Math-2

Lesson 7-4

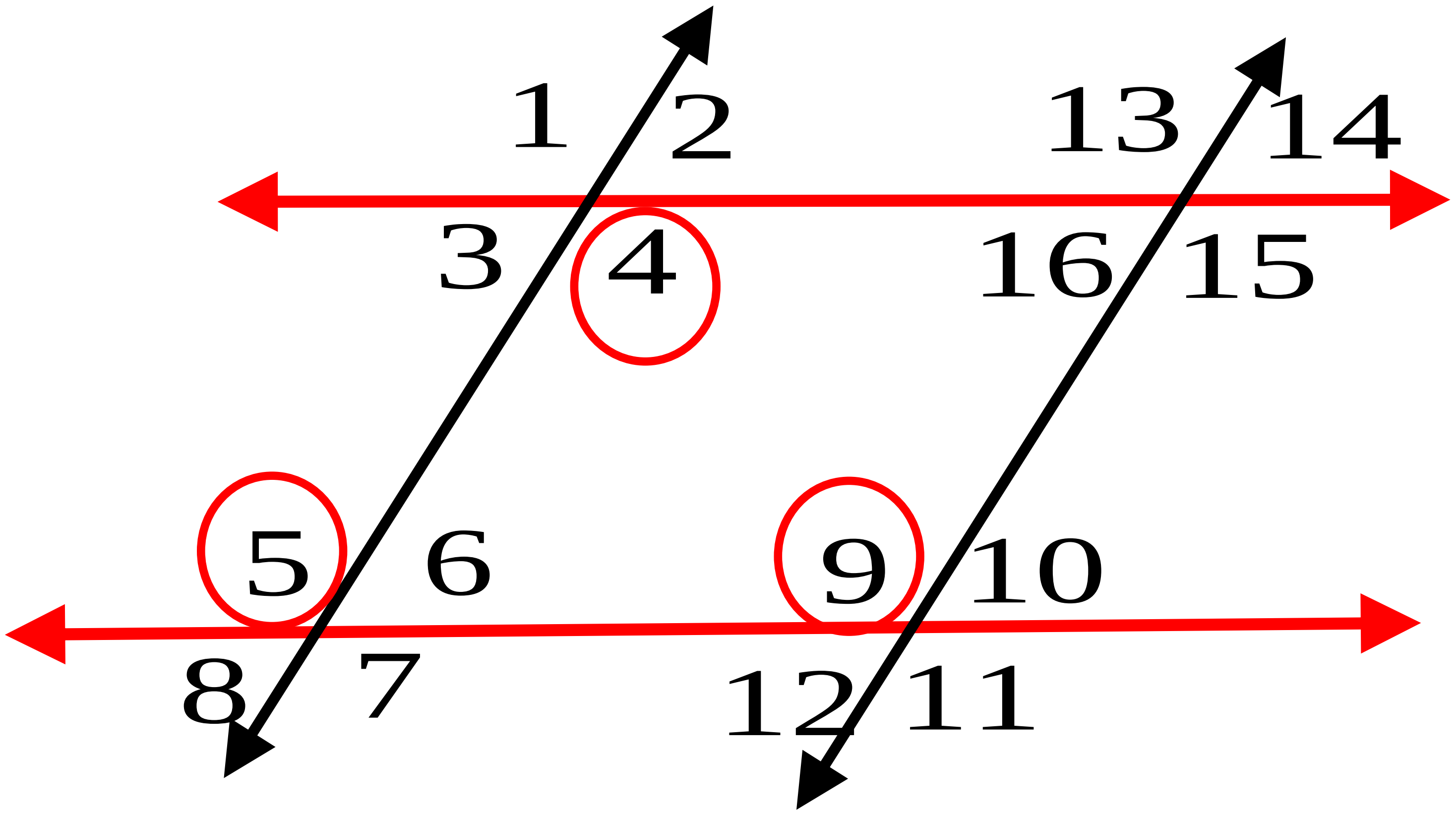
Properties of

Parallelograms

And

Isosceles Triangles

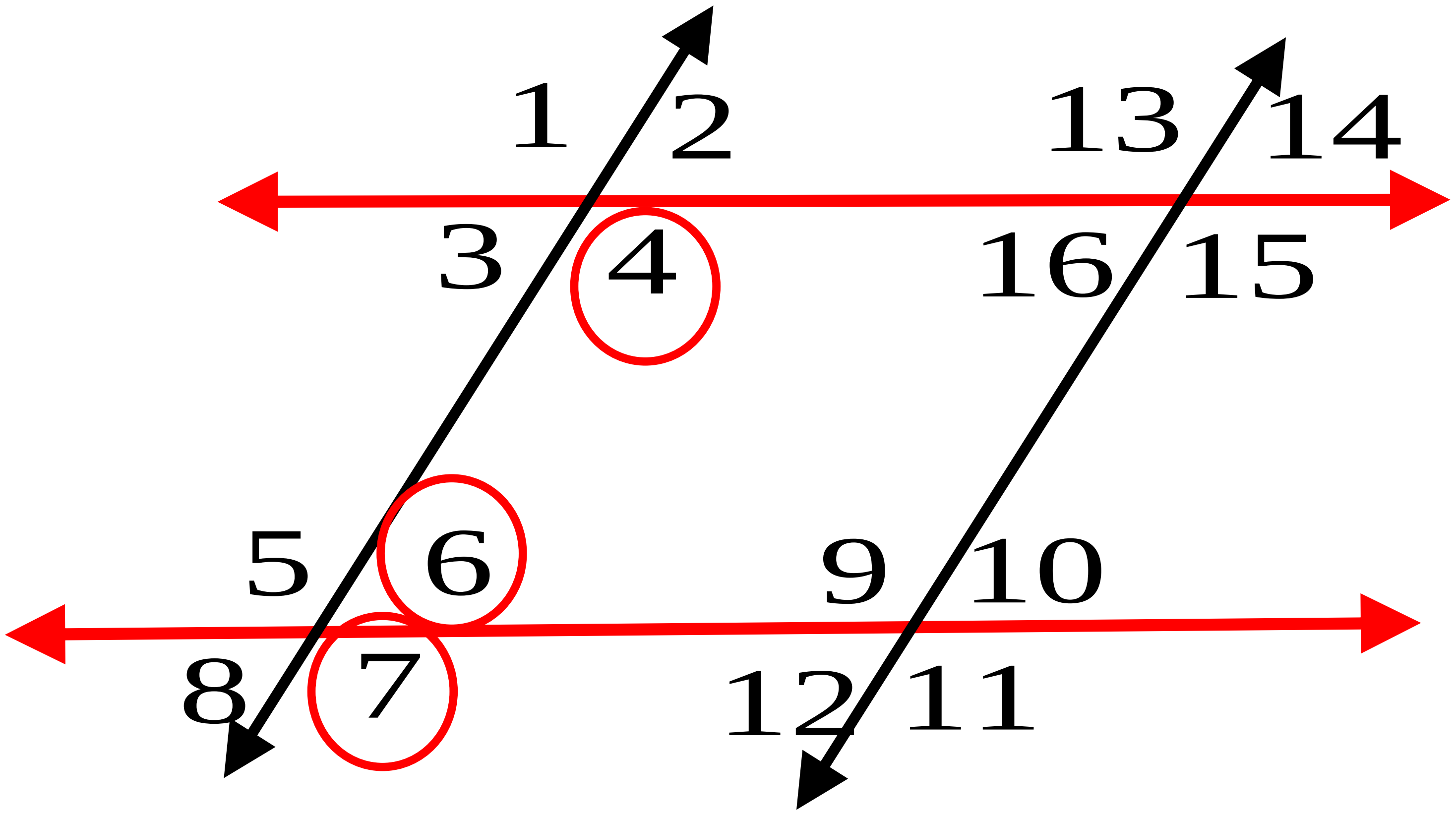
What sequence of angles would you "link" to prove $m\angle 4 = m\angle 9$



Alternate Interior

Corresponding

What sequence of angles would you "link" to prove: $m\angle 4 + m\angle 6 = 180$



Corresponding

Linear Pair

The two red lines are parallel, what can you say about ...

