Math-2
Lesson 8-2

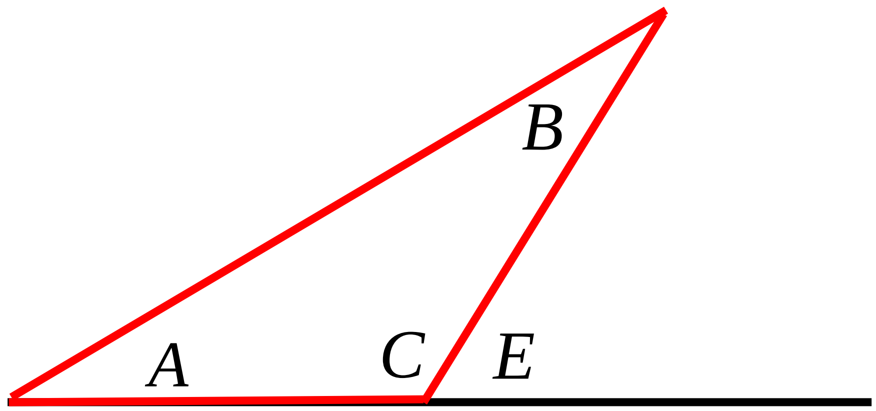
Exterior Angle Theorem,

Arcs, Central Angles, and

Inscribed Angles in Circles

<u>Exterior angle</u>: An angle formed by one side of a triangle and the extension of the adjacent side of the triangle.

Angle “E” is an exterior angle to triangle ABC.

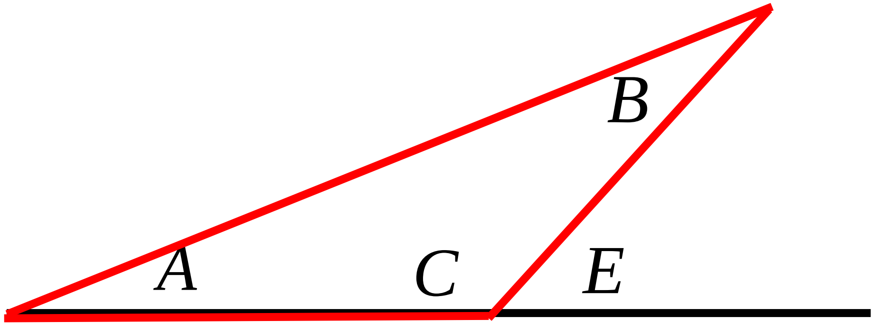


<u>Remote interior angle</u>: The two angles of a triangle that are on opposite sides of the triangle from the exterior angle.

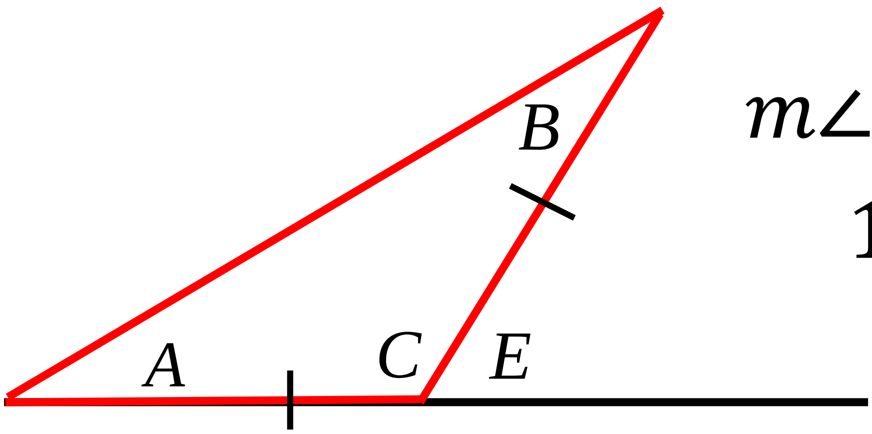
Angles “A” and “B” are “remote interior” angles to exterior angle “E”.

The "exterior angle" theorem

$m\angle A + m\angle B + m\angle C = 180$	"Triangle sum theorem"
$m\angle C + m\angle E = 180$	"Linear Pairs"
$m\angle A + m\angle B + m\angle C = m\angle C + m\angle E$	"substitution"
$m\angle A + m\angle B = m\angle E$	Property of equality (subtract $m\angle C$ from left/right)
The measure of an exterior angle equals the sum of the remote interior angles.	QED



Triangle ABC is Isosceles. The measure of exterior angle-E is 100. Find the measure of angle A.



