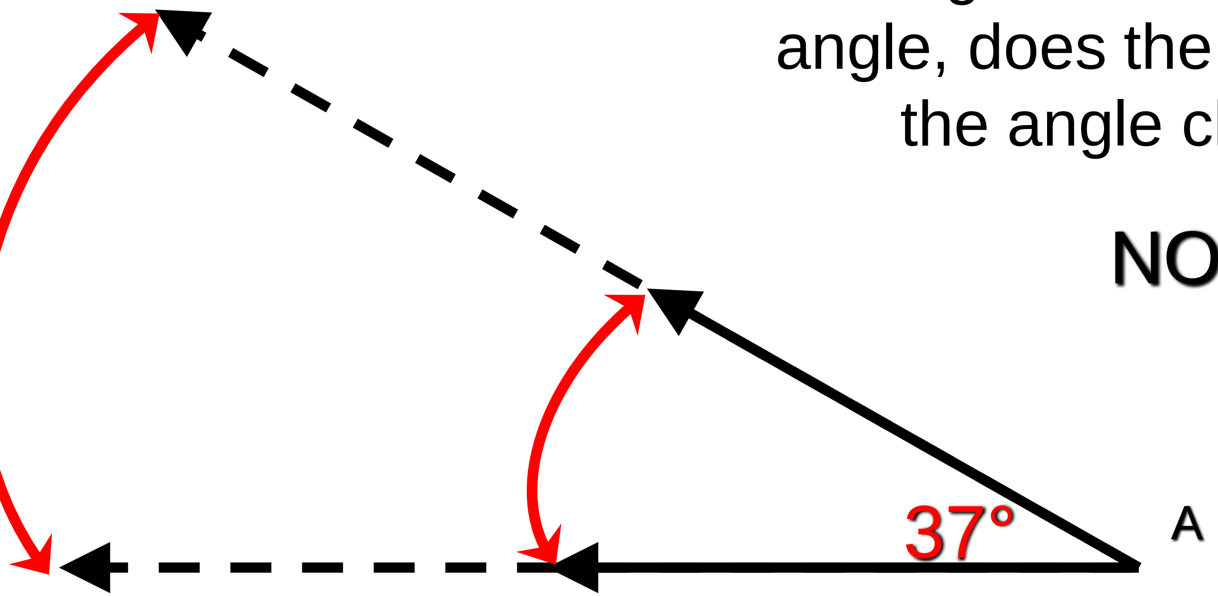
The idea of dividing a circle into 360 equal pieces dates back to the sexagesimal (60-based) counting system of the ancient Sumarians. Early astronomical calculations linked the sexagesimal system to circles.



Angles vs. arcs

If I lengthen the sides of the angle, does the measure of the angle change?

NO



Notice that the length of the arc depends upon how far the arc is from the vertex of the angle.

Degrees: The measure of an angle as a portion of 360° (the angle measure of a circle).

$$90^\circ = \tfrac{1}{4} * 360^\circ$$

Radian measure: the ratio of the arc length to the distance the arc is from the vertex of the angle.

$$\text{radian measure} = \frac{arc\ \text{length}}{\text{radius}}$$

Pi: an irrational number that is the ratio of the distance around the circle to the distance across the circle.

$$\pi = \frac{C}{D} \qquad \pi = \frac{C}{2r} \qquad C = 2\pi r$$

$$\boxed{\text{radian measure} = \frac{arc\ \text{length}}{\text{radius}}}$$

Radian measure for a complete circle.

$$radian\ \text{measure of a circle} = \frac{circumference}{\text{radius}}$$

$$radian\ \text{measure of a circle} = \frac{2\pi\,\cancel{\text{r}}}{\cancel{\text{r}}}$$

$$radian\ \text{measure of a circle} = 2\pi\ \text{radians}$$

Units of radians = inches/inches

Radian measure has <u>no units</u>! (nice)

What is the radian measure?