Linear Angle Pairs: supplementary

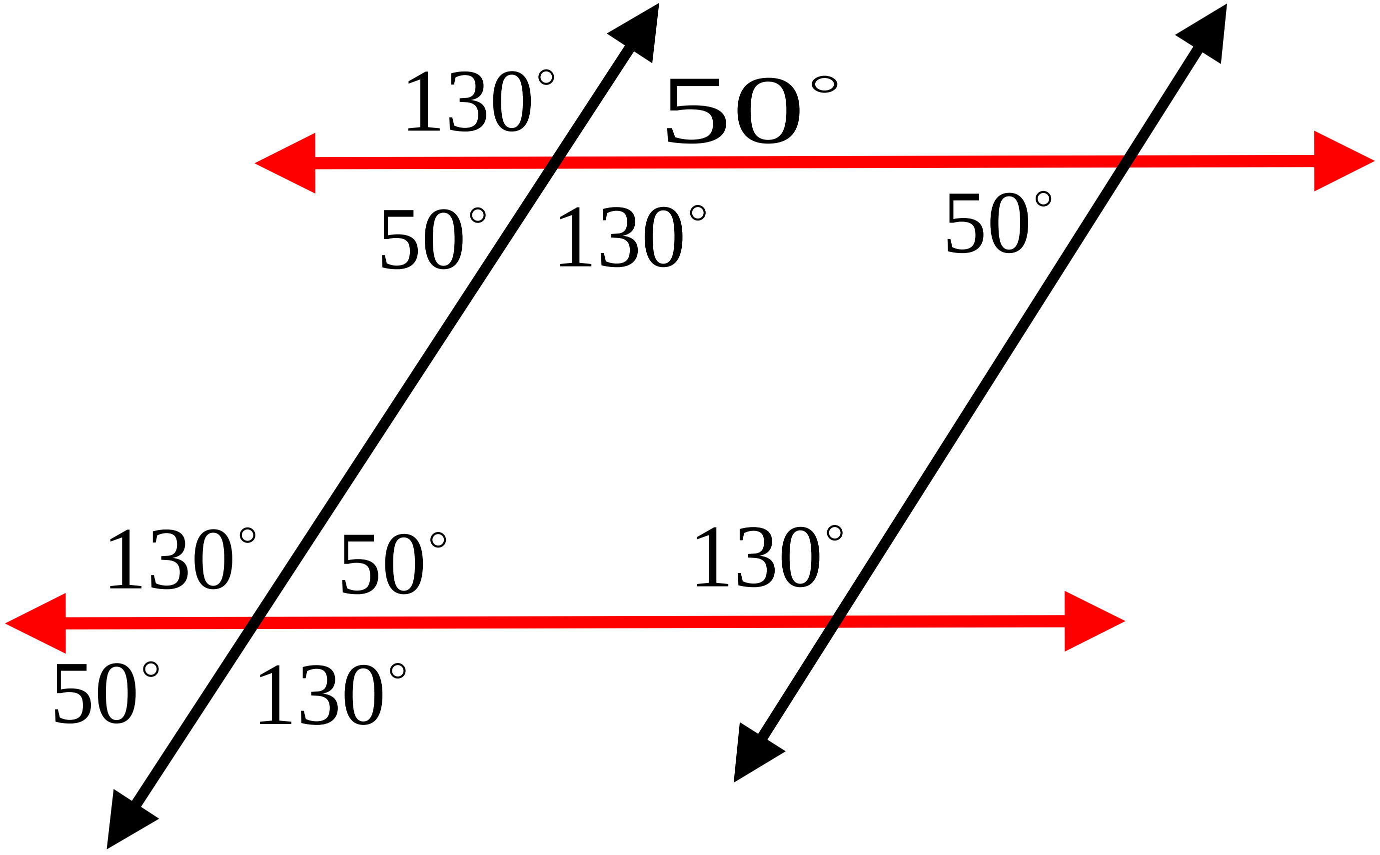
Vertical angle pair: congruent

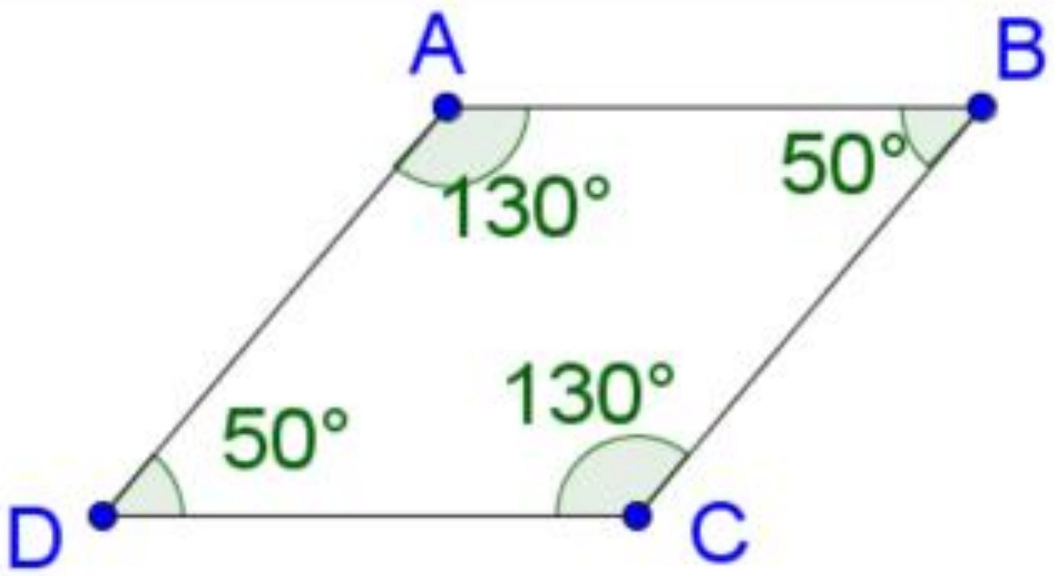
Alternate Interior Angles: congruent

Consecutive Interior Angles supplementary

Corresponding Angles: congruent

Alternate Exterior Angles: congruent





Parallelogram Properties :

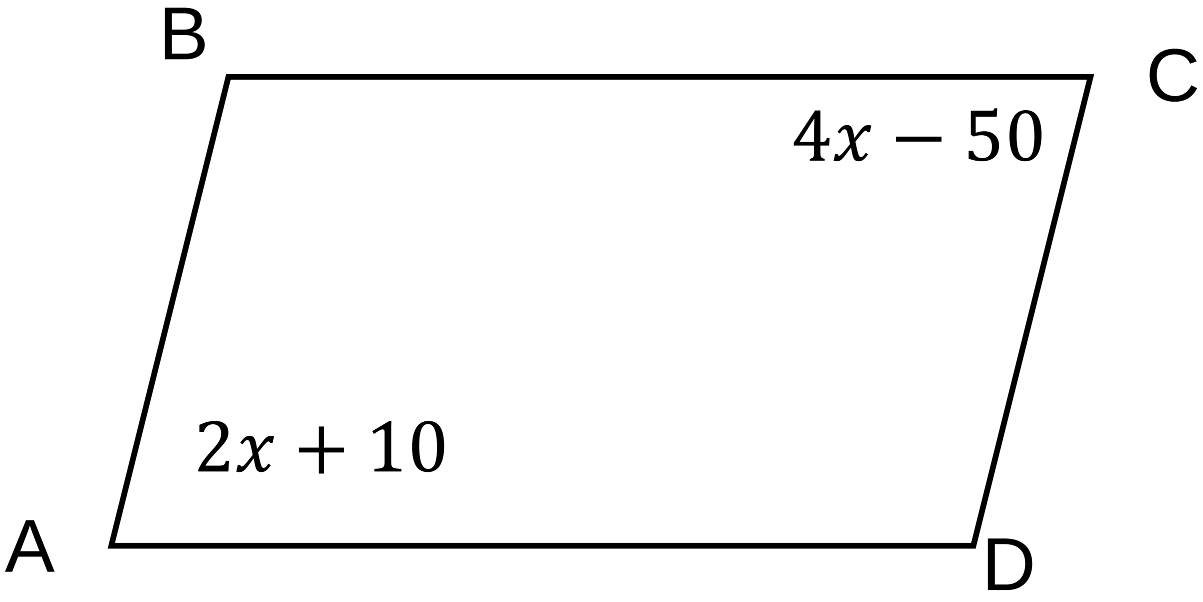
1. Opposite Angles are congruent.

$$m\angle A = m\angle C$$
$$m\angle B = m\angle D$$

2. Consecutive Interior Angles are supplementary.

$$m\angle A + m\angle B = 180$$

Math Problems from "Opposite Angles of Parallelograms are Congruent"



$x = ?$

$\angle A \cong \angle C$

$\text{m}\angle A = m\angle C$

$2\text{x} + 10 = 4x - 50$

$x = 30$

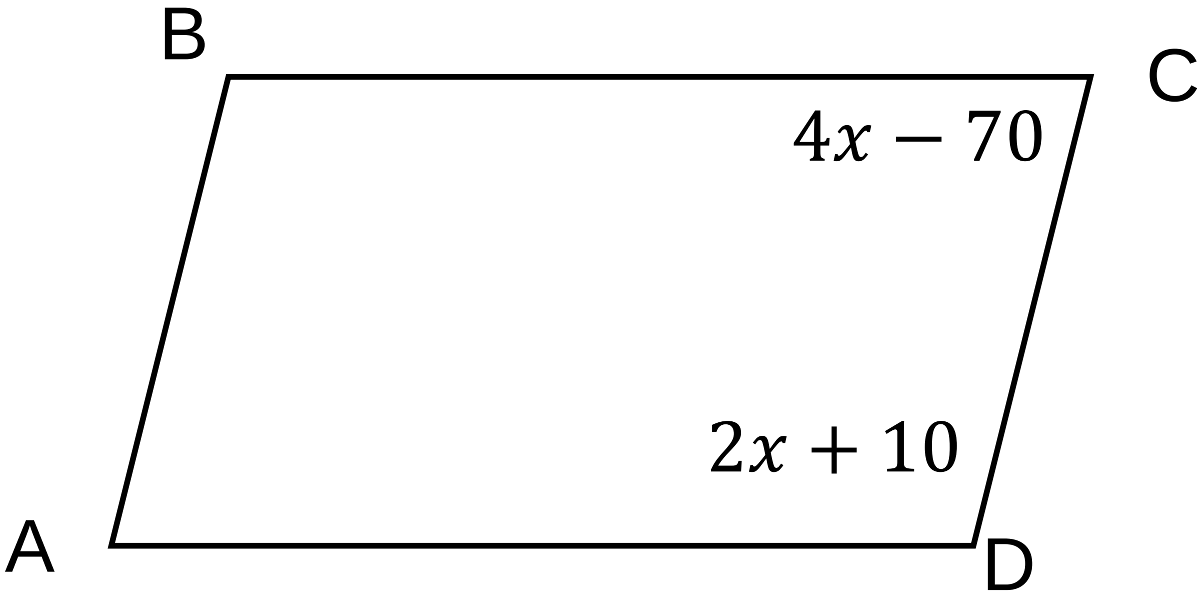
$m\angle A = ?$

$\text{m}\angle A = 2x + 10$

$\text{m}\angle A = 2(30) + 10$

$\text{m}\angle A = 70$

Math Problems from "Adjacent Angles of Parallelograms are Supplementary"



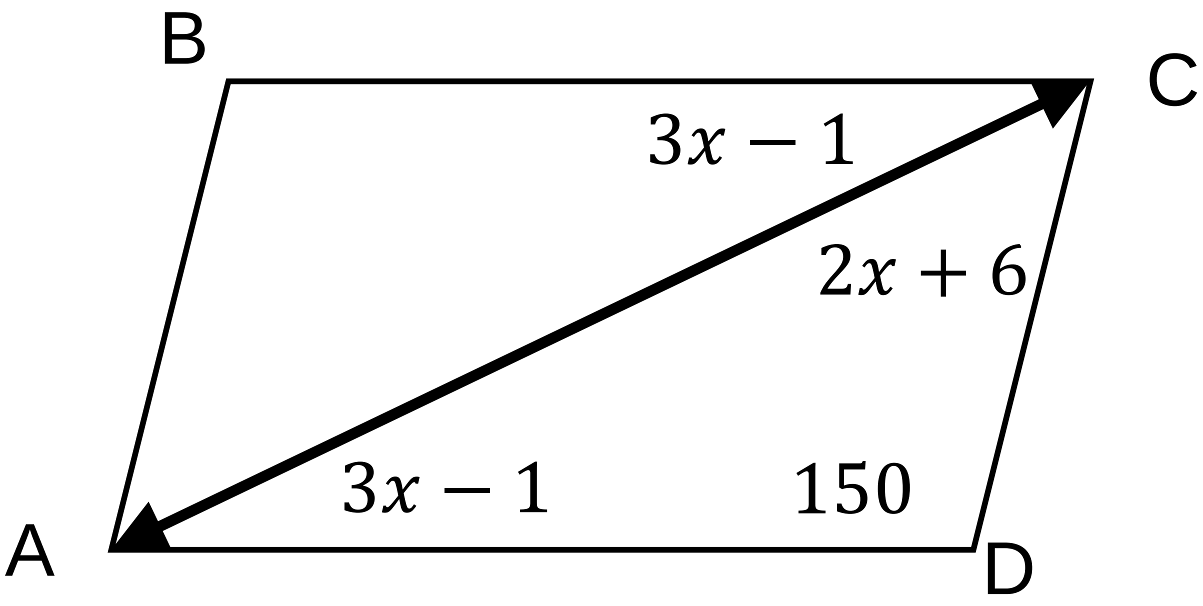
$x = ?$

$$m\angle D + m\angle C = 180$$

$$2x + 10 + 4x - 70 = 180$$

$$6x = 240$$

Math Problems from "Adjacent Angles of Parallelograms are Supplementary"



Segment AC is a *diagonal*.

