Math-2

Lesson 7-2

Distance (The Pythagorean Theorem)
And
Triangle Congruence

Distance

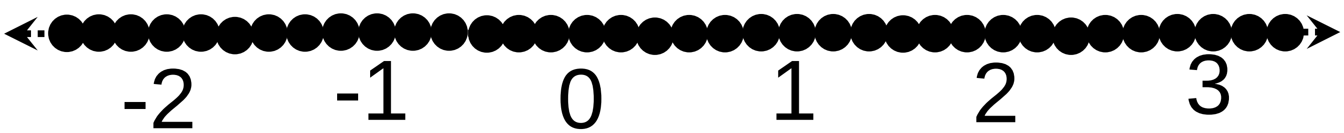
How do we represent the **Length of line segment** $\overline{AB}$ **?**

AB

We measure the Length of a line segment with a ruler.

Can a distance be negative?

distance formula $\text{distance}_{a\leftrightarrow b} = |a-b|$



The **distance between -2 and 3 on the number line**

$$\text{distance}_{-2\leftrightarrow 3} = |a-b| = |(-2)-(3)| = |-5| = 5$$

$$= |(3)-(-2)| = |5| = 5$$

Find the distance:

$$\text{distance}_{a \leftrightarrow b} = |a - b|$$

between 10 and 3

$$= |(10) - (3)| = |7| = 7$$

between -10 and -3

$$= |(-10) - (-3)| = |-10 + 3| = 7$$

between 10 and -3

$$= |(10) - (-3)| = |10 + 3| = 13$$

between -10 and 3

$$= |(-10) - (3)| = |-13| = 13$$

Does the order of the numbers matter?

Why or why not?

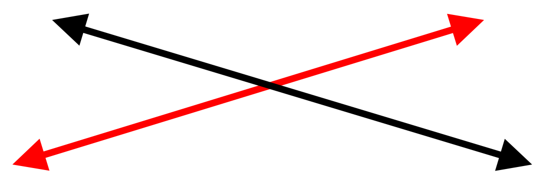
Theorem is a statement that has been proven to be true.

Theorems are usually written in
“IF hypothesis, THEN conclusion” format.

If the hypothesis is true then we know the conclusion is true.

We exchange the hypothesis and conclusion to get a converse.

Theorem: (it is a) dog, THEN (it) barks



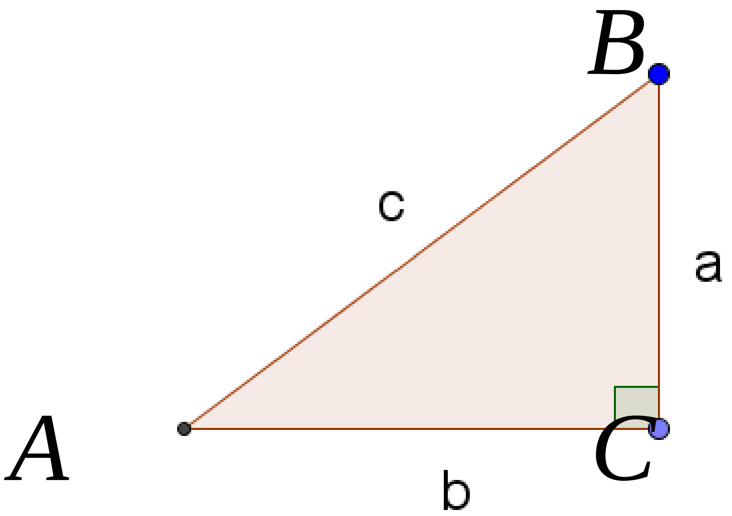
Converse of the Theorem: IF (it) barks, THEN (it is a) dog

The Pythagorean Theorem:
IF the triangle is a right triangle,
THEN the lengths of the sides are related by: $a^2 + b^2 = c^2$

The converse of this theorem is also the true (but this doesn’t work for all theorems).

IF the lengths of the sides of a triangle are related by $a^2 + b^2 = c^2$

THEN the triangle is a right triangle.