$360^o = \underline{\ 2\pi\ }$

$180^o = \underline{\ \pi\ }$

$60^o = \underline{\ \frac{\pi}{3}\ }$

$90^o = \underline{\ \frac{\pi}{2}\ }$

$30^o = \underline{\ \frac{\pi}{6}\ }$

$45^o = \underline{\ \frac{\pi}{4}\ }$

Degree-Radian Conversion Degree-Radian Conversion

$$180° = \pi \text{ radians}$$

To convert radians to degrees, multiply by $\dfrac{180^\circ}{\pi \text{ radians}}$.

To convert degrees to radians, multiply by $\dfrac{\pi \text{ radians}}{180^\circ}$.

$\left(\dfrac{\pi}{180^o}\right)$ These are "<u>conversion factors</u>" $\left(\dfrac{180^o}{\pi}\right)$

The ratio of these two numbers *<u>equals one</u>*.

Multiplication by "one" does not change the number.
It just changes what the number looks like.

Converting from Degrees to Radian Measure

$$\cancel{140^{o}}\left(\frac{\pi}{\cancel{180^{o}}}\right) = \frac{140}{180}\pi = \frac{14}{18}\pi = \frac{7}{9}\pi$$

Converting from Radian Measure to Degrees

$$\frac{\cancel{\pi}}{2}\left(\frac{180^{o}}{\cancel{\pi}}\right) = 90^{\circ}$$

Convert between radians and degrees using a "proportion".

$$\boxed{\frac{angle_{degrees}}{360} = \frac{angle_{radians}}{2\pi}}$$

$$\frac{7}{8}\pi$$

$$\frac{angle_{degrees}}{360} = \frac{{}^{7}/_{8}\cancel{\pi}}{2\cancel{\pi}}$$

$$360 * \frac{angle_{degrees}}{360} = 0.4375 \; * 360$$

$$angle_{degrees} = 157.5°$$

Your Turn: Convert between radians and degrees.

$$\frac{11}{3}\pi = ?$$

$$270^{o} = ?$$

Radian measure: the ratio of the arc length to the radius of the circle:

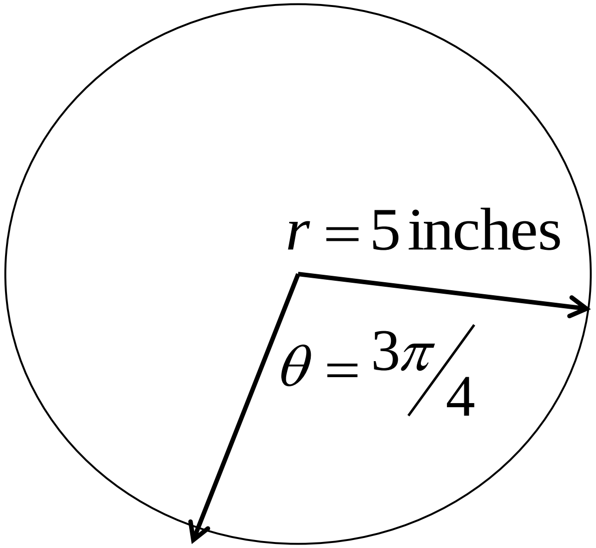
$$\boxed{\text{radian measure} = \frac{arc\ \text{length}}{\text{radius}}}$$

Theta: a Greek letter. Traditionally, we use Greek letters as variables for the measure of an angle.

$$\boxed{\theta = \frac{s}{r}} \qquad r\theta = s \qquad \boxed{s = r\theta}$$

Problem types you'll see:

What is length of the subtended arc?



