Math-2A
Lesson 6-11

Solving Systems of Equations by Graphing and Substitution

Sam takes his date to Baskin Robbins. They pig out and they each have a sundae and a milkshake. It costs her $9.

$$TC = COST_{milkshake} + COST_{sundae} \qquad 9 = 2m + 2s$$

Sarah follows Sam's lead and takes her date to Baskin Robbins. Not to be outdone by Sam, Sarah and her date really pig out and each has a sundae and 2 milkshakes. It costs her $13.

$$13 = 4m + 2s$$

How much does Baskin Robbins charge for their sundaes? What do they charge for their shakes?

What values of "m" and "s" make both statements true?

System of two linear equations: Two equations (of lines) that each have the same two variables. (in this case 'x' and 'y')

$$3x + y = 7$$
$$5x - 2y = -3$$

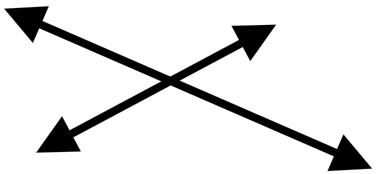
$$Ax + By = C \quad \text{(equation 1)}$$
$$Dx + Ey = F \quad \text{(equation 2)}$$

Solution to an Equation: all x-y pairs that make the equation a true statement (any point on the graph of the line).

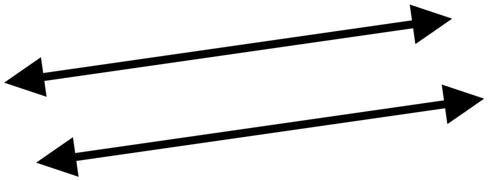
A solution of a system of two equations in two variables is an ordered pair of real numbers that is a solution of both equations.

Categories of Solutions:

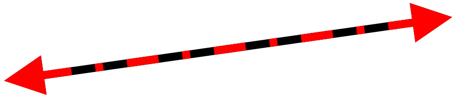
Ways 2 lines can be graphed:



Cross → one solution



Parallel →no solutions



Same line → infinitely many solutions

How do you know how many solutions there are? (1, 0, or infinite #)

$$y = 3x + 1$$
$$y = 2x + 1$$

Not same line, not parallel → one solution.

$$y = -2x + 3$$
$$y = -2x - 4$$

parallel → no solutions

$$2x + 2y = 2$$
$$x + y = 1$$

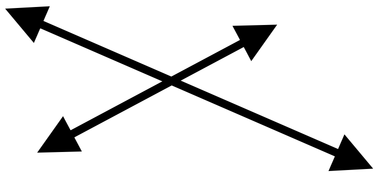
1st equation is a multiple of the 2nd equation → same line

→ infinite # of solutions.

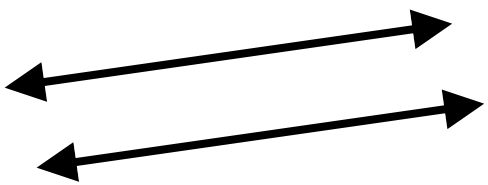
Which Category ?

$$y = 2x + 6$$

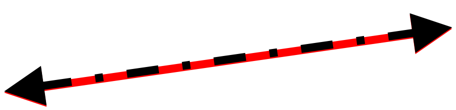
$$y = 4x - 2$$



Cross → one solution



Parallel →no solutions

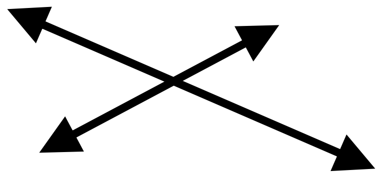


Same line → infinitely many solutions

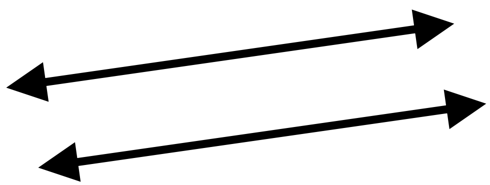
Which Category ?

$$y = 2x + 4$$

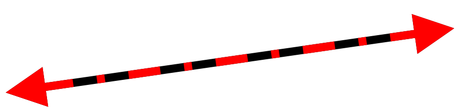
$$y = 2x - 7$$



Cross → one solution



Parallel →no solutions

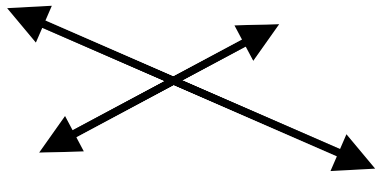


Same line → infinitely many solutions

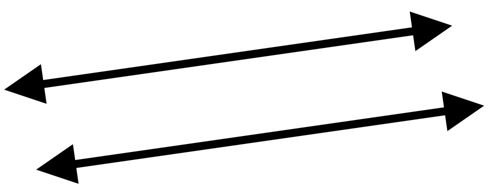
Which Category ?