$$a^2 + b^2 = c^2$$

$$(\quad)^2 + (\quad)^2 = (\quad)^2$$

$$(\sqrt{3})^2 + (5)^2 = c^2$$

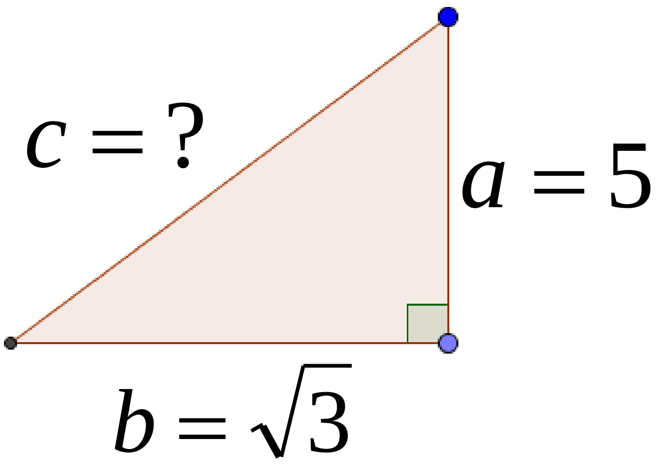
$$3 + 25 = c^2$$

$$28 = c^2$$

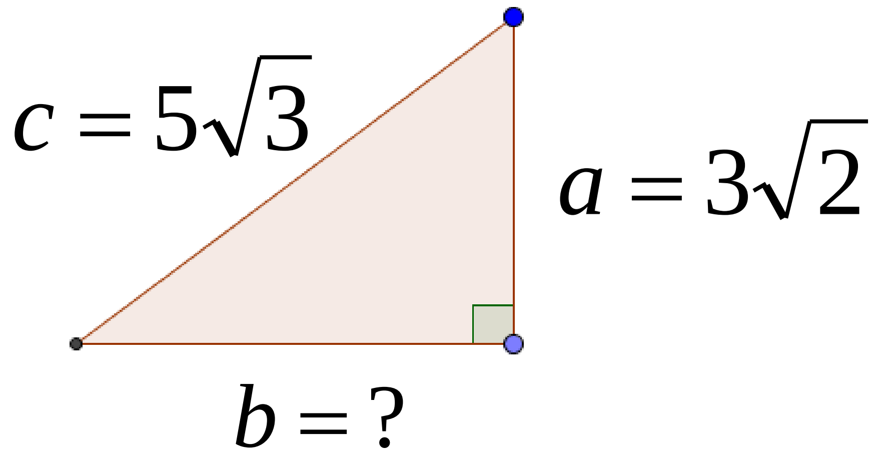
$$c = \sqrt{28}$$

$$c = \sqrt{4}\sqrt{7}$$

$$c = 2\sqrt{7}$$



$$a^2 + b^2 = c^2$$



$$(\quad)^2 + (\quad)^2 = (\quad)^2$$

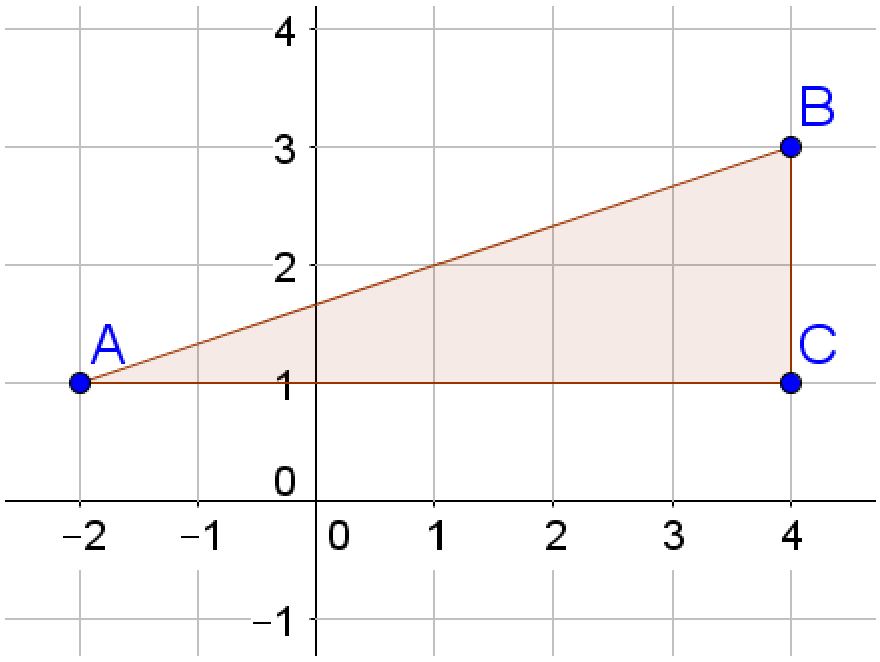
$$(3\sqrt{2})^2 + b^2 = (5\sqrt{3})^2$$

Power of a Product Property

$$(3)^2 * (\sqrt{2})^2 + b^2 = (5)^2 * (\sqrt{3})^2$$

$$18 + b^2 = 75$$

$$b = \sqrt{57}$$



AC = ?

AC = 6

BC = ?

BC = 2

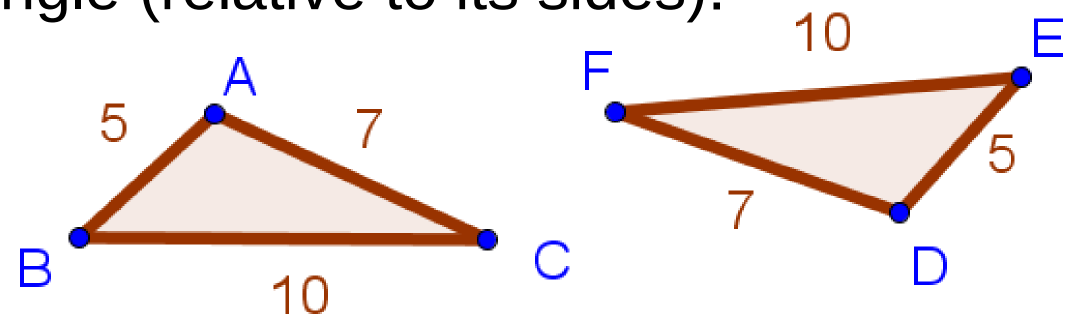
AB = ?

$$AB = \sqrt{(6)^2 + (2)^2}$$

$$AB = \sqrt{40}$$

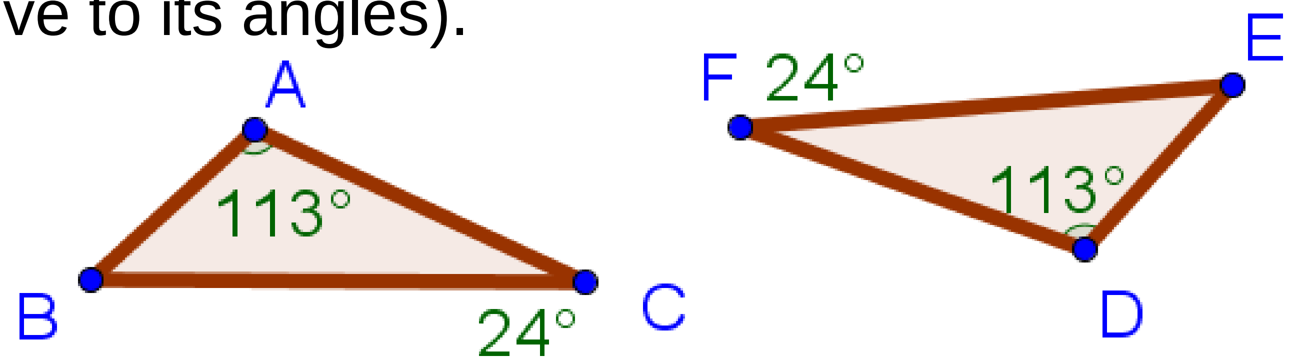
$$AB = 2\sqrt{10}$$

Corresponding Angles of Triangles: an angle in one triangle that has the same position (relative to its sides) as an angle in another triangle (relative to its sides).