$x = ?$

$$m\angle BCA + m\angle DCA = m\angle BCD$$

Angle Addition Postulate

$$m\angle ADC + m\angle BCD = 180$$

Adjacent Angles of Parallelograms

$$3\text{x} - 1 + 2x + 6 + 150 = 180$$

$$5\text{x} + 155 = 180 \qquad \boxed{\text{x} = 5}$$

$\angle BCA \cong \angle DAC$ Alternate Interior Angles

$$m\angle CAD + m\angle DCA + m\angle D = 180$$

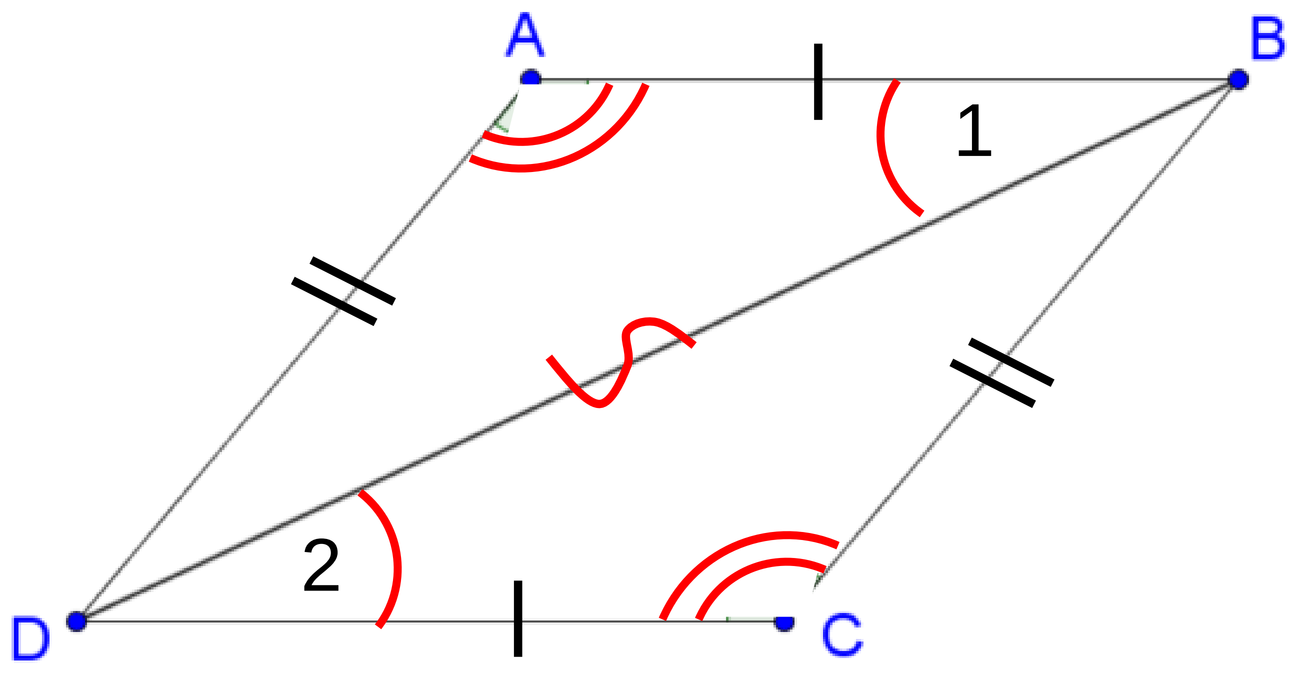
Triangle Angle Sum Theorem (we'll prove this later).

$$3\text{x} - 1 + 2x + 6 + 150 = 180$$

$$5\text{x} + 155 = 180$$

$$\boxed{\text{x} = 5}$$

If we could prove the diagonal forms two congruent triangles, we could use CPCTC to prove more properties of Parallelograms.



$$m\angle A = m\angle C$$

Opposite Angles are congruent.

$$\angle 1 \cong \angle 2$$

Alternate Interior Angles

$$BD = DB$$

Same segment → same length

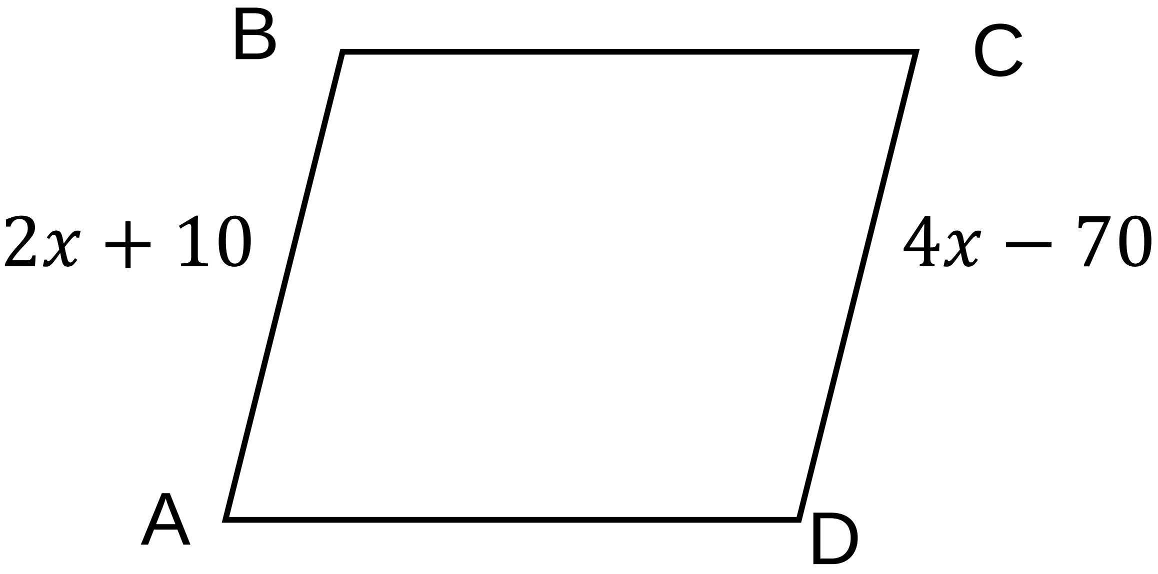
$$\Delta ABC \cong \Delta CDB$$

AAS Theorem

$AD = BC$ CPCTC

$AB = CD$ CPCTC

Math Problems from “Opposite Sides of Parallelograms are congruent”



$x = ?$

AB = CD

$2x + 10 = 4x - 70$

$2x = 80$

$x = 40$

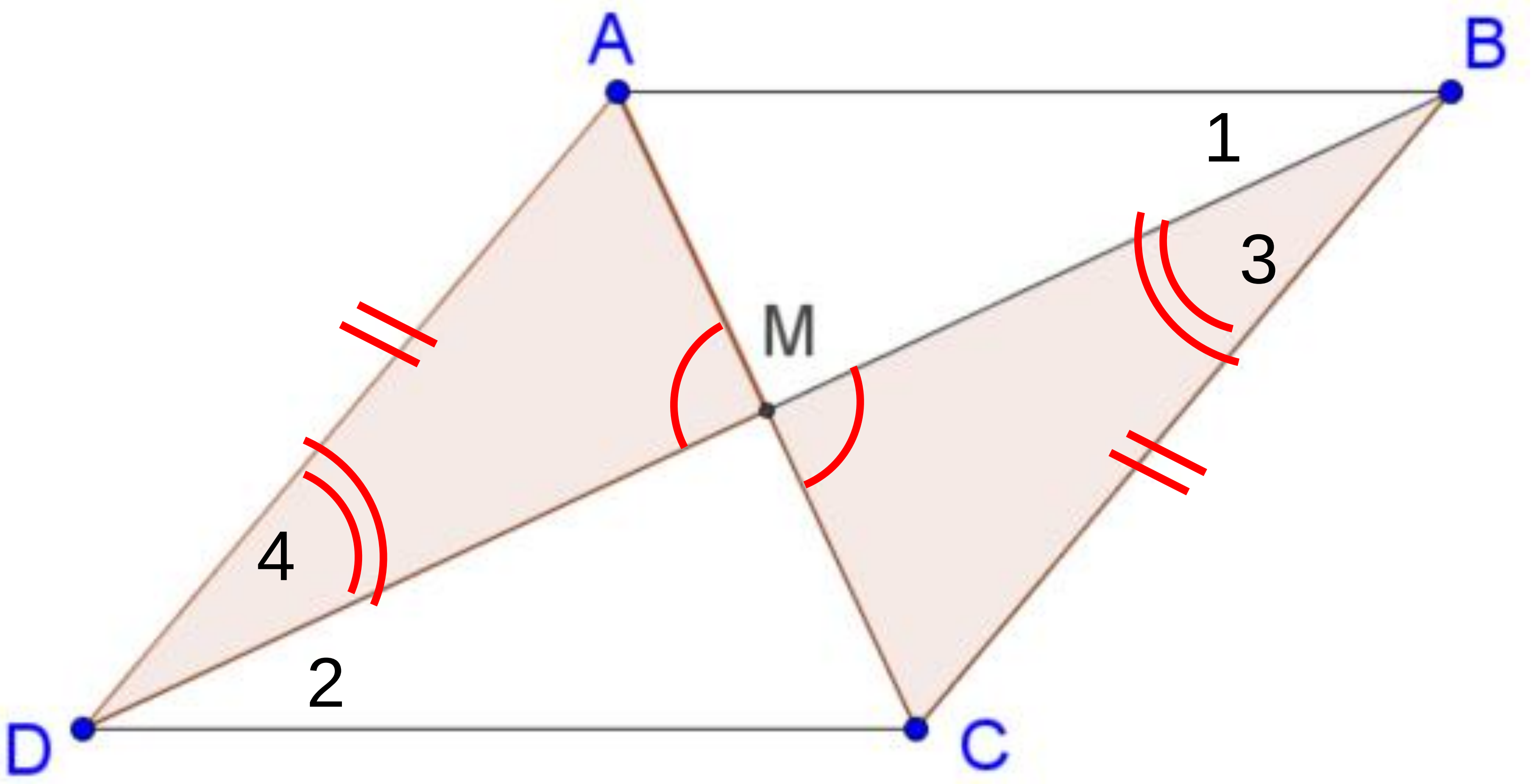
$AB = ?$

$AB = 2x + 10$

$AB = 2(40) + 10$

$AB = 90$

Can we prove that diagonals form two pairs of congruent triangles?