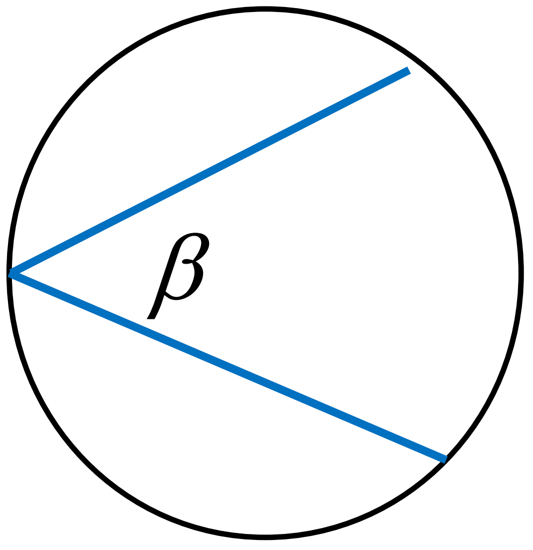
$$m\angle A = m\angle B$$

$$m\angle E = m\angle A + m\angle B$$

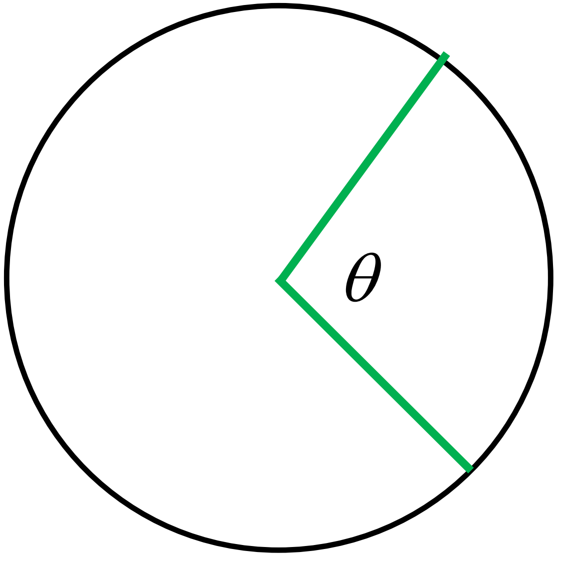
$$100 = 2 * m\angle A$$

$$50 = m\angle A$$

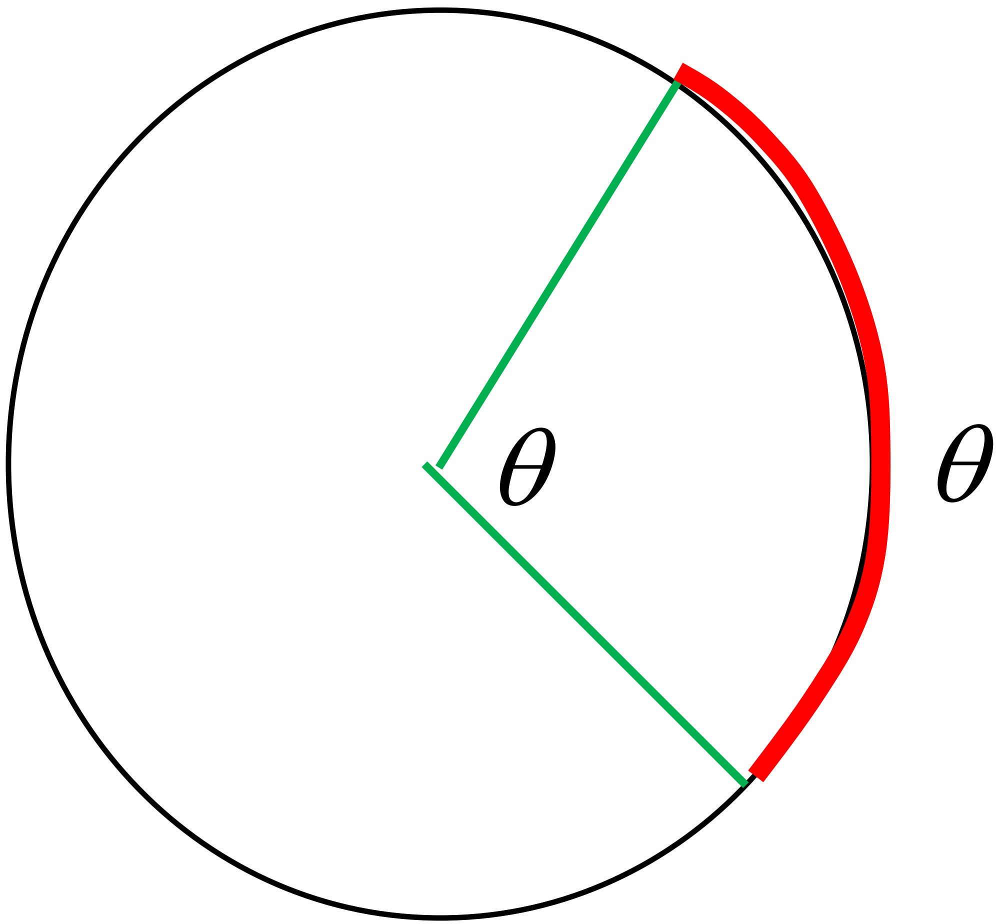
<u>Inscribed angle</u>: has its vertex <u>on</u> the circle.



<u>Central angle</u>: has its vertex <u>at</u> the <u>center of the circle</u>.