$\angle A$ corresponds to $\angle D$ since they are opposite the longest side of their respective triangle.

Corresponding Sides of Triangles: a side in one triangle that has the same position (relative to its angles) as a side in another triangle (relative to its angles).



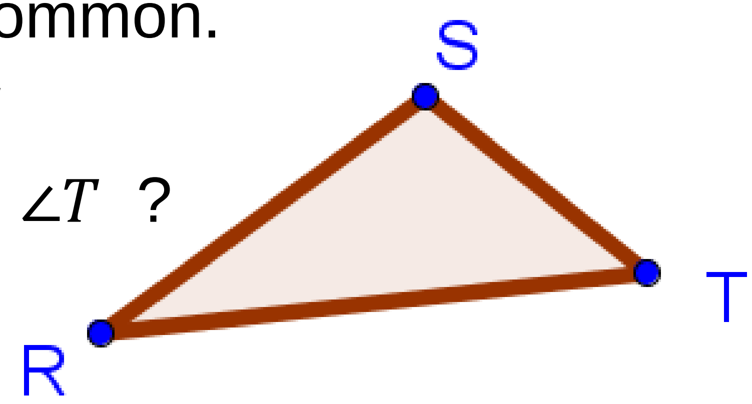
$\overline{BC}$ corresponds to $\overline{EF}$ since they are opposite the largest angle of their respective triangle.

- Included side: If two angles in a triangle are given, the included side is the side that is between the two angles or side that both of the angles have in common.

$\overline{RS}$ is the included side of $\angle R$ and $\angle S$

What is the included Side of $\angle S$ and $\angle T$?

$\overline{ST}$ is the included side of $\angle S$ and $\angle T$



Included angle: If two sides of a triangle are given, the included angle is the angle formed by those two sides.

$\angle T$ is the included angle of $\overline{RT}$ and $\overline{TS}$

What is the included angle of $\overline{SR}$ and $\overline{RT}$?

$\angle R$ is the included angle of $\overline{SR}$ and $\overline{RT}$

Congruence

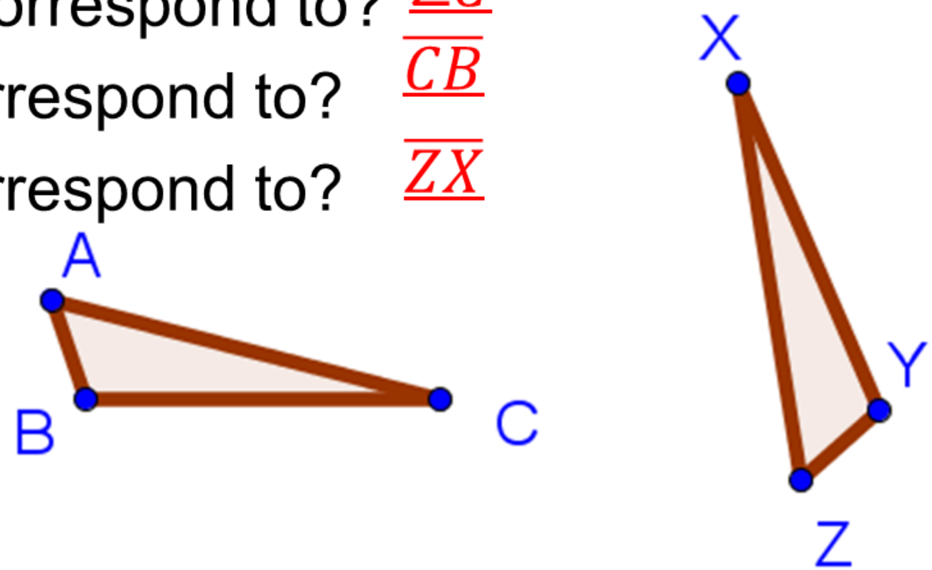
Angles: two angles are congruent if they have the same measure (degrees)

IF $\angle A \cong \angle B$ then $m\angle A = m\angle B$ (converse is true also)

Segments (sides): two line segments are congruent if they have the same length

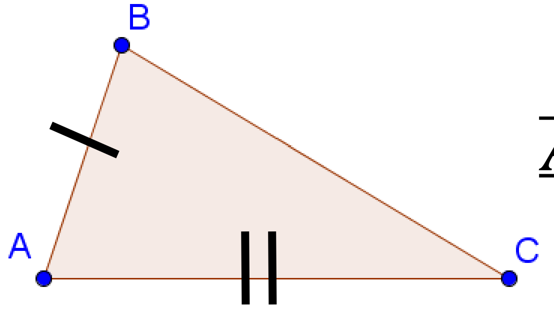
If $\overline{AB} \cong \overline{CD}$ then $AB = CD$ (converse is true also)

1) What angle does $\angle A$ correspond to? $\underline{\angle Z}$
2) What angle does $\angle X$ correspond to? $\underline{\angle C}$
3) What side does $\overline{XY}$ correspond to? $\underline{\overline{CB}}$
4) What side does $\overline{AC}$ correspond to? $\underline{\overline{ZX}}$

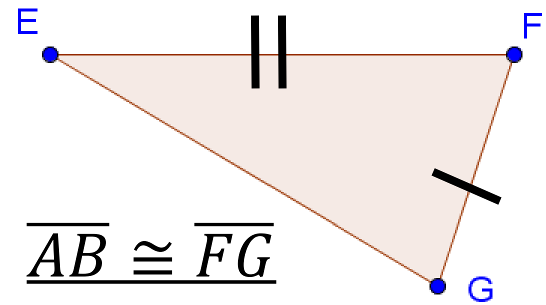


Congruence Symbols

Segment congruence symbols (without giving measures) are just “tic” marks.