$\overline{AD} \cong \overline{CB}$

Opposite Sides are congruent.

$\angle AMD \cong \angle CMB$

Vertical Angles

$\angle 3 \cong \angle 4$

Alternate Interior Angles

$\Delta AMD \cong \Delta CMB$

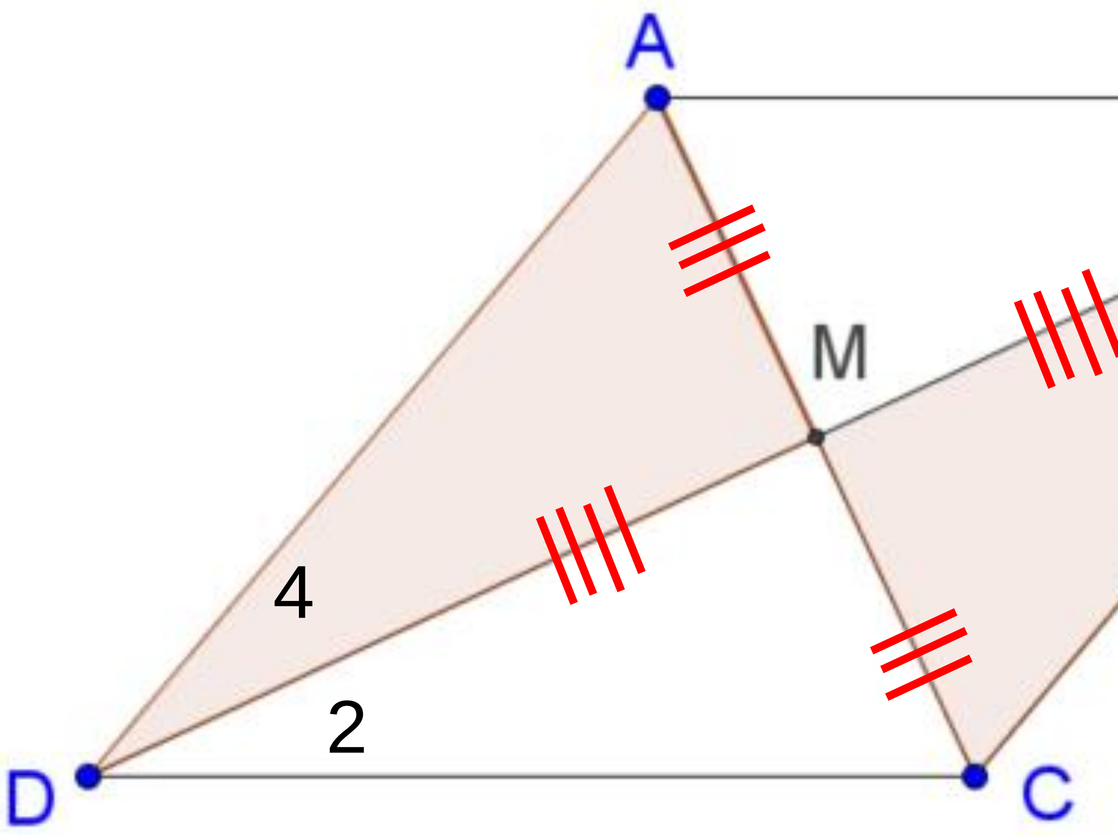
AAS Theorem

Using the other pairs of:

1) Opposite sides
2) Vertical angles
3) Alternate Interior Angles

$\Delta CMD \cong \Delta AMB$

AAS Theorem



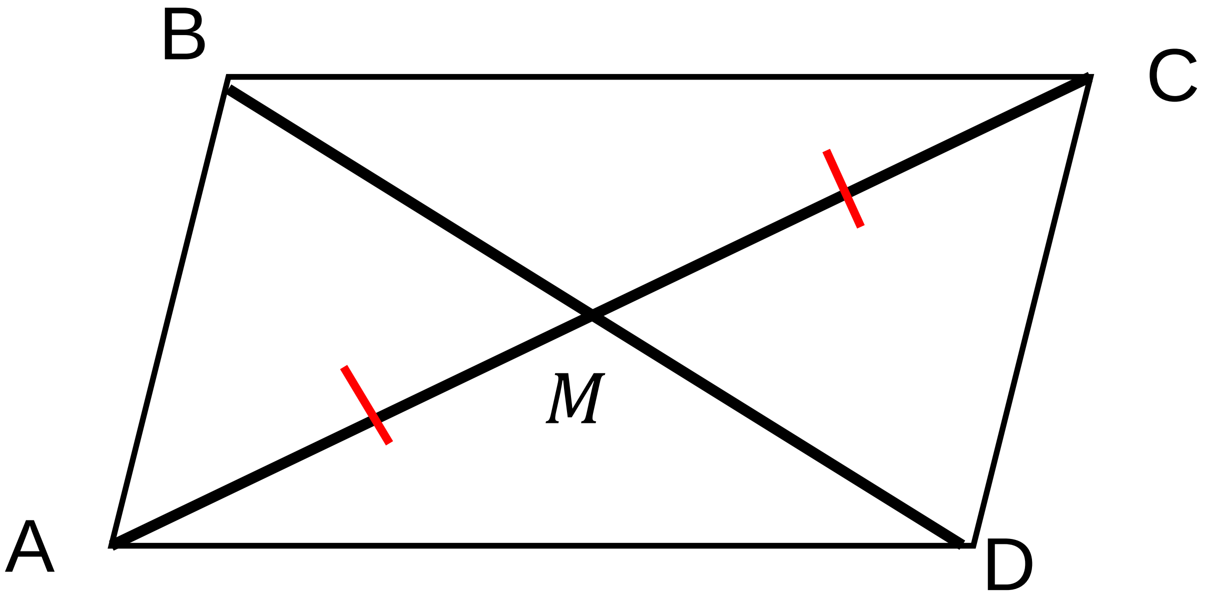
By CPCTC

$$\overline{DM} \cong \overline{MB}$$

$$\overline{AM} \cong \overline{CM}$$

Therefore, diagonals of parallelograms bisect each other.

Math Problems from “Diagonals of Parallelograms BISECT each other.”

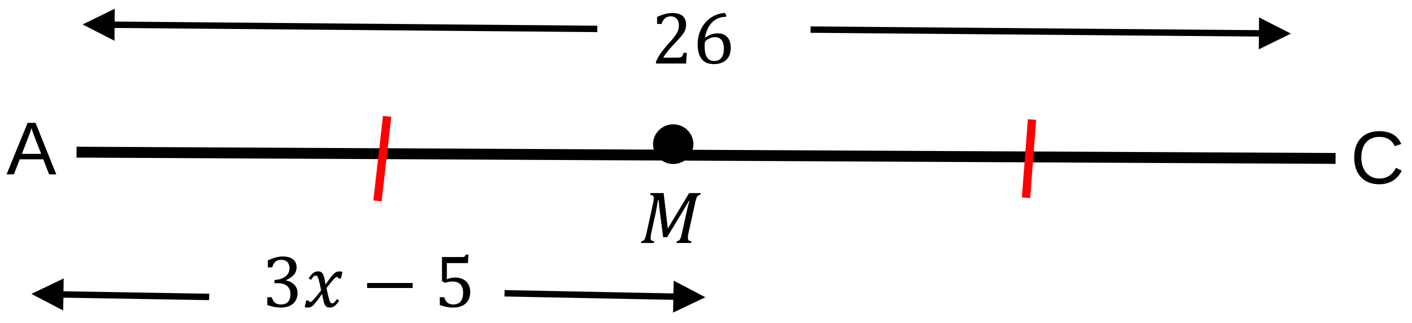


$$AC = 26$$

$$AM = 3x - 5$$

$$x = ?$$

1. Draw a picture of the diagonal and label the known measurements.



2. Write an equation that relates the lengths in the problem.

$$2 * AM = AC$$

$$2(3x - 5) = 26$$

3. Solve for ‘x’.

$$3x - 5 = 13$$

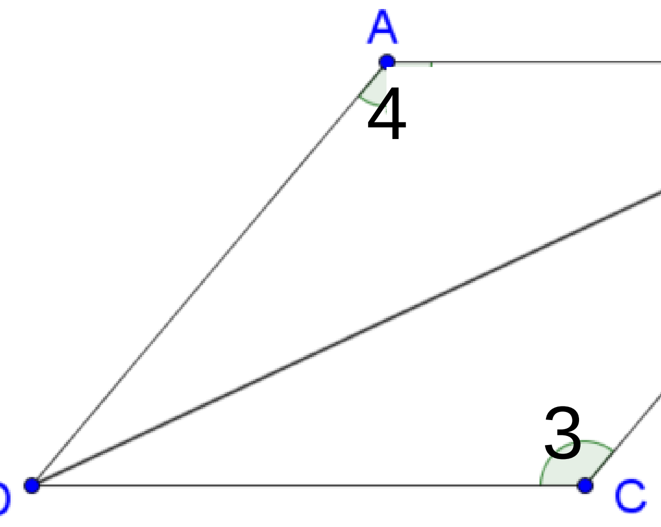
$$3x = 18$$

$$x = 6$$

Parallelogram Properties :