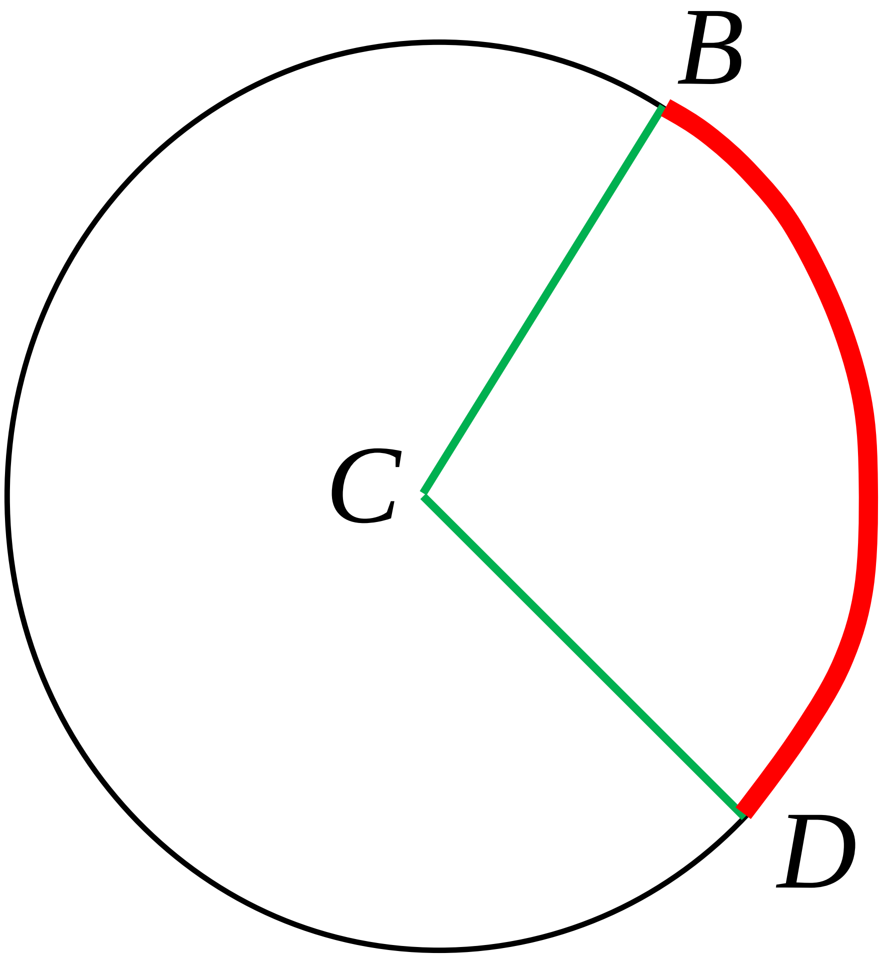


<u>Intercepted arc</u>: the arc of the circle that is in the interior of the angle. It has the <u>same degree-measure as the central angle.</u>



Naming Arcs

The arc subtended by Center Angle C is $\overset{\frown}{BD}$



Spoken: "arc BD"

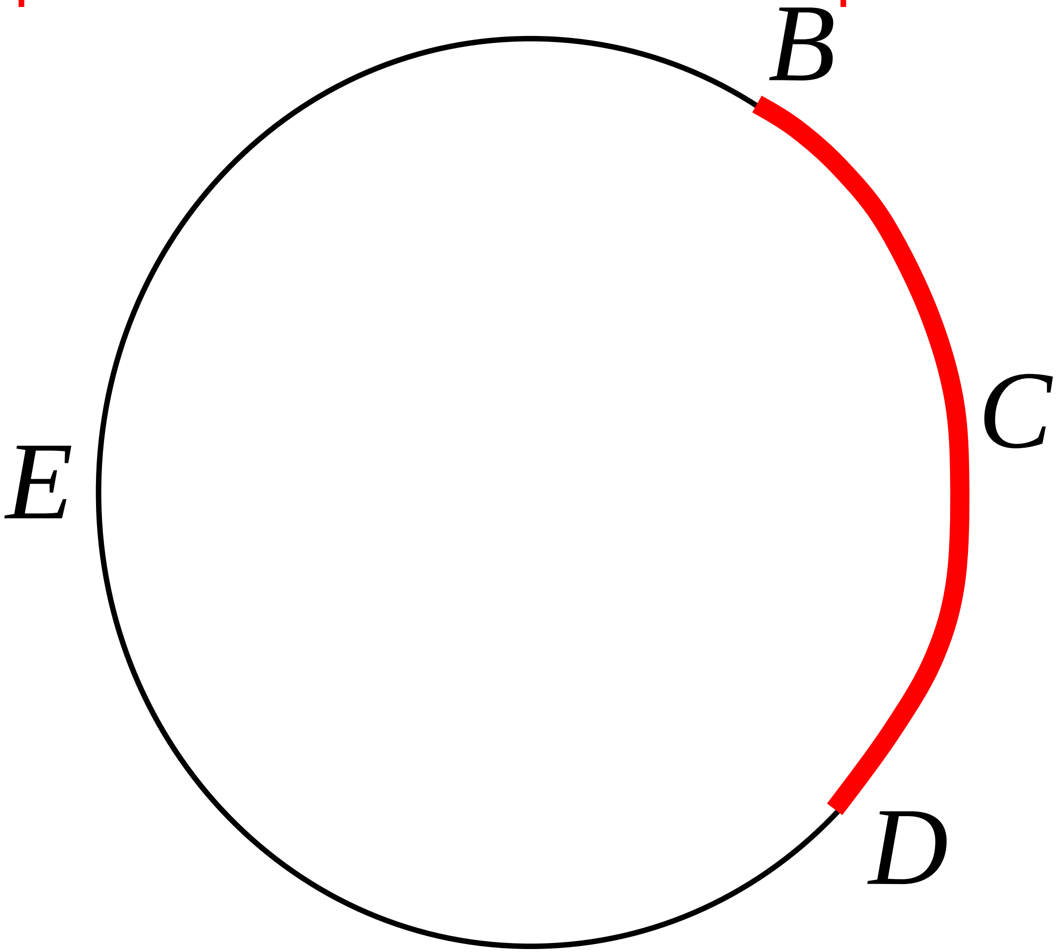
$m\overset{\frown}{BD}$

Spoken: "the measure of arc BD"

<u>Minor Arcs</u>: arcs that are less than half the circle.

<u>Major Arcs</u>: arcs that are more than half the circle.