$\overline{AC} \cong \overline{EF}$

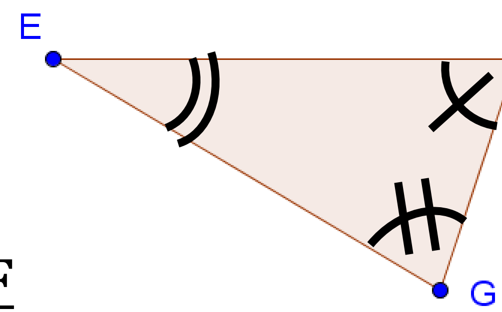


$\overline{AB} \cong \overline{FG}$

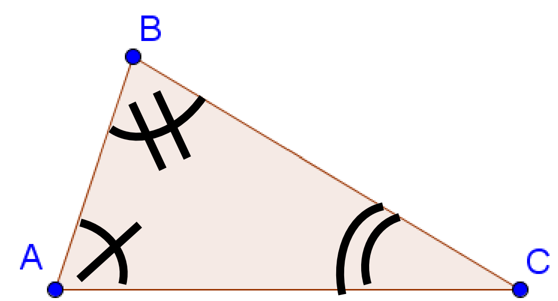
Angle congruence symbols can either be the “arc” symbol with either:

(1) tic-marks

(2) Repeated arcs



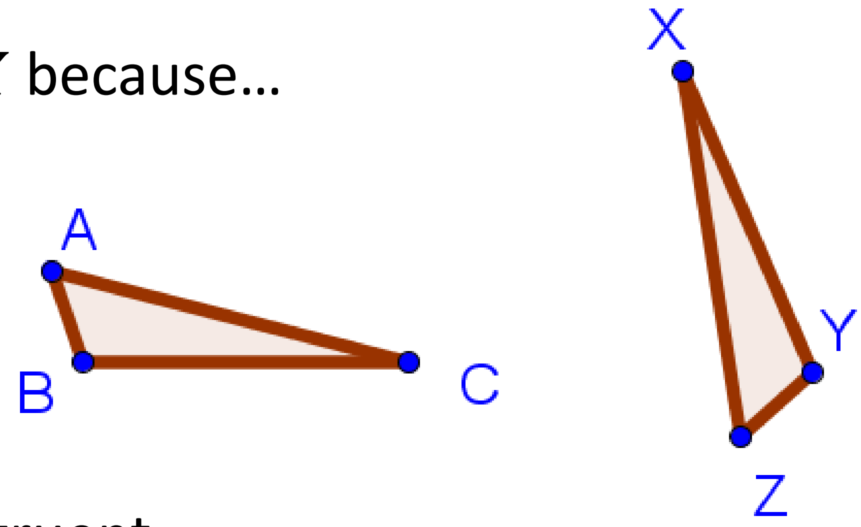
$\angle A \cong \angle F$



Congruence Statements

When stating **congruence**, the order **is** important

- The vertices of must be put in order so that the vertices in one triangle correspond to the vertexes in the other triangle. If Corresponding parts are congruent, then the triangles are congruent.
- For example: $\Delta ABC \cong \Delta ZYX$ because…
  - $\angle A \cong \angle Z \quad \angle B \cong \angle Y$
  - $\angle C \cong \angle X \quad \overline{AB} \cong \overline{ZY}$
  - $\overline{BC} \cong \overline{YX} \quad \overline{CA} \cong \overline{XZ}$



- "corresponding parts of congruent

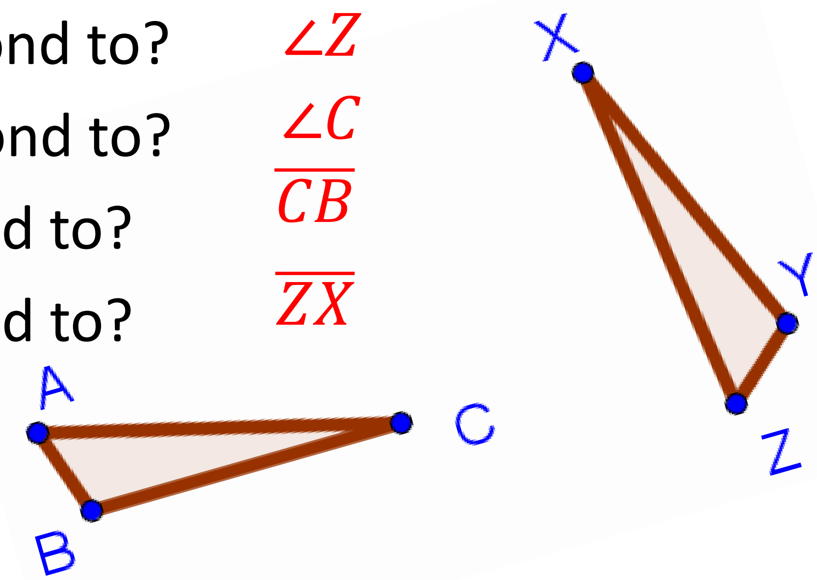
triangles are congruent" or "CPCTC"

1) What angle does $\angle A$ correspond to? $\angle Z$

2) What angle does $\angle X$ correspond to? $\angle C$

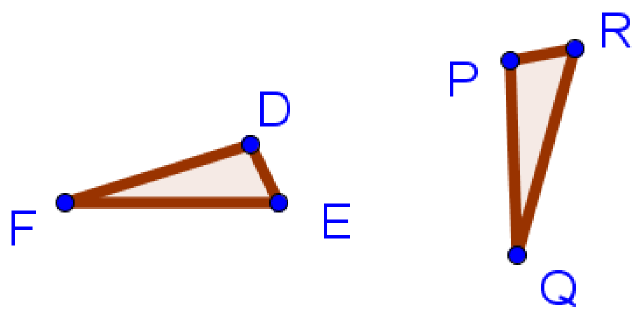
3) What side does $\overline{XY}$ correspond to? $\overline{CB}$

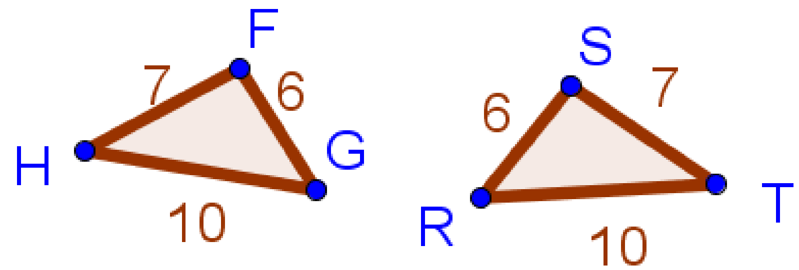
4) What side does $\overline{AC}$ correspond to? $\overline{ZX}$



Each pair of triangles is congruent:

Write a congruence statement for each pair of triangle.