1. Opposite Angles are congruent. $m\angle 3 = m\angle 4$

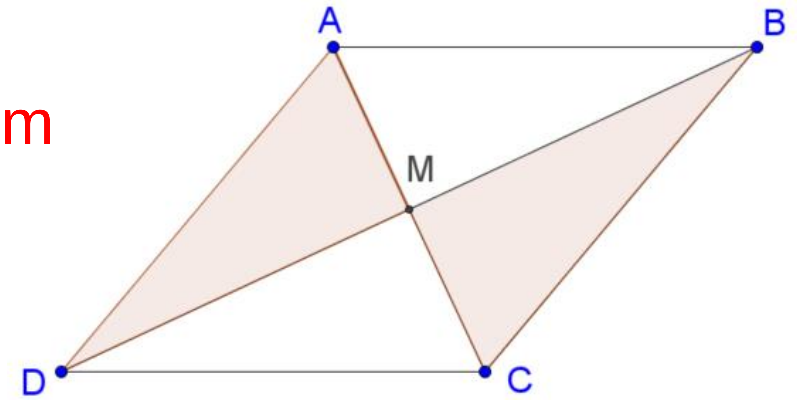
2. Consecutive Interior Angles are supplementary. $m\angle 1 + m\angle 2 + m\angle 3 = 180$



3. A diagonal of a parallelogram forms two congruent triangles. $\triangle DAB \cong \triangle CBD$

4. Opposite Sides of parallelograms are congruent. $AB = CD$

5. Opposite triangles formed by the diagonals (plural) form congruent triangles. $\triangle AMD \cong \triangle CMB$



6. Diagonals of parallelograms bisect each other.

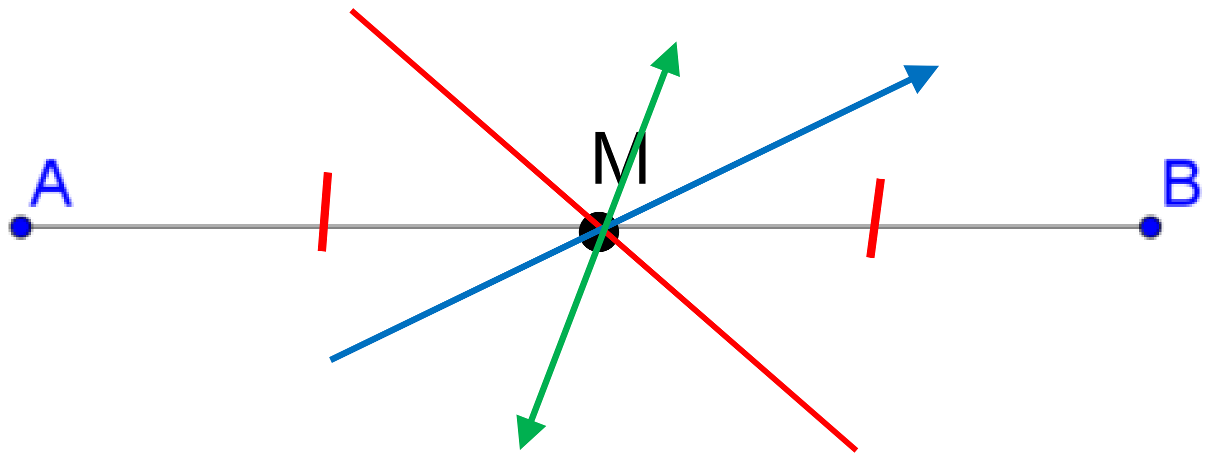
$AM = MC \qquad AC = 2 * MC$

Segment Bisector: if a line segment is intersected by a ray, segment or line at the midpoint of the segment, then the ray, segment line is a segment bisector.

a) Another segment

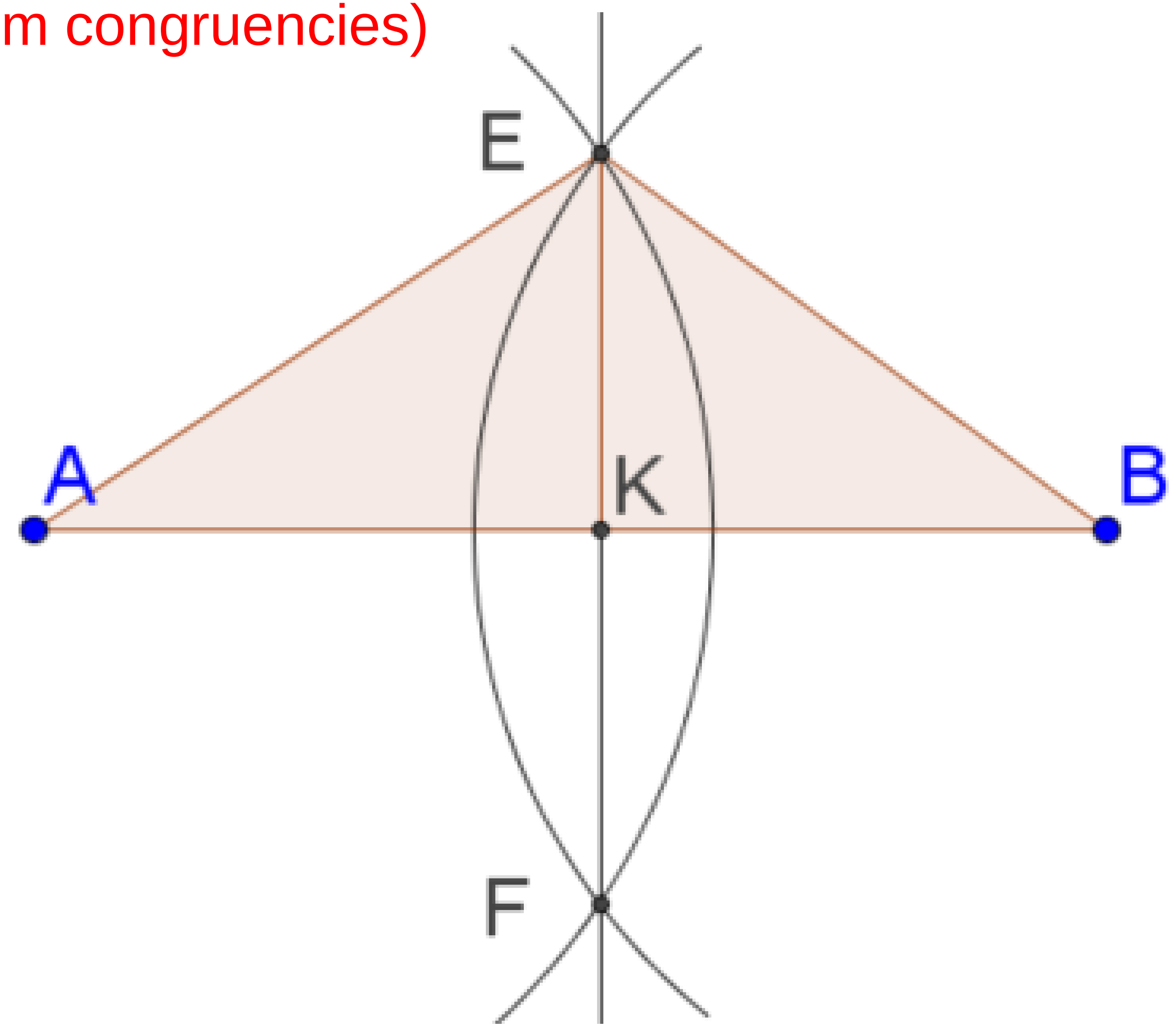
b) A ray

c) A line.



$\overline{EF}$ is a <u>perpendicular bisector of</u> $\overline{AB}$.

Are there any equations (that come from congruencies) that we can write from this result?



$$m\angle AKE = m\angle BKE = 90$$

<u>perpendicular</u> bisector

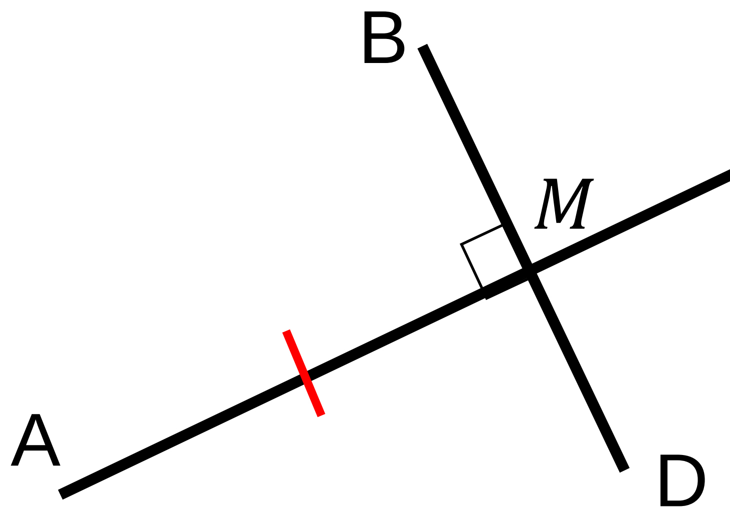
$$AK = BK$$

<u>perpendicular</u> bisector

$$AB = 2 * AK$$

<u>segment</u> addition

Math Problems from “Perpendicular Bisectors”