$$2x + 3y = 6$$

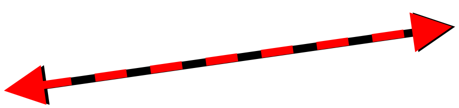
$$4x + 6y = 12$$



Cross → one solution



Parallel →no solutions



Same line → infinitely many solutions

Methods of Solving Systems

1. Graphing: The points of intersection are the solutions.

2. Substitution:

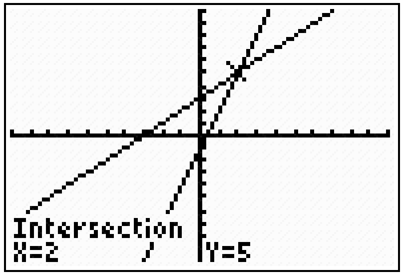
3. Elimination: we'll do this later.

Solving by graphing:

Is very easy if you use a graphing calculator.

$$y = x + 3$$

$$y = 3x - 1$$



Is doable if you use graph paper BUT you must be VERY accurate to get the correct solution.

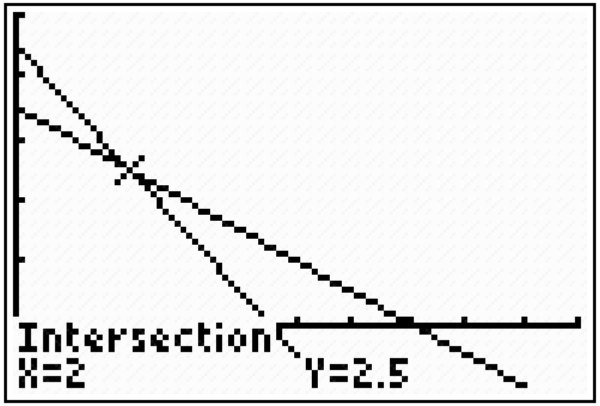
Original Word Problem:

What values of “m” and “s” make both statements true?

$$9 = 2m + 2s$$
$$13 = 4m + 2s$$

$$m = -s + 4.5$$
$$m = -0.5s + 3.25$$

m



s

Milkshakes: $2.00

Sundae: $2.50

$$6x+2y=3$$
$$y=-3x+1$$

<u>Solve by graphing</u>

→ No solutions

$$6x+2y=3$$
$$y=-3x+1$$

→ $y=-3x+\frac{3}{2}$

The lines were parallel.

x – 3y = 5
-x + 5y = 3

Solution: (17, 4)

y = 2x + 6
y = 5x + 8

Solution: (-2/3, 13/3)

$$2x-y=2$$
$$4x+2y=8$$

Solution: (3/2, 1)

Substitution Method

1. Solve one equation for one of the variables (already done if in “y =“ form).

$y = -2x + 8$

$y = 3x - 2$

2. Substitute the value of the variable into the other equation.

$(\ \ ) = -2x + 8$

$3x - 2 = -2x + 8$

$+2x \qquad +2x$

3. Solve for the single variable.

$5x - 2 = 8$

$+2 \quad +2$

$5x = 10$

$\div 5 \quad \div 5$

$x = 2$

4. Substitute the value of the solved-for variable into <u>either equation</u> to find the other varable.

$y = 3x - 2$

$y = 3(\ \ ) - 2$

$y = 3(2) - 2$

$y = 6 - 2$

$y = 4$

5. Test your solution (2, 4) in the <u>other equation.</u>

$y = -2x + 8$

$(4) = -2(2) + 8$

Solve the System of Equations Using the Substitution Method

$y = -3$

$y = -6x + 21$

(4, -3)

$y = -8x + 22$

$y = 4x - 2$

(2, 6)

$y = 6x - 3$

$y = -4x - 3$

(0, -3)

How do you know how many solutions there are? (1, 0, or infinite #)

$$6x + 2y = 3 \qquad 6x + 2(-3x + 1) = 3 \qquad 2 = 3$$

$$y = -3x + 1 \qquad 6x - 6x + 2 = 3$$

All the variables "disappeared" and the equation is <u>false</u>:

→ No solutions

How can that be?

$$6x + 2y = 3 \quad \rightarrow \quad y = -3x + \tfrac{3}{2}$$

$$y = -3x + 1$$

The lines were parallel.

How do you know how many solutions there are? (1, 0, or infinite #)

$$6x + 2y = 4$$
$$y = -3x + 2$$

$$6x + 2(-3x + 2) = 4$$
$$6x - 6x + 4 = 4$$

$$4 = 4$$

All the variables "disappeared" and the equation is <u>true</u>:

→ Infinitely many solutions

How can that be?

$$6x + 2y = 4 \quad \longrightarrow \quad y = -3x + 2$$

$$y = -3x + 2$$

Different versions of the same equation!