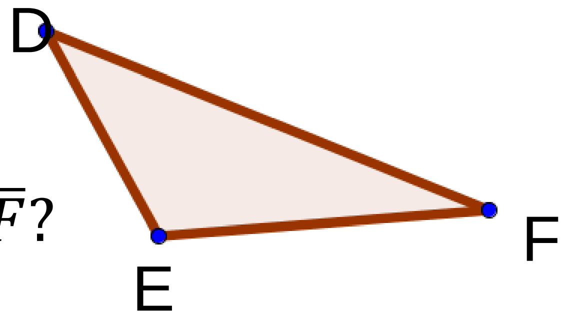


$\Delta DEF \cong \underline{\Delta PRQ}$

$\Delta RST \cong \Delta GFH$

1) $\angle D$ is the included angle of which two sides?

$\overline{DF}$ and $\overline{DE}$



2) What is the included angle of sides $\overline{DF}$ and $\overline{EF}$?

$\angle F$

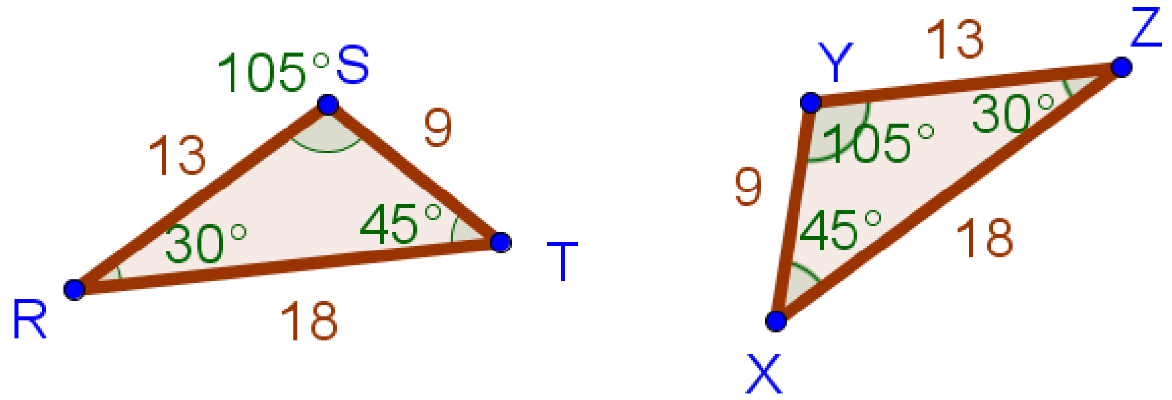
3) $\overline{DF}$ is the included side of which two angles?

$\angle D$ and $\angle F$

4) What is the included side of $\angle D$ and $\angle E$ $\overline{DE}$

Triangle Congruence

Why are ΔRST and ΔZYX congruent? (That is, how do we prove it



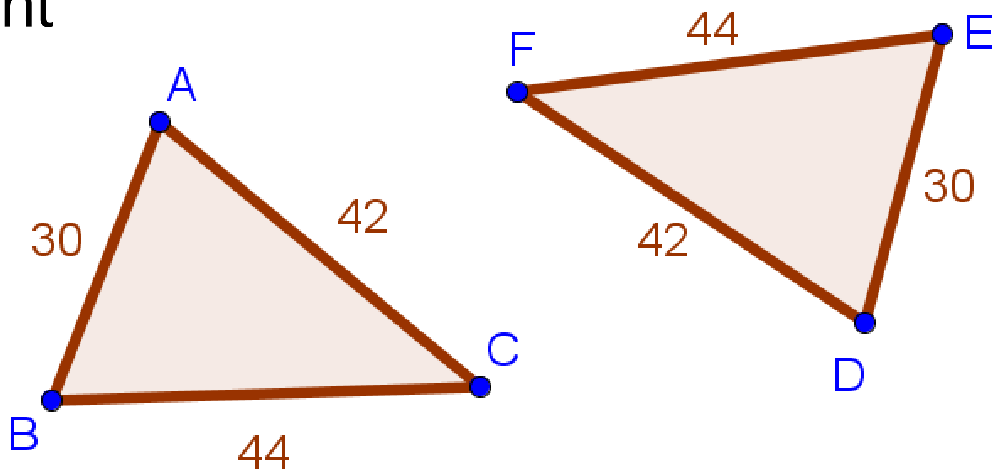
- All corresponding parts are congruent (CPCTC)
- This is just the definition of congruence

Do we need all 6 pairs of angles and sides to be congruent to prove the triangles are congruent?...

We can prove Triangle Congruence using congruence of only three pairs of corresponding parts.

Side-Side-Side (SSS) Congruency Axiom: if all three pairs of corresponding sides of a triangle are congruent, then the triangles are congruent

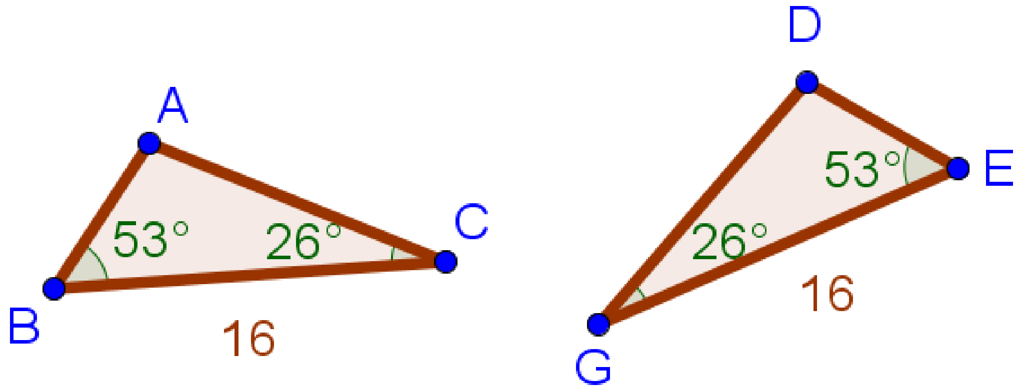
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$
- $\overline{CA} \cong \overline{FD}$



- Therefore,
- $\Delta ABC \cong \Delta DEF$ by **SSS**

Angle-Side-Angle (ASA) Congruency Axiom: if two angles and their included side are congruent, then the two triangles are congruent.