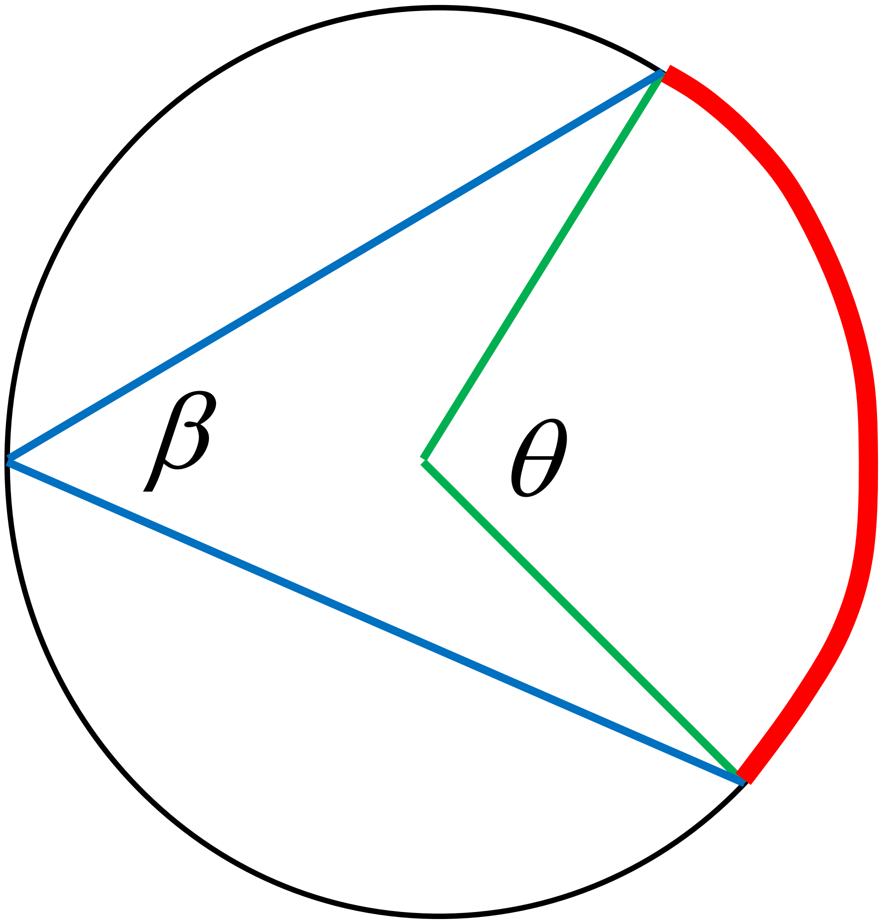
To distinguish between <u>minor arc BD</u> and <u>major arc BD</u>, we could add a letter between 'B' and 'D' to indicate a point in between that the arc passes through.



$\overset{\frown}{BCD}$ (minor arc)

$\overset{\frown}{BED}$ (major arc)

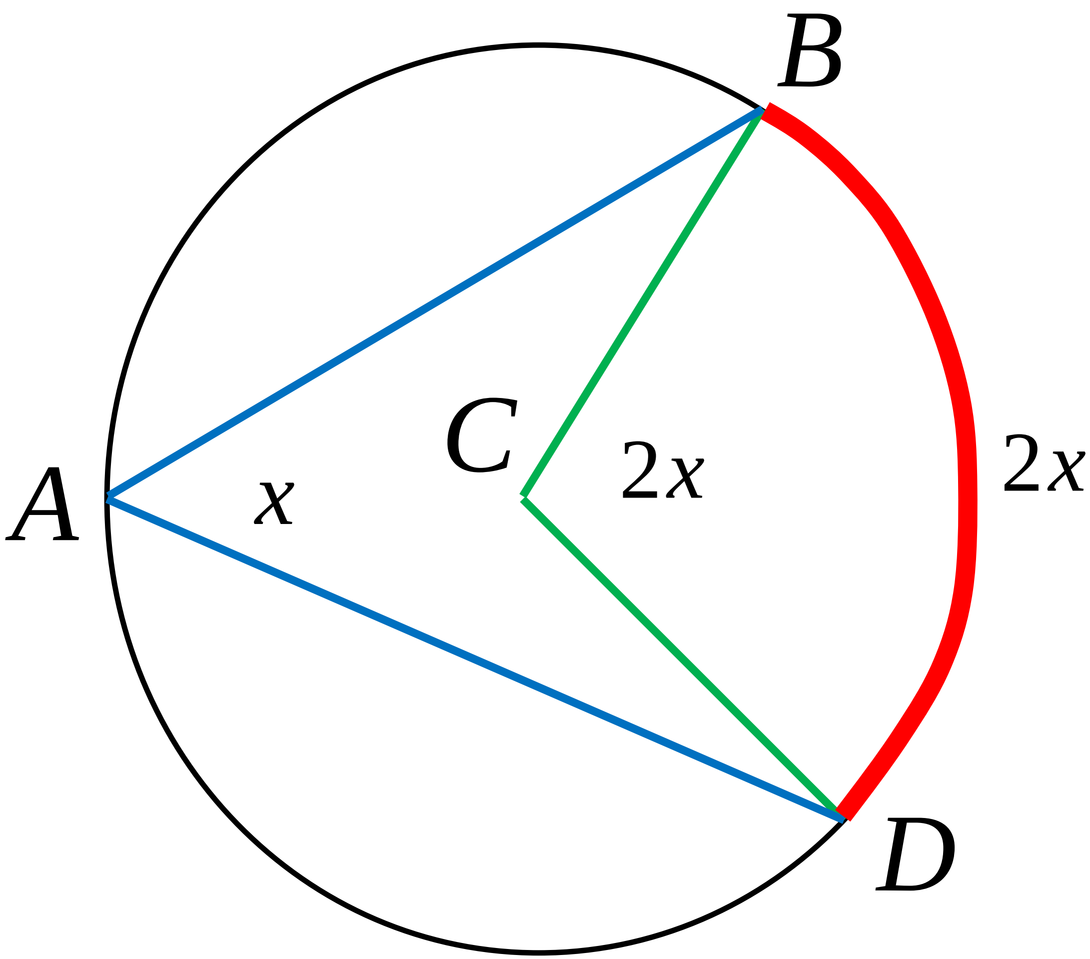
The “Central” Angle and the “inscribed” angle intercept (“cut”) the same arc.



Which angle has the larger measure?

