# Math-2

## Lesson 2-6

## Rational Exponents

$\sqrt[4]{x^{13}}$ We can simplify this in two ways

1. $\sqrt[4]{x * x * x * x * x * x * x * x * x * x * x * x * x}$

$x$

$x$

$x$

$$\sqrt[4]{x^{13}} \to x * x * x * \sqrt[4]{x} \to x^{3}\sqrt[4]{x}$$

2. $\sqrt[4]{x^{12} * x^{1}} \to x^{12/4} * \sqrt[4]{x} \to x^{3}\sqrt[4]{x}$

We can write radical as powers!! $\sqrt[4]{x^{13}} \to x^{13/4}$

# Radicals CAN be written as Powers

$$6\sqrt[5]{x}$$

Index

Radicand

Coefficient

$$6(x)^{1/5}$$

Exponent

Base

Coefficient

Coefficient → Coefficient

Radicand → Base

Index → Denominator of the Exponent

**The <u>index number</u> is the <u>denominator of the exponent</u>.**

# Your turn:

Write the following in “radical form”

5th Root of 18 $= \sqrt[5]{18}$

4th Root of 25 $= \sqrt[4]{25}$

What type of number does 5th sound like?

$$\frac{1}{5}$$

# Are <u>radicals</u> related to <u>powers</u>?

$$3^{1/2} = \sqrt[2]{3}$$

$$5^{1/3} = \sqrt[3]{5}$$

$$\sqrt[2]{x} = x^{1/2}$$

$$\sqrt[3]{7} = 7^{1/3}$$

None of these have coefficients!

$$3\sqrt[2]{y} = 3y^{1/2}$$

$$5\sqrt[3]{7} = 5(7)^{1/3}$$

Multiplication (by a coefficient) is "repeated addition." This explains why coefficients of radicals become coefficients of powers.

$$3\sqrt[2]{y} = \sqrt{y} + \sqrt{y} + \sqrt{y}$$

$$\sqrt{y} = y^{1/2}$$

$$3y^{1/2} = y^{1/2} + y^{1/2} + y^{1/2}$$

What happens if there is a product under the radical?

$$\sqrt[2]{xy} = (xy)^{1/2}$$

$$5\sqrt[3]{3x} = 5(3x)^{1/3}$$

$$2\sqrt[4]{21mn} = 2(21mn)^{1/4}$$

How did we show that the index number applied to the entire product (radicand) when re-written in "power form"?

Power of a product → product inside parentheses with an exponent.

**What happens if there is a <u>power</u> under the radical?**

$$\sqrt[5]{x^2 y} = (x^2 y)^{1/5} = x^{2/5} y^{1/5}$$

$$6\sqrt[3]{3m^2} = 6(3m^2)^{1/3} = 6(3^{1/3})m^{2/3}$$

**How did we show that the <u>index number</u> applied to the <u>entire product (including the power)</u> when re-written in "power form"?**

**Power of a product → product <u>inside parentheses</u> with an exponent.**

“Exponential Form” that has both
a numerator and denominator

The exponent can be written as a rational number.

$$x^{5/2} = \sqrt[2]{x^5}$$

Numerator:
Exponent of the base.

Denominator:
Root of the base.

$$\sqrt[3]{2^2} = 2^{2/3}$$

Radical Form | Exponential Form

**Write the following radicals as powers.**

$$\sqrt[2]{3m} \rightarrow (3m)^{1/2}$$

$$4\sqrt[3]{5y} \rightarrow 4(5y)^{1/3}$$

$$3m\sqrt[4]{6n} \rightarrow 3m(6n)^{1/4}$$

$$\sqrt[5]{x^3y^2} \rightarrow (x^3y^2)^{1/5} \rightarrow x^{3/5}y^{2/5}$$

$$5\sqrt[4]{3m^2} \rightarrow 5(3m^2)^{1/3} \rightarrow 5(3^{1/3})m^{2/3}$$

Rewrite in “radical form”

$$m^{1/5} \rightarrow \sqrt[5]{m}$$

$$3nm^{1/4} \rightarrow 3n\sqrt[4]{m}$$

$$2(18n^2)^{1/6} \rightarrow 2\sqrt[6]{18n^2}$$

$$5(4x^2y^6)^{1/3} \rightarrow 5\sqrt[6]{4*x^2*y^6} \rightarrow 5y\sqrt[3]{4x^2}$$

# Multiply Powers Property

$$y^2 * y^3 = ? \quad = y^{2+3} \quad = y^5$$

When multiplying “same based powers” add the exponents.