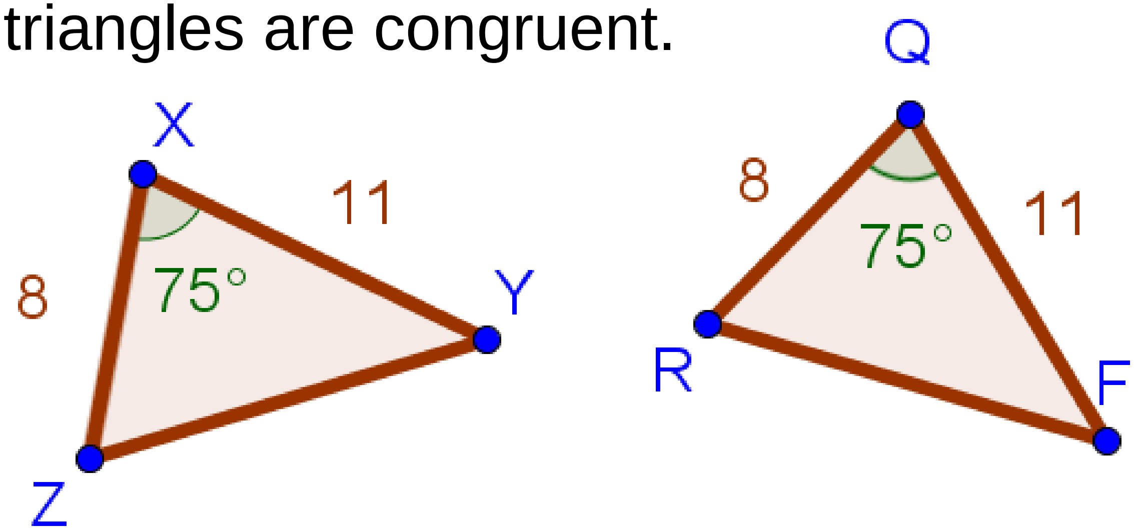
- $\angle ABC \cong \angle DEG$
- $\overline{BC} \cong \overline{EG}$
- $\angle BCA \cong \angle EGD$



- Therefore,
- $\Delta ABC \cong \Delta DEG$ by **ASA**

Side-Angle-Side (SAS) Congruency Axiom: if two pairs of corresponding sides and the pair of included angles are congruent, then the triangles are congruent.

- $\overline{XZ} \cong \overline{QR}$
- $\angle ZXY \cong \angle RQF$
- $\overline{XY} \cong \overline{QF}$

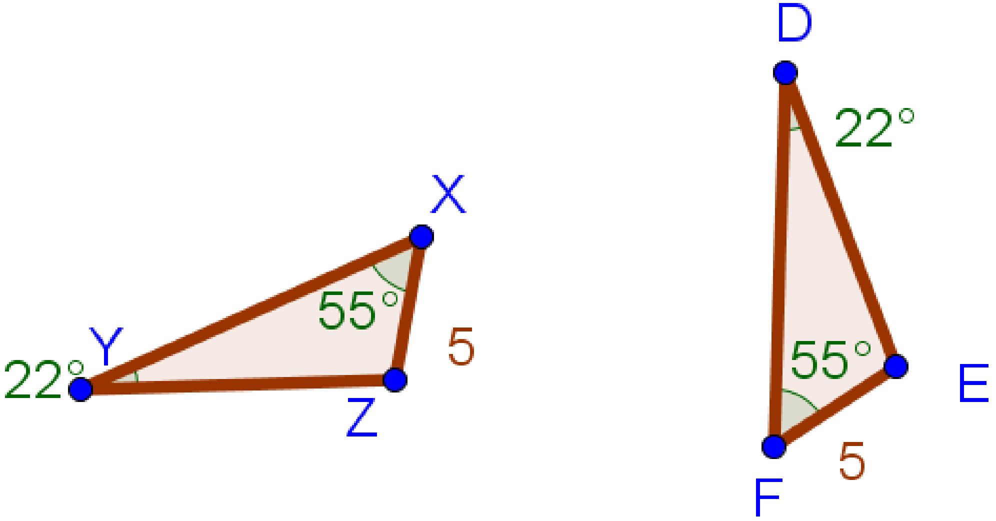


Therefore,

- $\Delta XYZ \cong \Delta QFR$ by **SAS**

Angle-Angle-Side (AAS) Congruency Axiom: If two pairs of corresponding angles are congruent and one pair of corresponding sides are congruent (which are NOT the included side), then the two triangles are congruent.

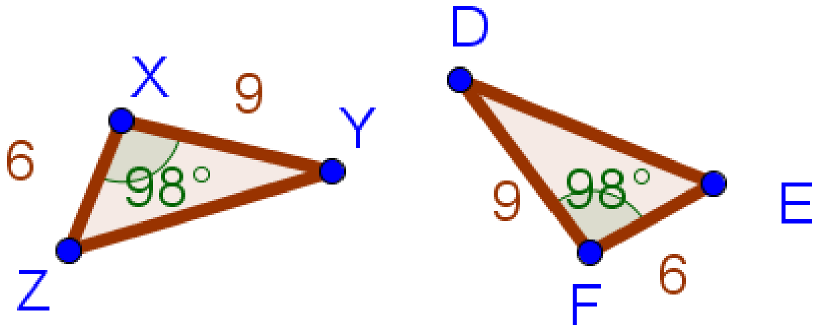
- $\angle ZXY \cong \angle EFD$
- $\angle XYZ \cong \angle FDE$
- $\overline{XZ} \cong \overline{FE}$