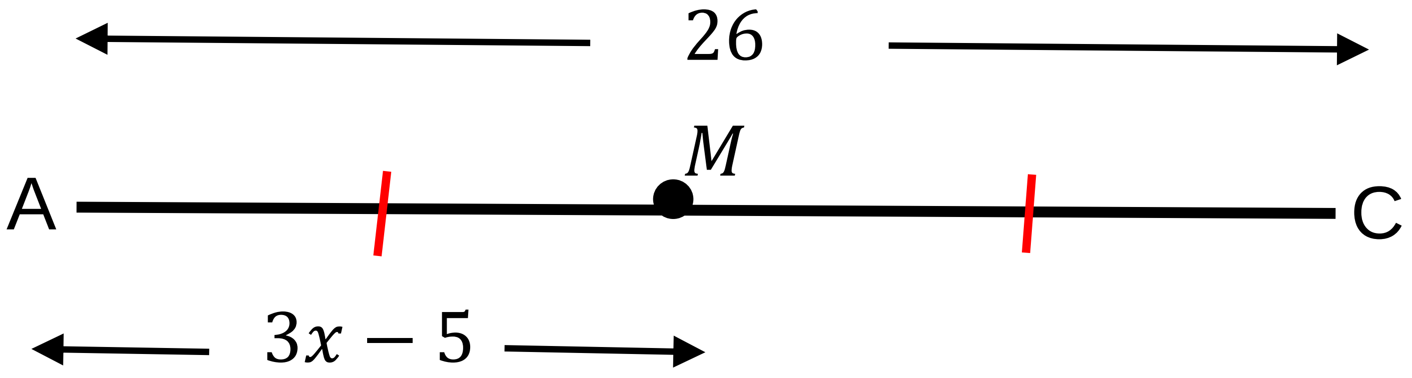


$$AC = 26$$

$$AM = 3x - 5$$

$$x = ?$$

1. Draw a picture of the segment and label the known measurements.



2. Write an equation that relates the lengths in the problem.

$$2 * AM = AC$$

$$2(3x - 5) = 26$$

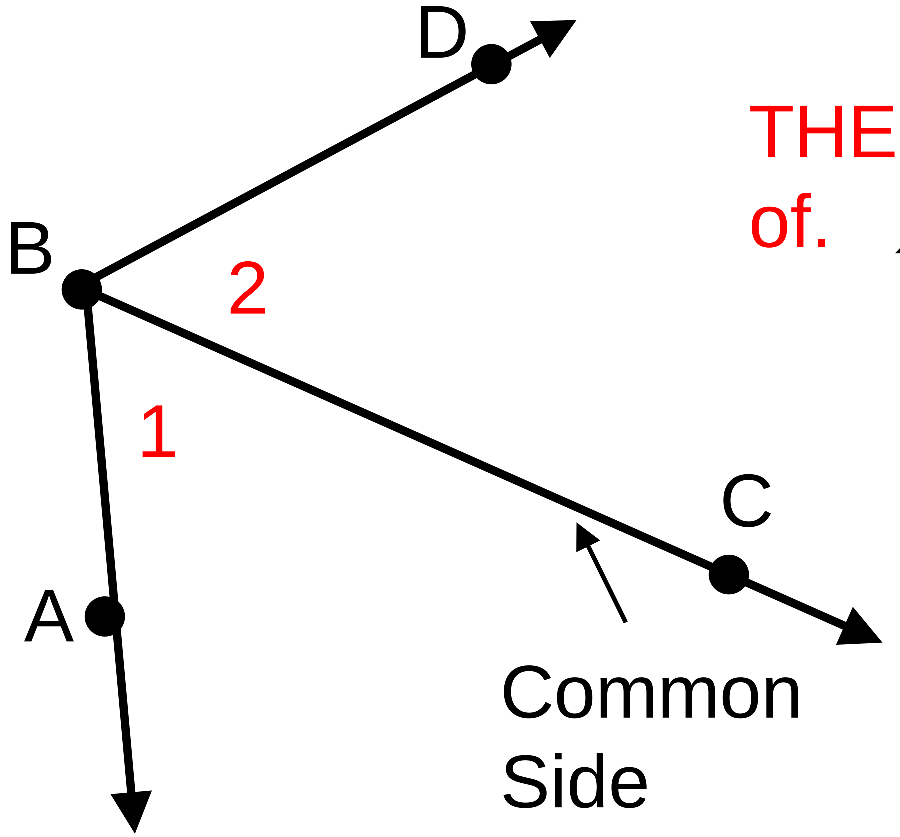
3. Solve for ‘x’.

$$3x - 5 = 13$$

$$3x = 18$$

$$x = 6$$

<u>Angle Bisector</u>: a common side of two adjacent angles that divides the angle into two angles of equal measure.



If $m\angle 1 = m\angle 2$

THEN $\overline{BC}$ is an angle bisector of. $\angle ABD$

Are there any equations that we can write from this result?

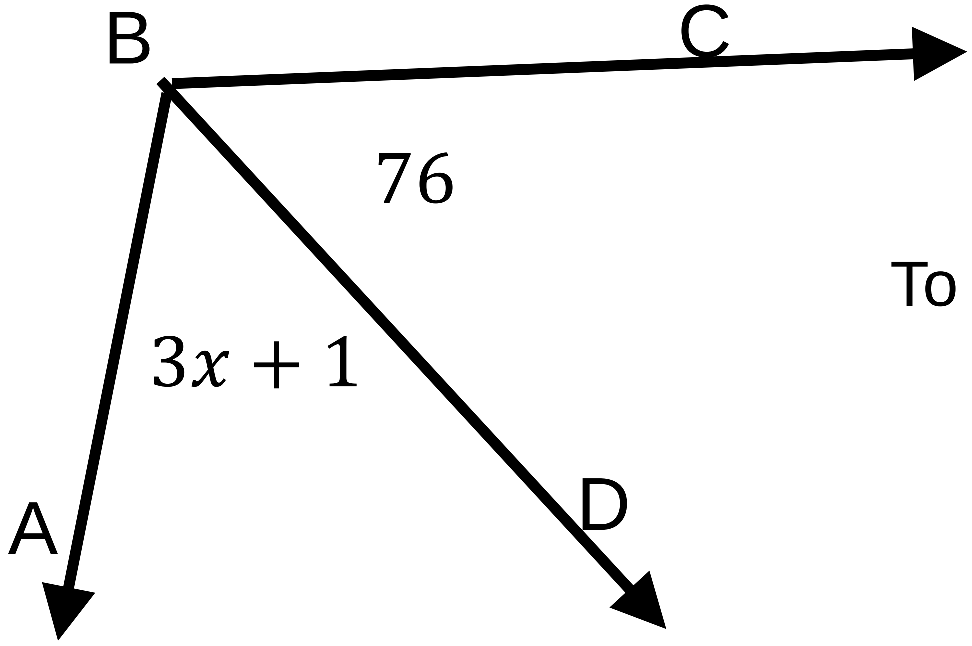
$$m\angle ABC = m\angle DBC$$

angle bisector

$$m\angle ABD = 2 * m\angle DBC$$

angle bisector

Math Problems from “Angle Bisectors”



$\overrightarrow{BD}$ is an angle bisector of $\angle ABC$

$x = ?$

To solve for an unknown value, you need an <u>equation</u>.

$m\angle ABD = m\angle CBD$

$3x + 1 = 76$

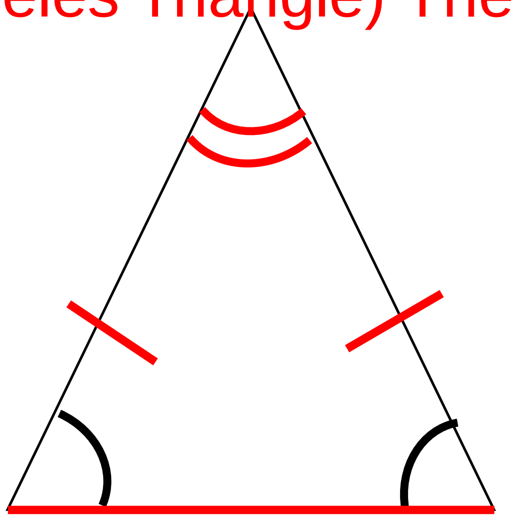
$3x = 75$

$x = 25$

Isosceles Triangle: A triangle with two congruent sides.

Legs: (Of an Isosceles Triangle) The two congruent sides.

Vertex Angle: (Of an Isosceles Triangle) The included angle of the legs.



Base: (Of an Isosceles Triangle) The opposite the vertex angle.

Base Angles: (Of an Isosceles Triangle) The angles that include the base.

<u>Given:</u> ΔABC is an Isosceles Triangle and $\overline{AM}$ is an angle bisector of vertex angle A.

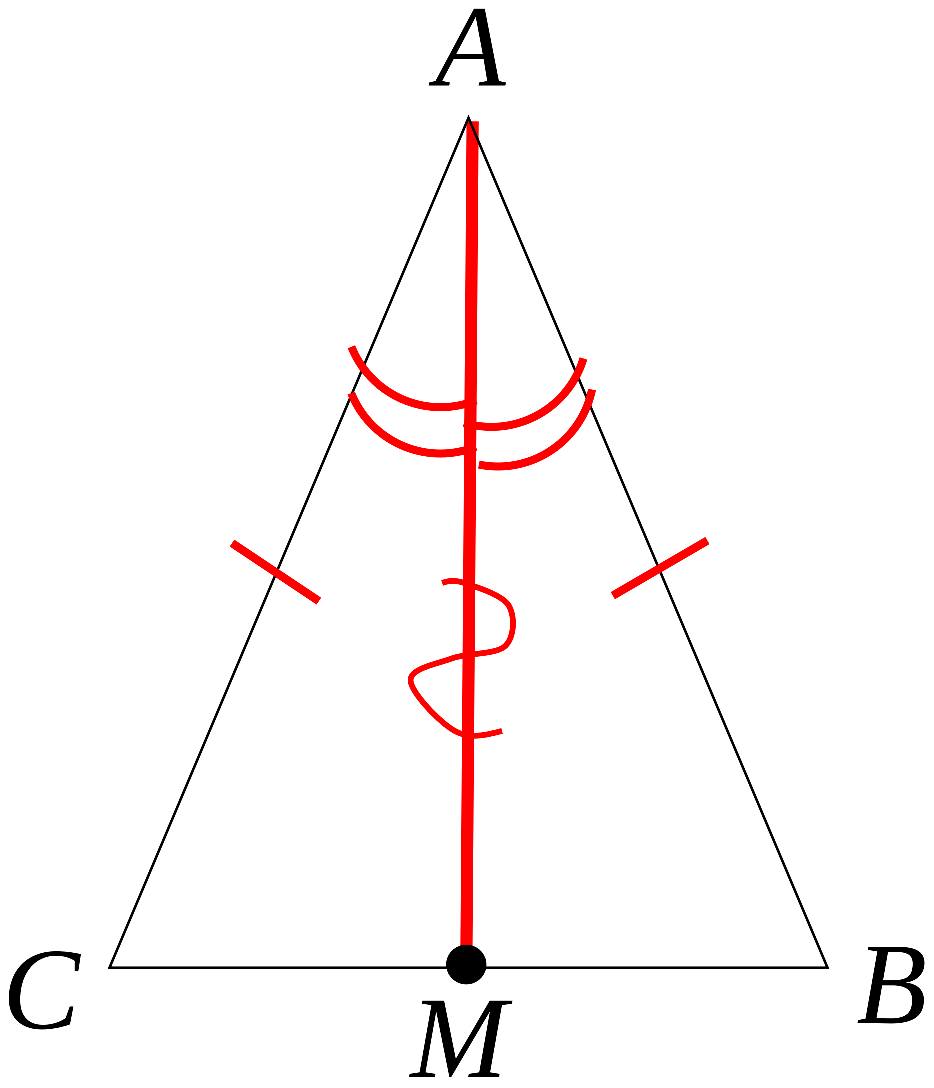
What other congruencies result from this statement?