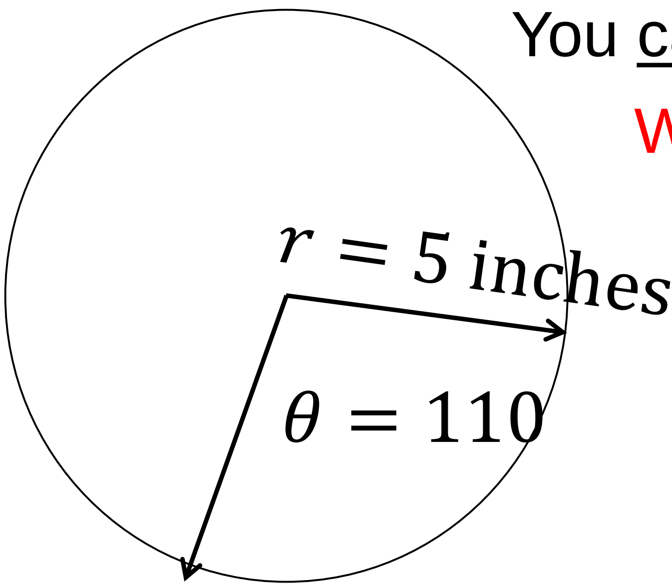
$$\boxed{s = r\theta}$$

$$s = 5 * {}^{3\pi}\!/_{4}$$

$$s = \frac{15\pi}{4} \text{ inches}$$

You cannot use: $\boxed{s = r\theta}$

Why not?



This formula is a re-arrangement of the definition of radian measure. You cannot use degrees with this formula.

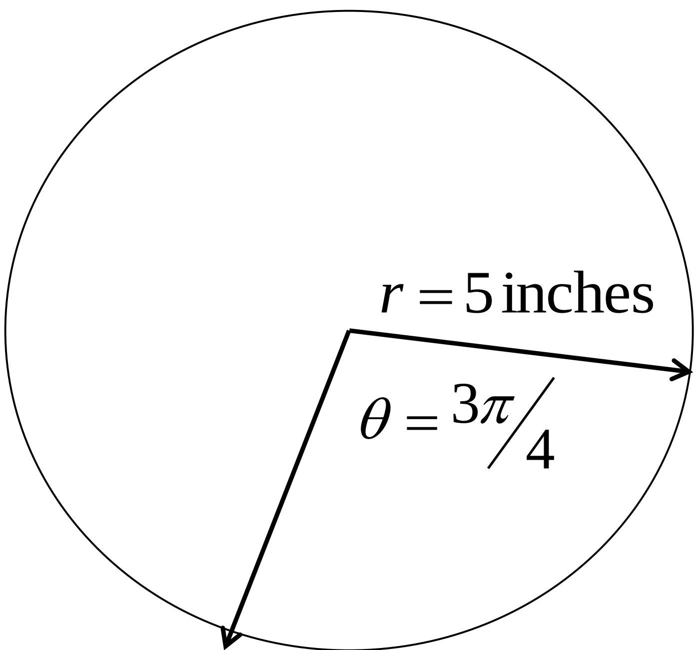
Solving "subtended arc" problems: (1) use a proportion OR (2) convert the angle measure to radians and use the formula.

$$\boxed{\frac{\text{part}}{\text{whole}_{(\text{arc lengths})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}}$$

$$\frac{s}{2 * \pi * r} = \frac{\theta}{360 \text{ or } 2\pi}$$

$$\boxed{\frac{\text{arc of the sector}}{\text{total arc of the circle}} = \frac{\text{angle of the sector}}{\text{total angle of the circle}}}$$

$$\frac{s}{\cancel{2} * \cancel{\pi} * 5} = \frac{3\pi/4}{\cancel{2\pi}}$$



$$s = \frac{5 * 3}{4}\pi$$

$$\boxed{s = \frac{15\pi}{4} \text{ in}}$$

We want our answers in reduced fraction form, with (Pi) π in the answer.

Find the length of the crust of a slice of pizza.