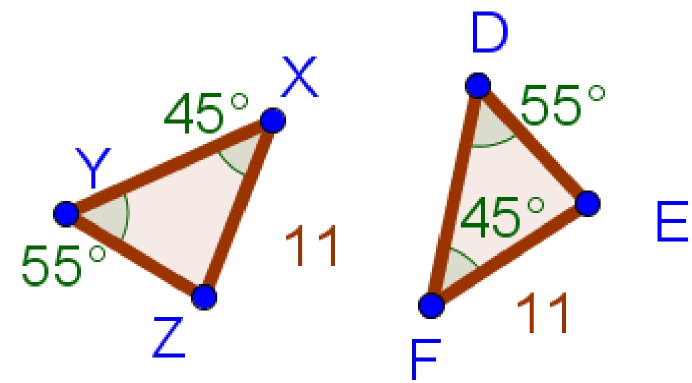
- Therefore,
- $\Delta XYZ \cong \Delta FDE$ by **AAS**

- Determine which congruence condition proves the congruence for each the following pairs of triangles.
- Write a congruence statement for each of the following pairs of triangles.

1)

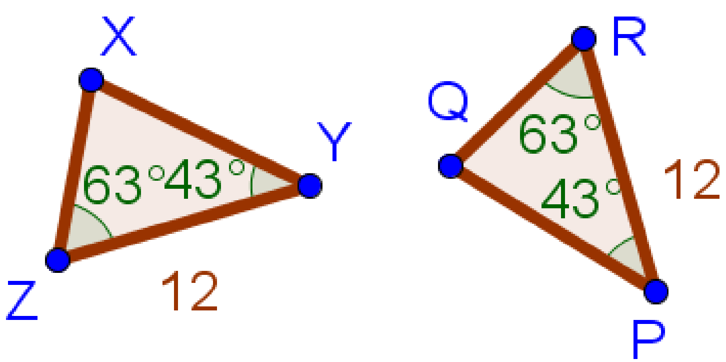


2)

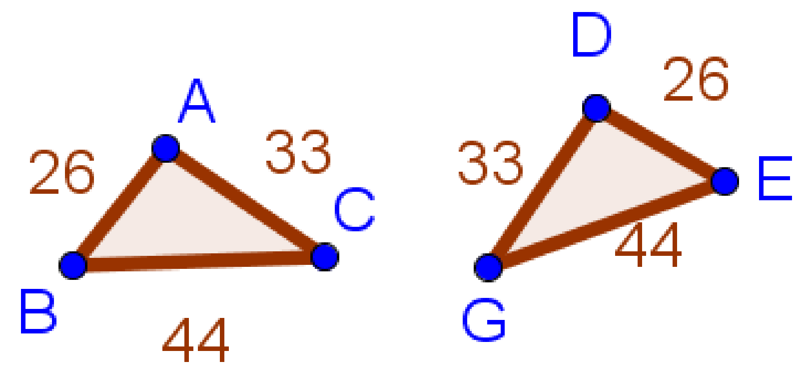


- Determine which congruence condition proves the congruence for each the following pairs of triangles.
- Write a congruence statement for each of the following pairs of triangles.

3)

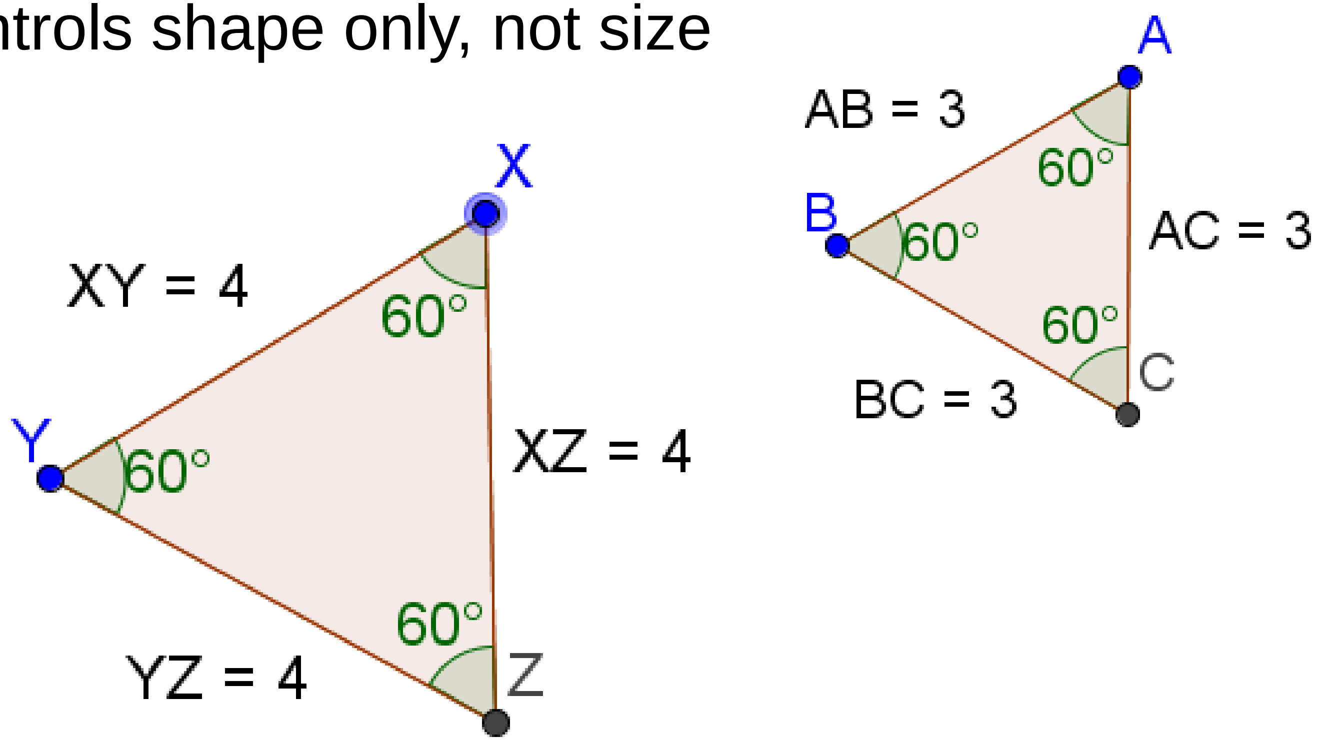


4)