$\angle CAM \cong \angle BAM$ ($\overline{AM}$ bisects $\angle A$)

$AC = AB$ (Isosceles Triangle)

$AM = AM$ (congruent to itself)

$\Delta CAM \cong \Delta BAM$ (SAS)



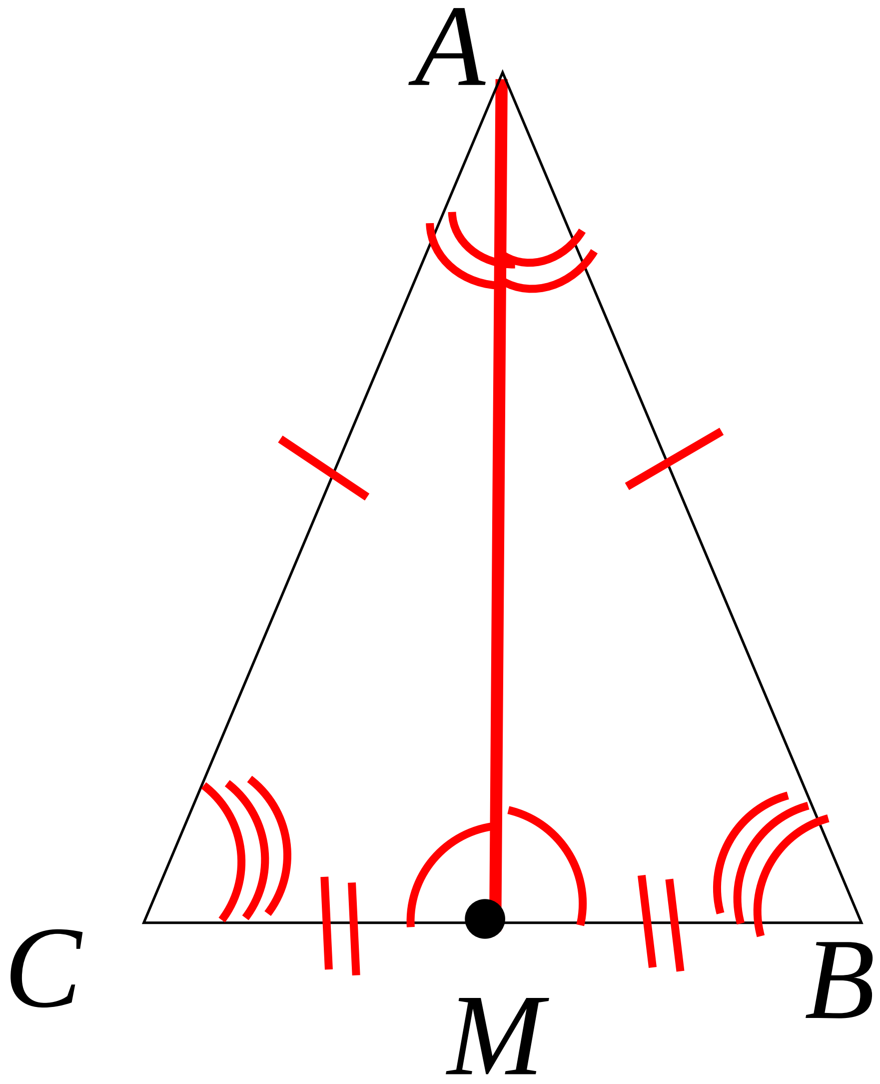
$$\Delta CAM \cong \Delta BAM$$

Congruent triangles give us SIX Pairs of congruencies.

$$CM = BM$$

$$m\angle CMA = m\angle BMA$$

$$m\angle ACM = m\angle ABM$$



Properties of Isosceles Triangles

1. The vertex and bisector forms two congruent triangles.

$$\Delta CAM \cong \Delta BAM$$

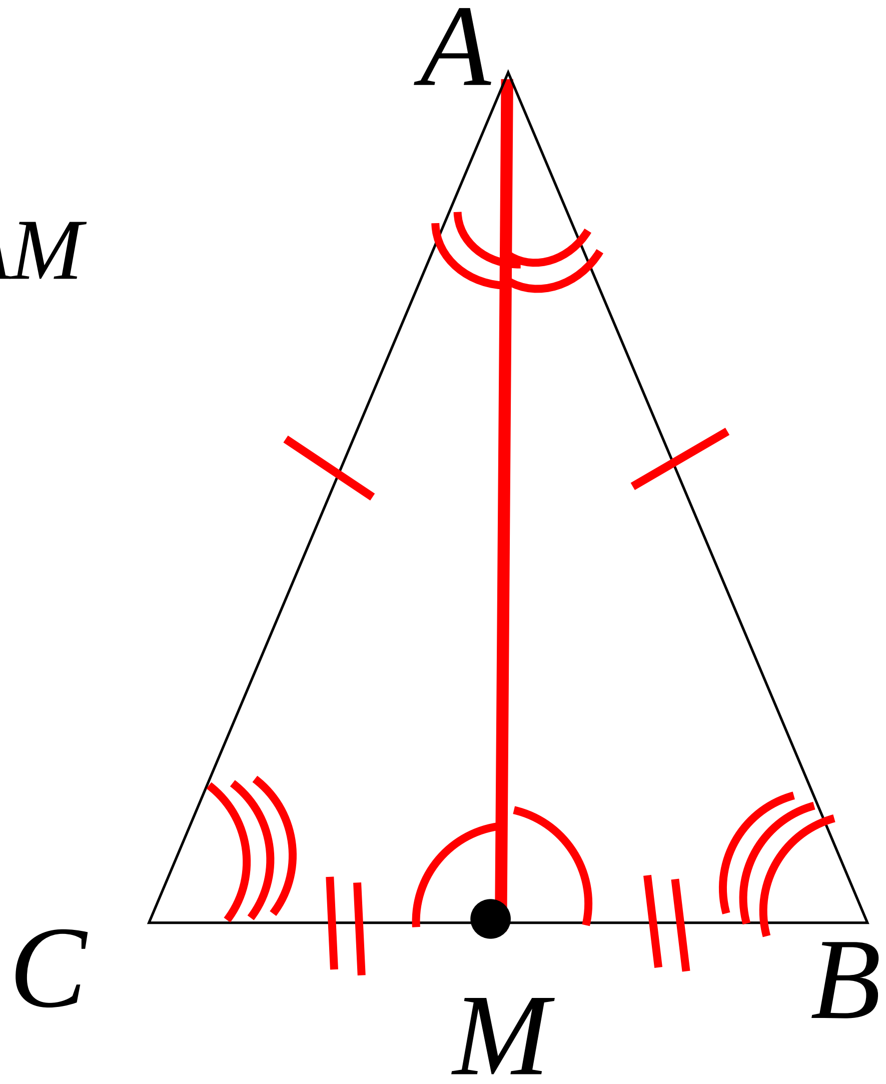
2. The vertex angle bisector is a perpendicular bisector of the base.

$$m\angle CMA = m\angle BMA = 90$$

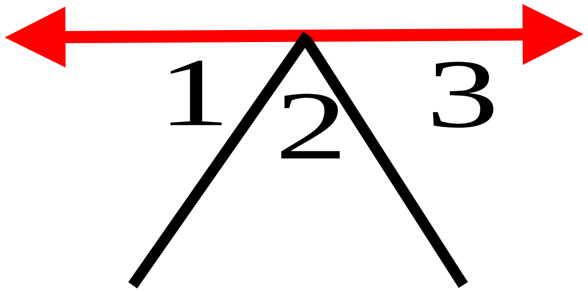
$$CM = BM$$

3. Base Angles are congruent.

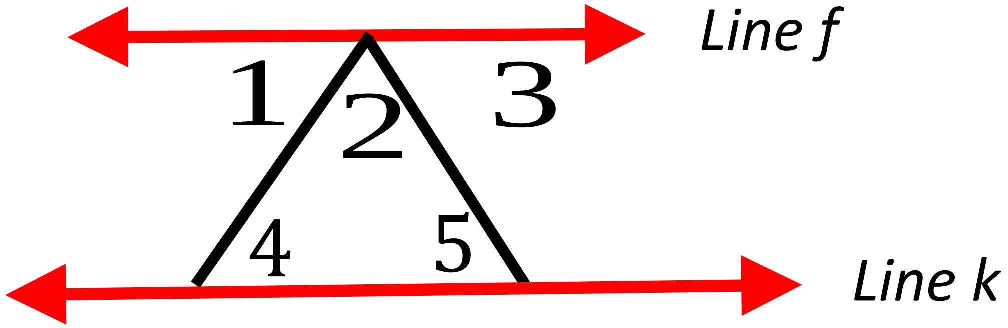
$$m\angle ACM = m\angle ABM$$



Triangle Sum Theorem: If $\angle A, \angle B, and\ \angle C$ are the interior angles of a triangle, then their measures add up to $180°$.