14 inch pizza (diameter)

Slice is 1/8 of the pizza

$$\boxed{\frac{\text{part}}{\text{whole}_{\text{(arc lengths)}}} = \frac{\text{part}}{\text{whole}_{\text{(angles)}}}}$$

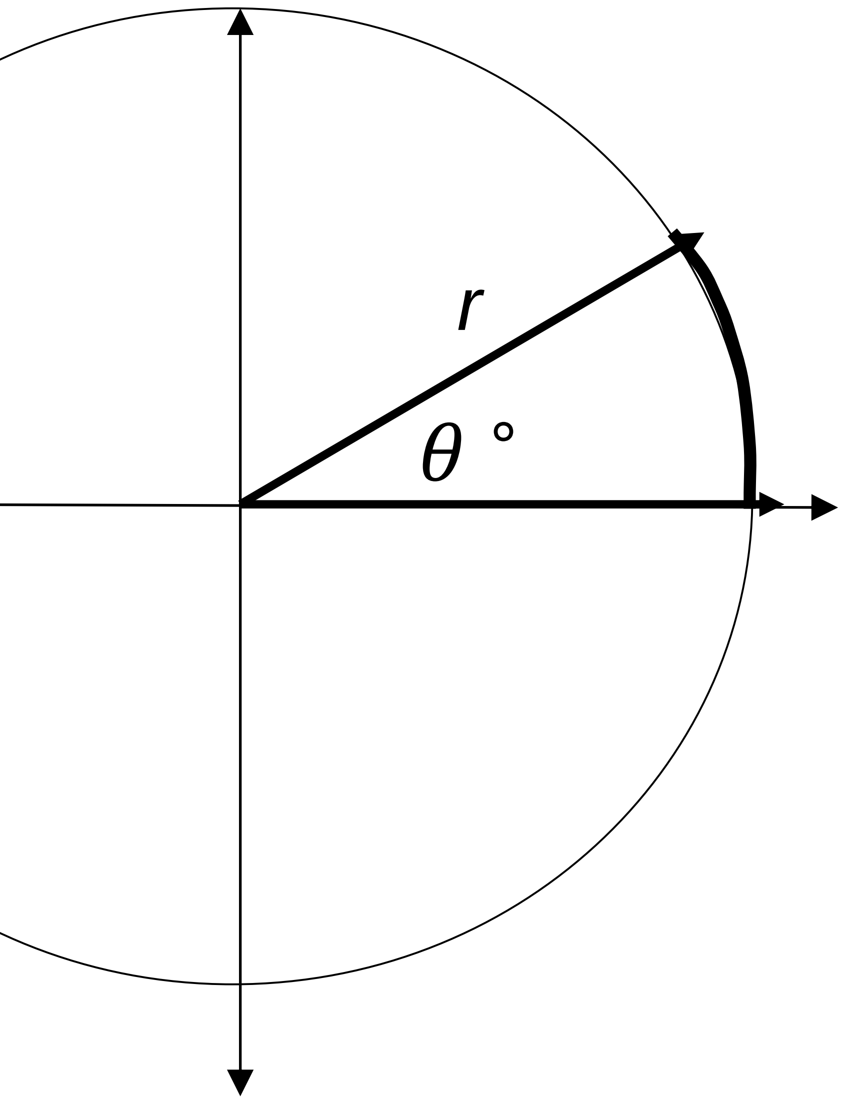
$$\frac{s}{2\pi r} = \frac{1}{8} * 360^{o}$$

$$\frac{s}{2\pi r} = 2 * *\pi * 7 * \frac{1}{8} * 360^{o}$$

The area of a Sector, is a fraction of the area of a circle.

Write a proportion.

$$\frac{\text{part}}{\text{whole}_{(\text{areas})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}$$