Inscribed/Center Angle/Inscribed Arc Theorem

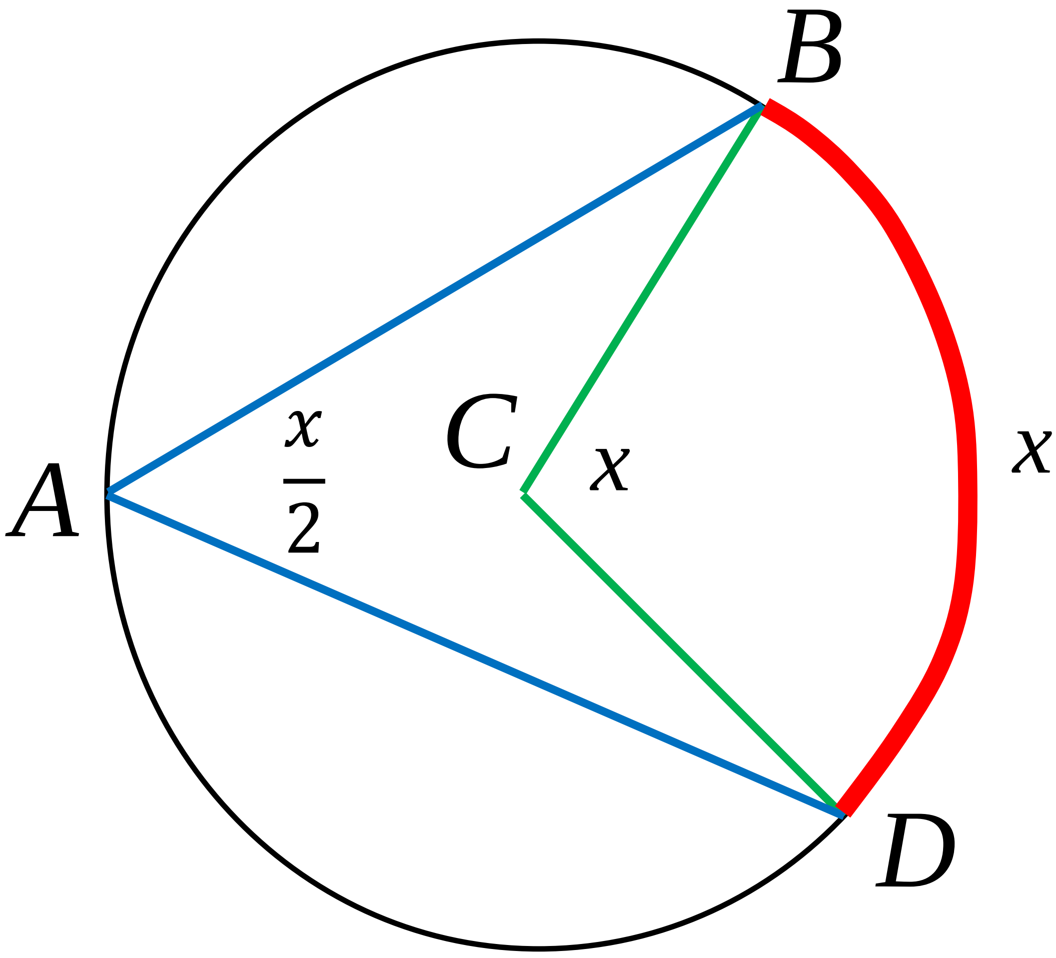
If an inscribed angle and a central angle subtend the same arc, then the measure of the central angle equals twice the measure of the inscribed angle.



If a central angle subtends an arc, then the measure of the arc equals twice the measure of the inscribed angle.

Inscribed/Center Angle/Inscribed Arc Theorem

If an inscribed angle subtends an arc, then the measure of the inscribed angle equals half the measure of the central angle (or subtended arc).



Find the measure of the angle.

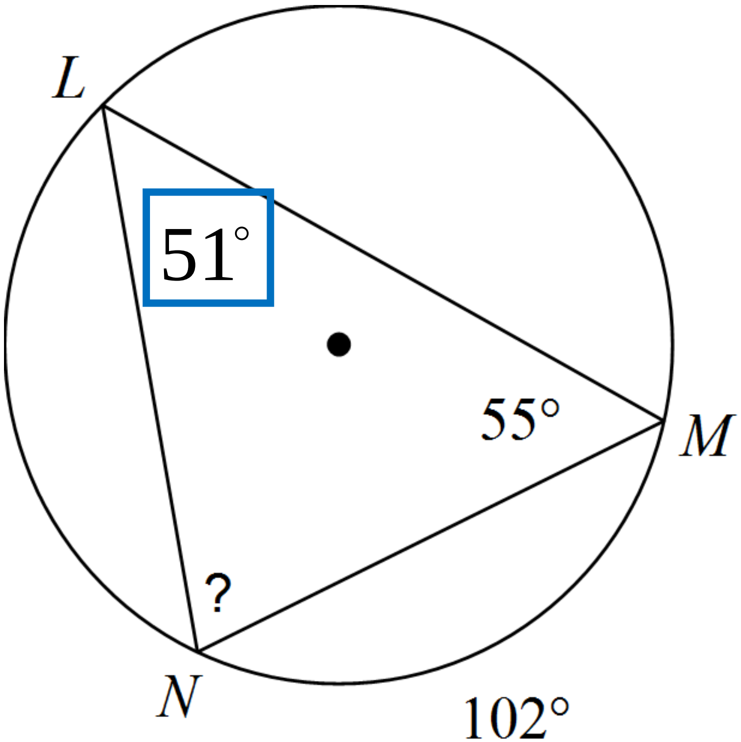
To solve for an unknown value, you need an equation.