Angle-Angle-Angle (AAA) Condition (is NOT a congruency)

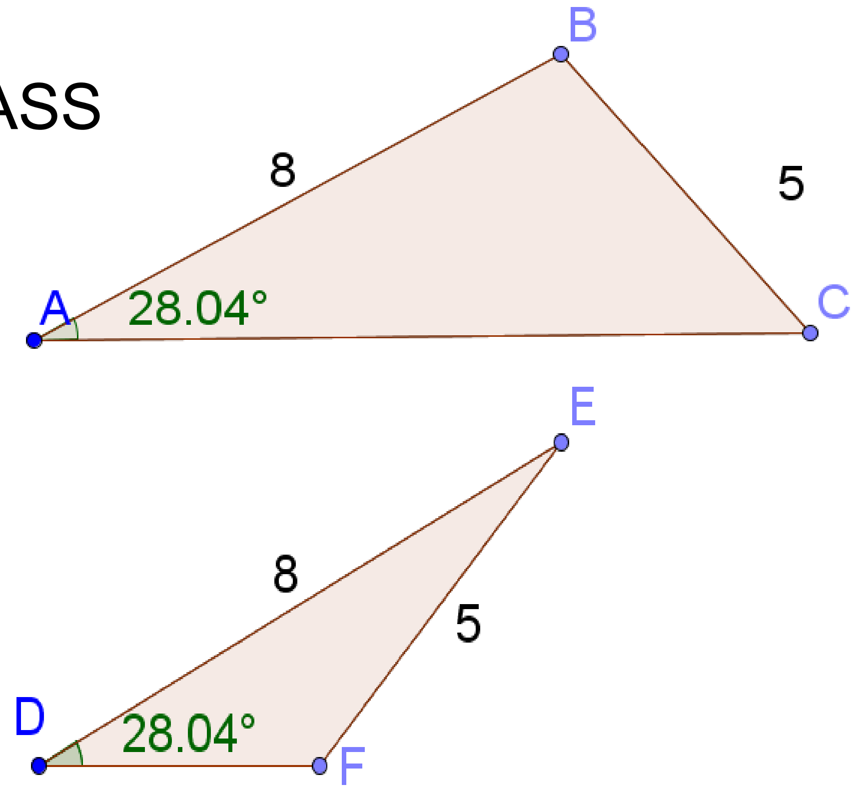
AAA controls shape only, not size



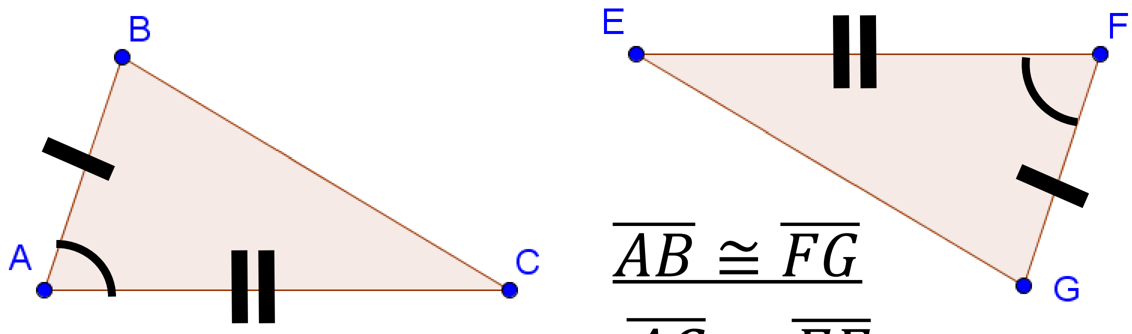
Angle-Side-Side (ASS) Condition (is NOT a congruency)

Let's look at an example of ASS

- $\angle A \cong \angle D$
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$



Write a congruence statement that identifies the *additional information needed to prove* these two triangles are congruent by **SAS.**



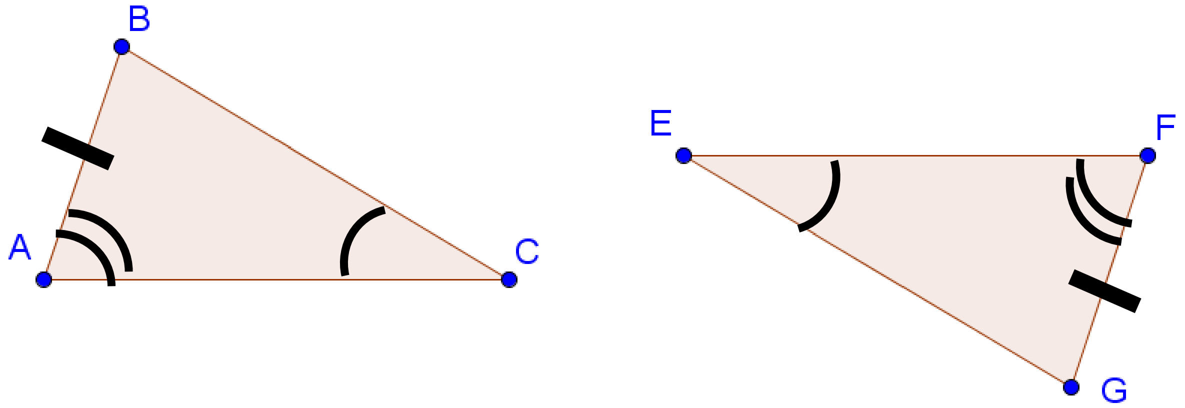
$\overline{AB} \cong \overline{FG}$

$\overline{AC} \cong \overline{EF}$

$\angle A \cong \angle F$

Write a triangle congruence statement: $\Delta ABC \cong \Delta FGE$

Are the triangles congruent? If so, write a congruence statement for the two triangles, and identify why are they congruent.



$$\Delta ABC \cong \Delta FGE \quad by\ AAS$$