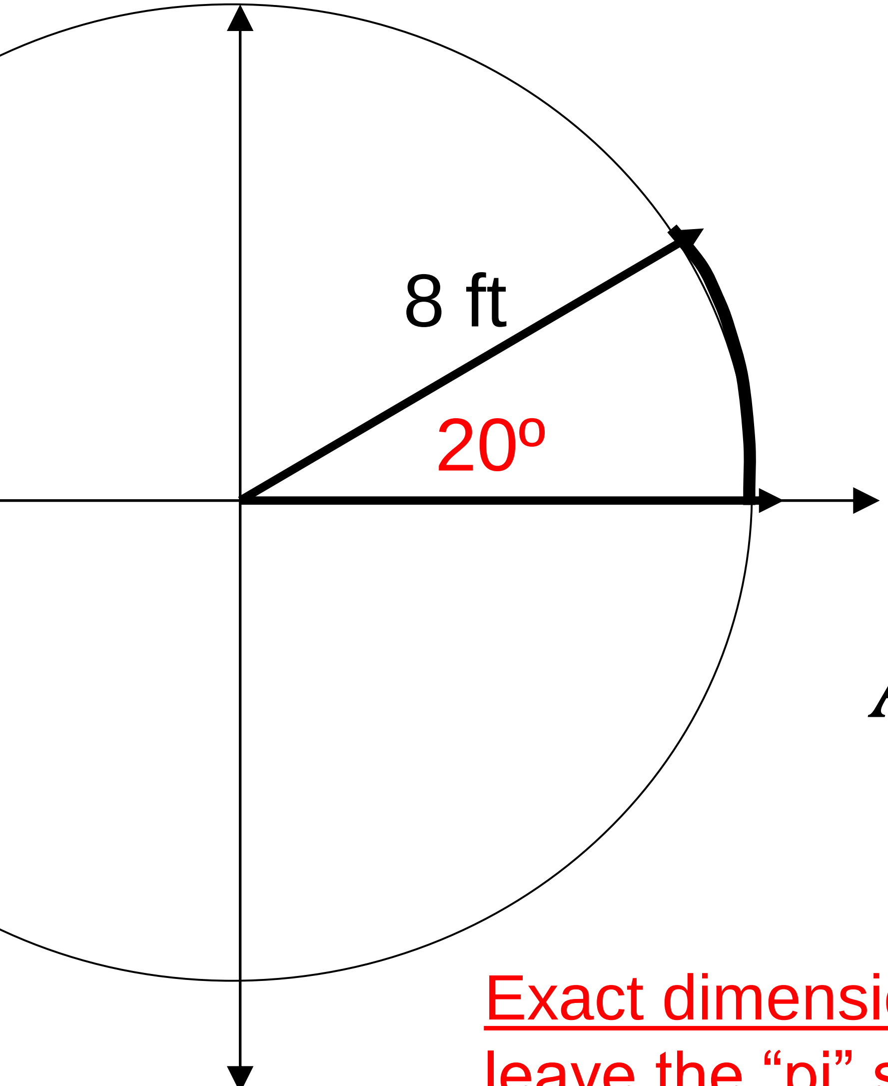
$$\frac{A_{\text{sector}}}{\pi * r^2} = \frac{\theta}{360 \text{ or } 2\pi}$$

Exact dimension → Exact answer

leave the “pi” symbol in your answer

A circle has an 8 foot radius. What is the area of a 20° sector?

Write a proportion.



$$\frac{\text{part}}{\text{whole}_{(\text{areas})}} = \frac{\text{part}}{\text{whole}_{(\text{angles})}}$$

$$\boxed{\frac{A_{\text{sector}}}{\pi * r^2} = \frac{\theta}{360 \text{ or } 2\pi}}$$

$$\frac{A_{sector}}{\pi * 8^2} = \frac{20}{360}$$

$$\frac{A_{sector}}{64\pi} = 0.5555\bar{5}$$

$$A_{sector} = 64 * 0.5555\bar{5} * \pi$$

$$A_{\text{sector}} = \frac{32}{9}\pi \text{ ft}^2$$

Exact dimension → Exact answer

leave the “pi” symbol in your answer

Find the area of the crust of a slice of pizza.

14 inch pizza (diameter)

Slice is 1/8 of the pizza

$$A = \frac{1}{8} * \pi * r^2$$

$$A = \frac{1}{8} * \pi * 7^2$$

$$A = \frac{49}{8}\pi \ \ in^2$$