1. Inscribed Angle. → Inscribed/Central Angle/Inscribed Arc Thm.

$m\angle L = ?$ $2m\angle L = m\widehat{NM}$ $m\angle L = 0.5 * 102^{\circ}$

$m\angle L = 51$

1. Triangle → Triangle Sum Theorem



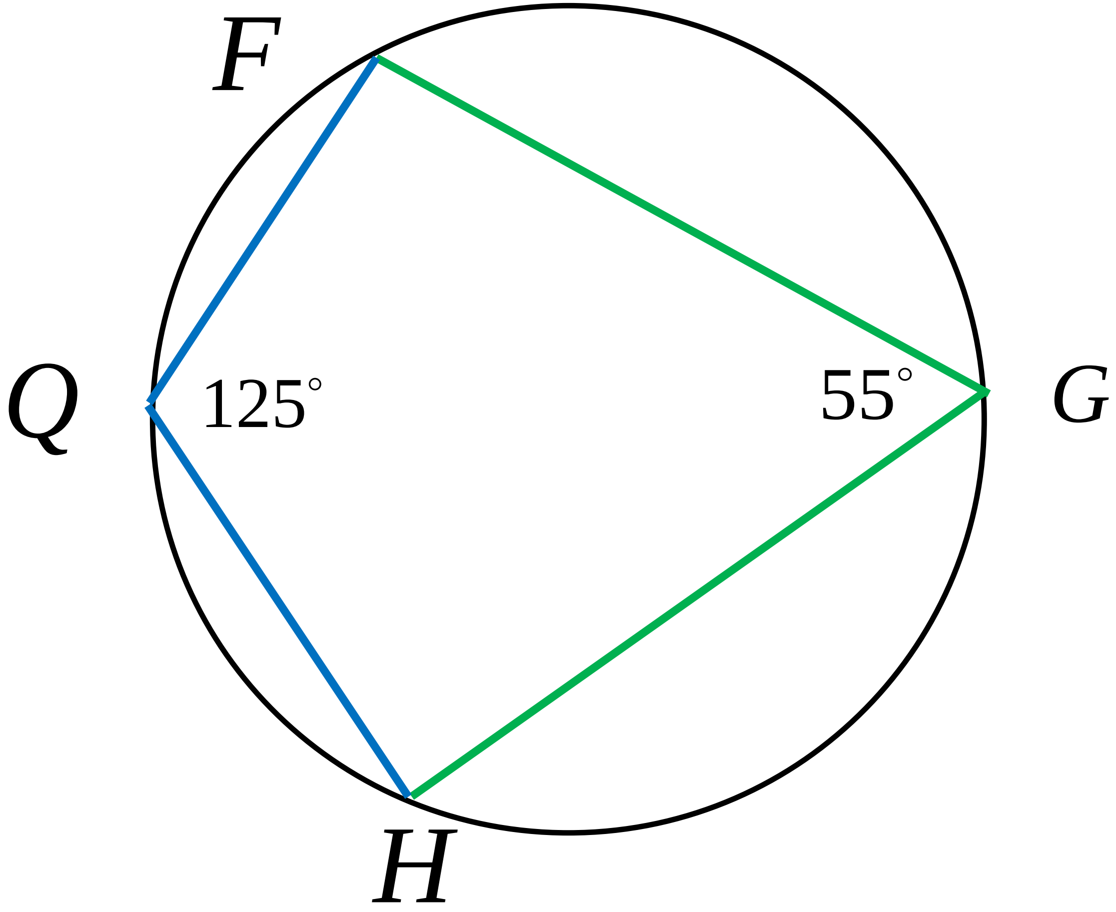
$m\angle N = ?$

$m\angle N = 180 - 55 - 51$

$m\angle N = 74$

A useful result of inscribed angles that cut "opposite" arcs.

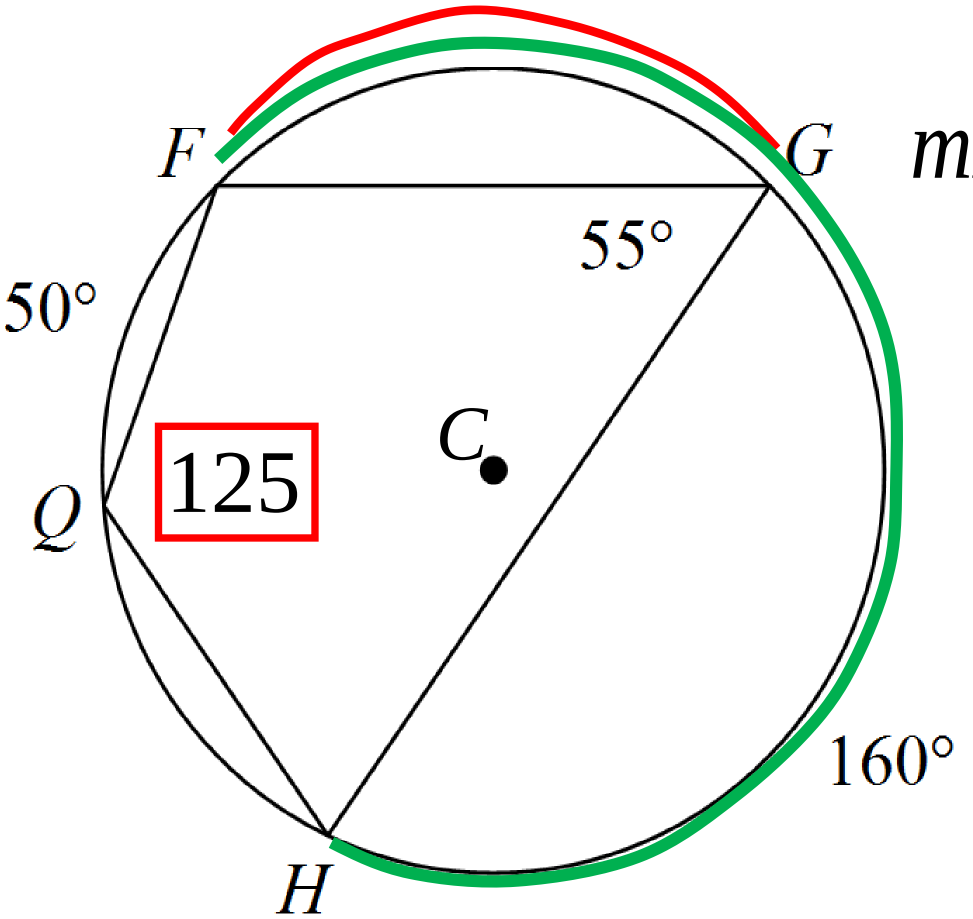
Inscribed angles that "cut opposite arcs are supplementary (add up to 180).