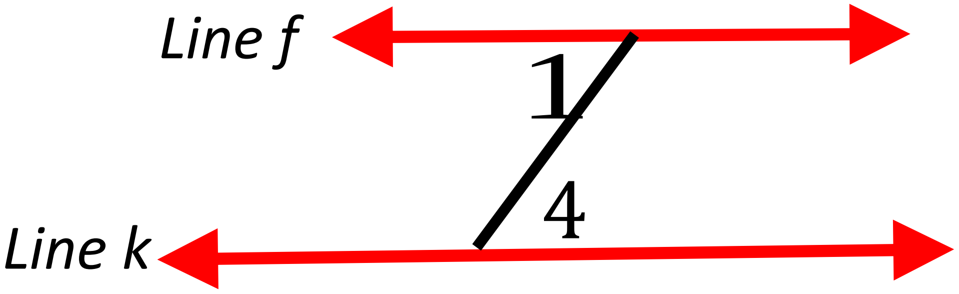


$m\angle 1 + m\angle 2 + m\angle 3 =$ <u>180°</u>

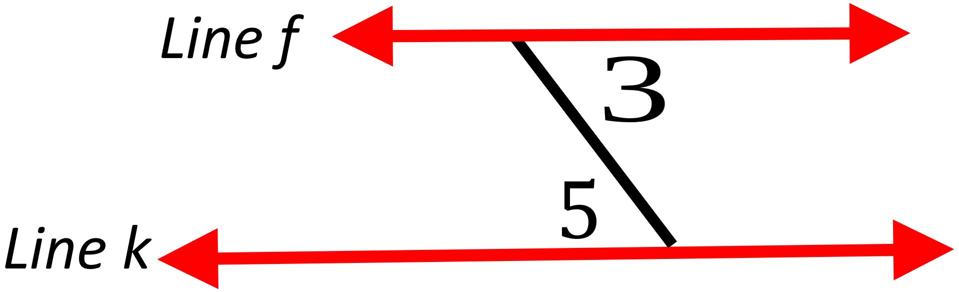


$m\angle 4 + m\angle 2 + m\angle 5 =$ <u>180°</u>

Line k ∥ *Line f*



$m\angle 1$ <u>=</u> $m\angle 4$



$m\angle 3$ <u>=</u> $m\angle 5$

Math Problems from “The Triangle Sum Theorem.”

1. Write an equation that relates the measures of the angles.

$$m\angle A + m\angle B + m\angle C = \underline{\;180°\;}$$

2. Substitute the measures of the angles into the equation.

$$2x - 1 + 3x + 7 + 4x + 3 = 180°$$

3. Solve for ‘x’.

$$9x + 9 = 180°$$

$$9x = 171°$$

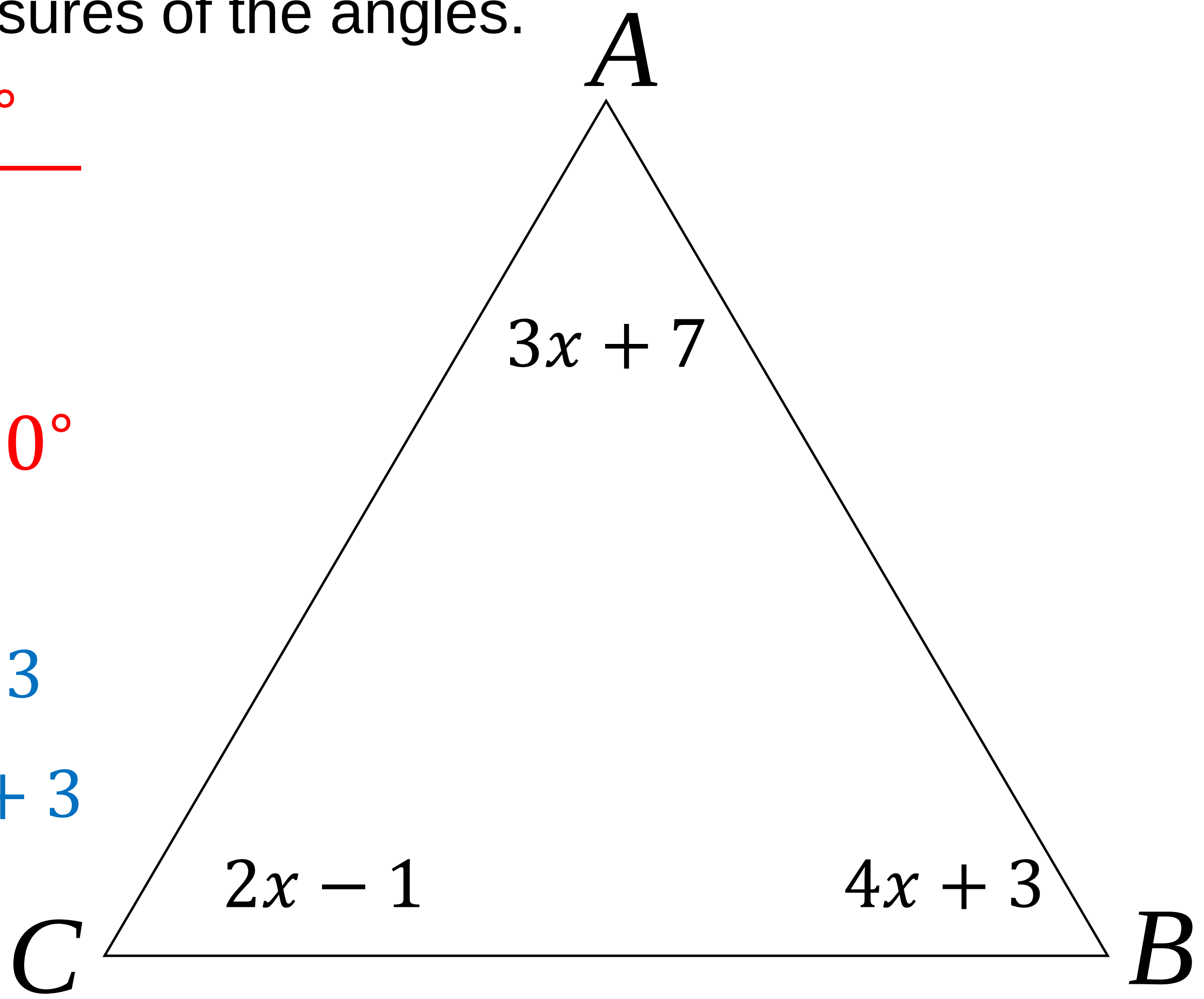
$$x = 19$$

$$\text{m}\angle B = ?$$

$$\text{m}\angle B = 4x + 3$$

$$\text{m}\angle B = 4(19) + 3$$

$$\text{m}\angle B = 79°$$



Constructing a Perpendicular Bisector

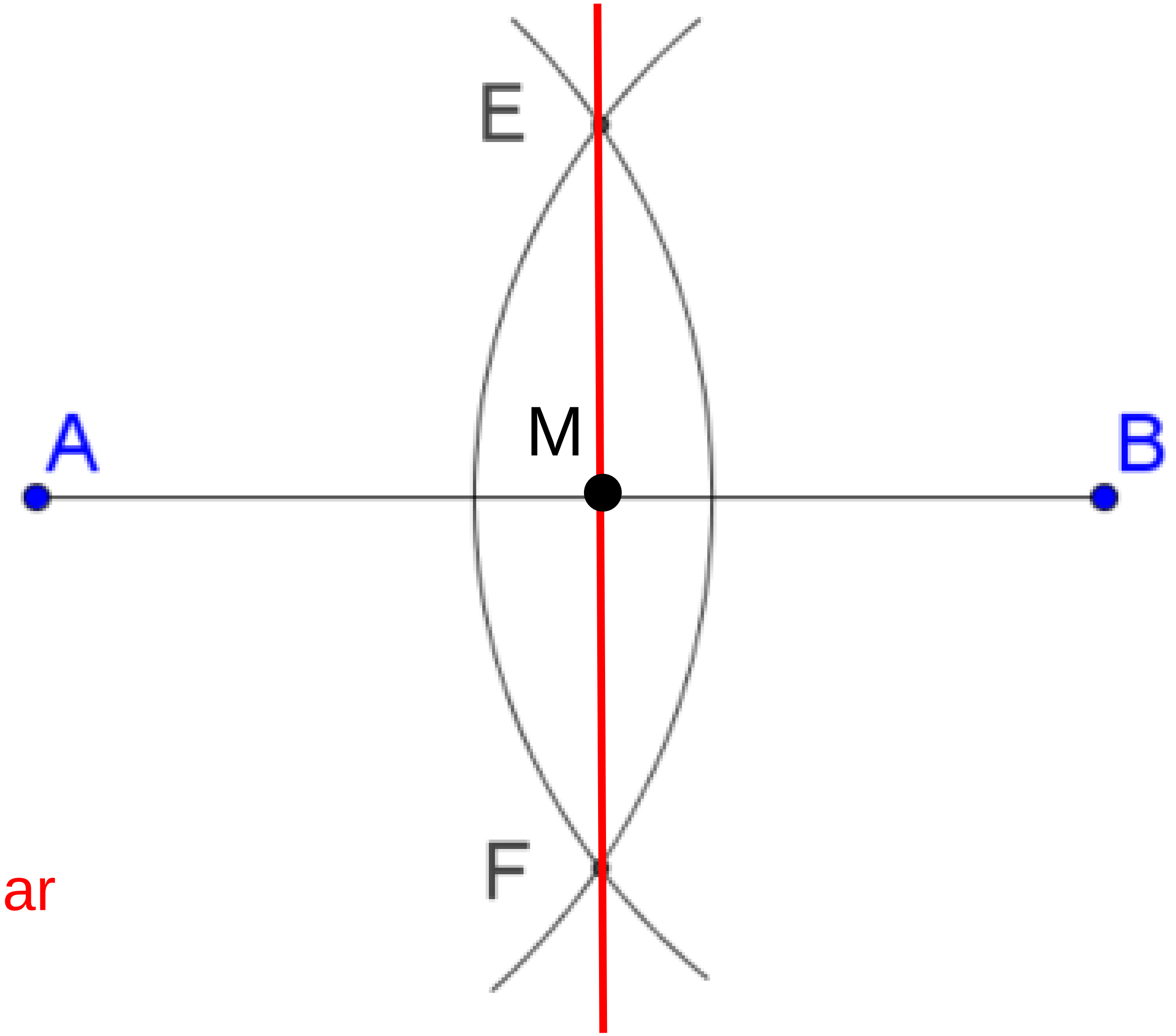
<u>Given a line segment AB</u>

1) Using a compass draw two arcs of equal radius using the endpoints as the center of each are.

2) Construct a point where the two arcs intersect.

3) Construct a line through these two points.

4) $\overline{EF}$ Is the perpendicular bisector of $\overline{AB}$



Constructing an Angle Bisector

Given $\angle B$

1) Using a compass draw an arc using point B as the center.

2) Construct two points (points A and C) where the arc intersects the side of the angles

3) Construct $\overline{AC}$

4) Construct a perpendicular bisector of $\overline{AC}$

5) $\overline{BM}$ is the angle bisector of $\angle ABC$