<u>Multiply Powers Property</u> Add exponents

$$x^{\frac{2}{3}} * x^{\frac{3}{4}} \rightarrow x^{\frac{2}{3}+\frac{3}{4}}$$

Yes, you must be able to add fractions

Working with just the exponent →

$$\frac{2}{3} + \frac{3}{4}$$

Multiply by "1" in the form of…→

$$\frac{4}{4} * \frac{2}{3} + \frac{3}{4} * \frac{3}{3} \rightarrow \frac{8}{12} + \frac{9}{12} \rightarrow \frac{17}{12}$$

Rewrite the power →

$$\rightarrow x^{\frac{17}{12}}$$

# Exponent of a Power Property

$$\left(y^2\right)^3 = ? \qquad = y^{2*3} \qquad = y^6$$

When multiplying “same based powers” add the exponents.

# Exponent of a Power Property

$$\left(y^{1/2}\right)^{2/3} = y^{\frac{1}{2}*\frac{2}{3}} = y^{\frac{2}{6}} = y^{\frac{1}{3}}$$

$$\left(\frac{x^2}{y^{3/2}}\right)^{2/3} = \frac{x^{\frac{2}{1}*\frac{2}{3}}}{y^{\frac{3}{2}*\frac{2}{3}}} = \frac{x^{\frac{4}{3}}}{y^1}$$

# Exponent of a Power Property

Multiply exponents

$$\left(x^{\frac{3}{4}}y^{5}\right)^{\frac{1}{3}} \rightarrow x^{\frac{3}{4}*\frac{1}{3}}y^{\frac{5}{1}*\frac{1}{3}} \rightarrow x^{\frac{1}{4}}y^{\frac{5}{3}}$$

$$3x\left(y^{1/5}\right)^{2/3} = 3\ x\ y^{\frac{1}{5}*\frac{2}{3}} = 3xy^{\frac{2}{15}}$$

# Negative Exponent Property

Grab and drag same-based powers to be next to each other.

$$\frac{x^2 y^{2/3}}{y^{-1/2}} = x^2 y^{2/3} y^{1/2} = x^2 y^{\frac{2}{3}+\frac{1}{2}} = x^2 y^{\frac{4}{6}+\frac{3}{6}} = x^2 y^{\frac{7}{6}}$$

$$\frac{2x^{\frac{1}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2}{x^{\frac{2}{3}} x^{-\frac{1}{3}}} \rightarrow \frac{2}{x^{\frac{1}{3}}}$$

Not allowed to have rational exponents in the denominator

$$\rightarrow 2x^{-1/3}$$

Not allowed to have negative exponents.

# Rational Exponents in the Denominator

$$\frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}}$$

Rational exponent in the denominator means irrational denominator, which we rationalize

$$\frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}} * \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{y}}{y}$$

Rational exponent in the denominator → what is the next bigger natural number from ½ ?

1

What number do you add to ½ to get 1?

$$\frac{1}{y^{1/2}} * \frac{y^{1/2}}{y^{1/2}} = \frac{y^{1/2}}{y}$$

In order to add a number to an exponent you have to multiply by a same-based power with the exponent you are trying to add.

# Negative Exponent Property

$$\frac{2x^{\frac{1}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2}{x^{\frac{2}{3}}x^{-\frac{1}{3}}} \rightarrow \frac{2}{x^{\frac{1}{3}}}$$

What is the next bigger whole number than 1/3 ?

1

What number do you add to 1/3 to get 1?

2/3

$$\rightarrow \frac{2}{x^{\frac{1}{3}}} * \frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}} \rightarrow \frac{2x^{\frac{2}{3}}}{x^{\frac{1}{3}+\frac{2}{3}}}$$

$$\rightarrow \frac{2x^{\frac{2}{3}}}{x}$$

Multiply by one “in the form of” a same-base power whose exponent is 2/3 (both numerator and denominator)

Your turn:

$$x^{2/3}x^{3/4} \quad = x^{\frac{2}{3}+\frac{3}{4}}$$

$$= x^{\left(\frac{2}{3}*\frac{4}{4}\right)+\left(\frac{3}{4}*\frac{3}{3}\right)} \quad = x^{\frac{8}{12}+\frac{9}{12}} = x^{\frac{17}{12}}$$

# Negative Exponent Property

$$\frac{x^2}{y^{-1/2}} = x^2 y^{1/2}$$

We don't want negative exponents in our answers

$$\frac{y^{-3}}{x^{-3/2}} = \frac{x^{3/2}}{y^3}$$