Find the measure of the angle.

$m\overset{\frown}{FG} = ?$ $m\angle Q = ?$ $\boxed{125}$ $m\overset{\frown}{FGH} = ?$

$= 2(125^\circ) = 250^\circ$



$m\overset{\frown}{FG} = m\overset{\frown}{FGH} - m\overset{\frown}{GH}$

$= 250^\circ - 160^\circ$

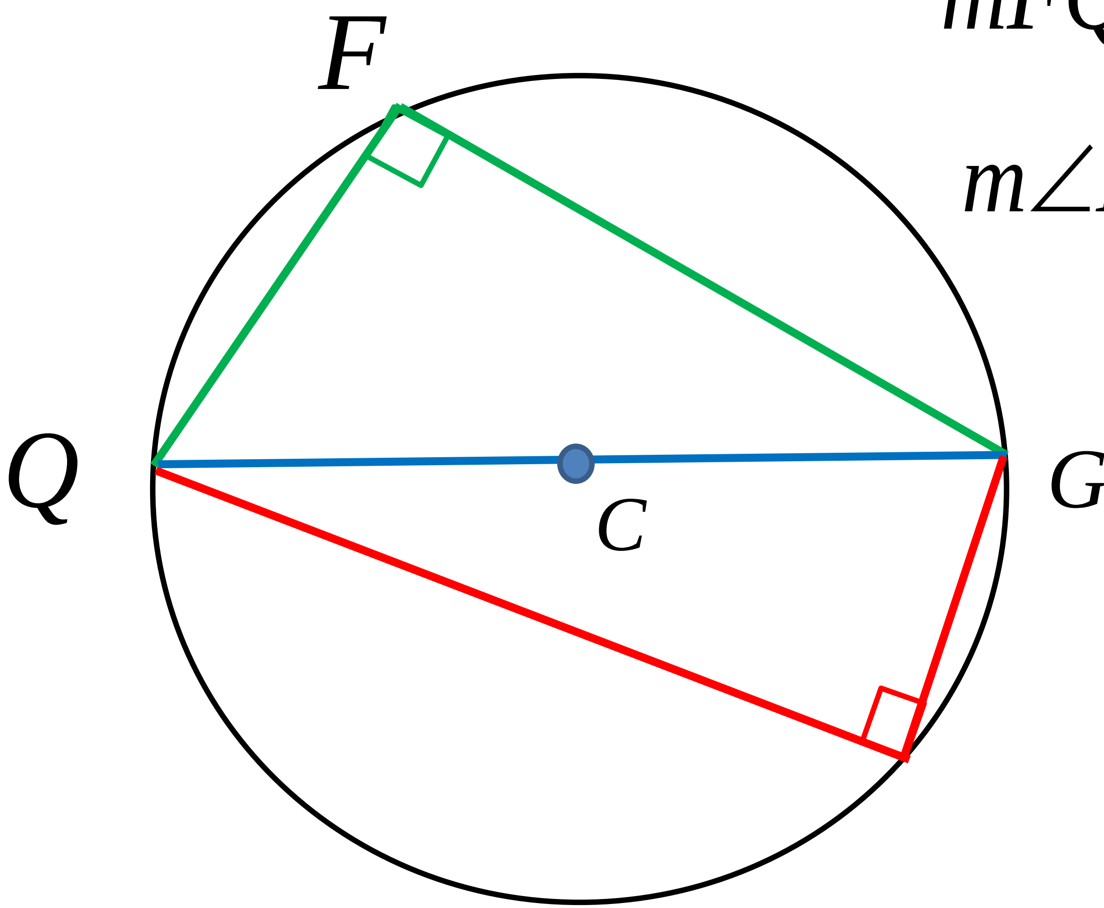
$= 90^\circ$

A useful result of an inscribed angle that cuts a diameter:

Segment QG is a diameter of circle C.

$$m\overset{\frown}{FQH} = ? = 180^{\circ}$$

$$m\angle F = ? \; = 90^{\circ}$$



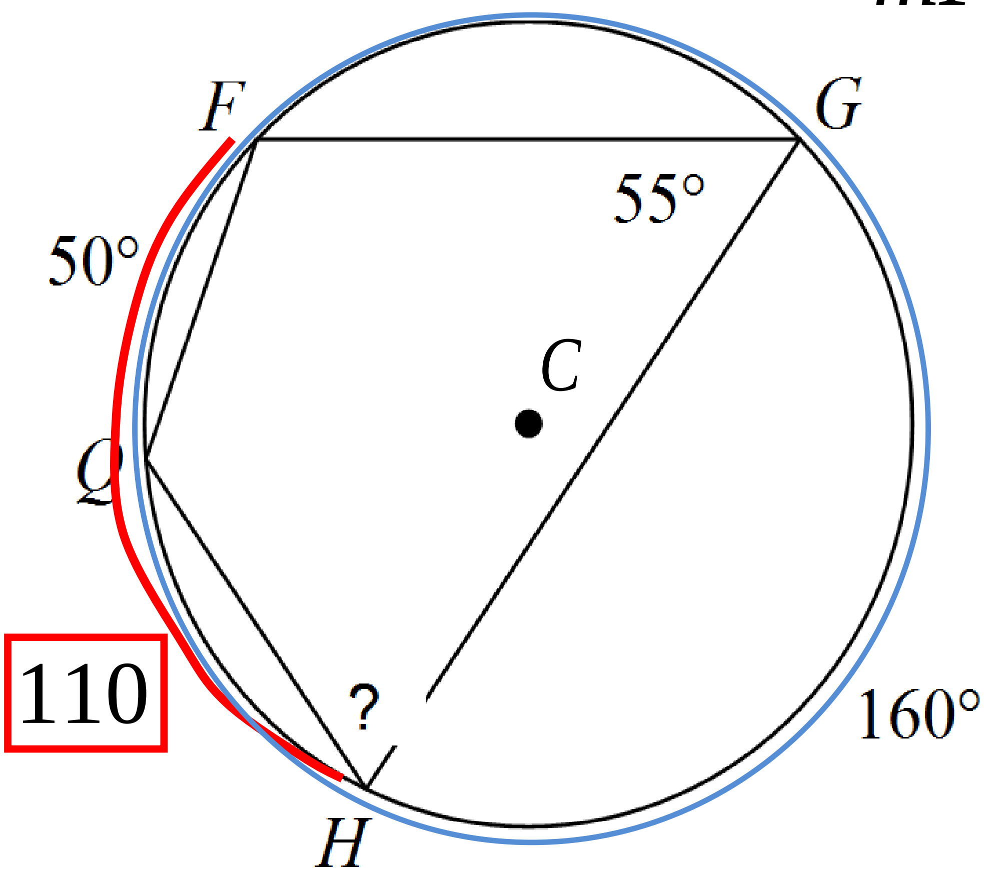
An inscribed angle that "cuts a diameter" always has a measure of 90.

Find the measure of the angle.

To solve for an unknown value, you need an ___________.

1. Inscribed Angle. → Inscribed/Central Angle/Inscribed Arc Theorem

$m\angle Q = ?$



$$m\overset{\frown}{FQH} = ? = 2(55^\circ) = 110^\circ$$

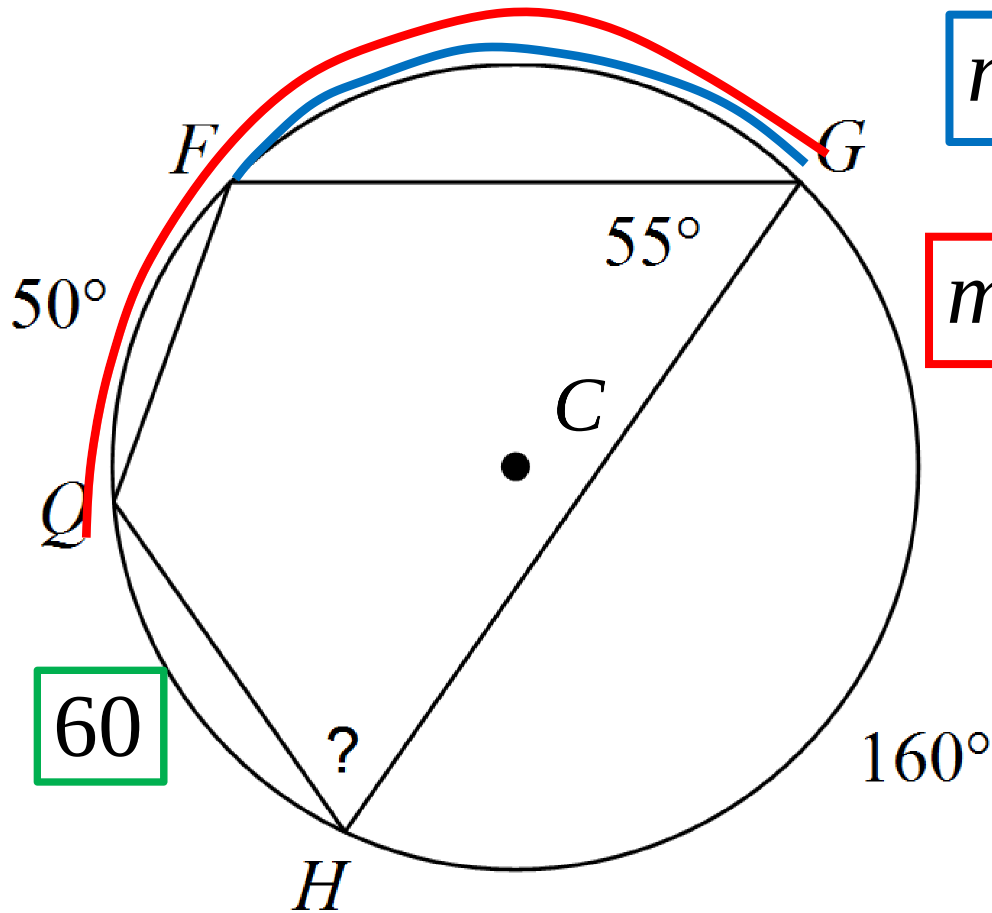
$$\boxed{m(Circle) = 360}$$

$$m\overset{\frown}{FGH} = ? = 360 - 110$$
$$= 250$$

$$m\angle Q = \frac{1}{2} * 250$$

$$m\angle Q = 125$$

$$m(arc\ FG) = 360 - 60 - 50 - 160$$



$$m(arc\ FG) = 90$$

$$m(arc\ QFG) = 50 + 90$$

$$m(arc\ QFG) = 140$$

$$m\angle QHG = 70$$