Math-2
Lesson 2-4

Radicals

$\sqrt{3}$ What number is equivalent to the square root of 3?

$x = \sqrt{3}$ Square both sides of the equation

$(x)^2 = \left(\sqrt{3}\right)^2$ $x^2 = 3$

$x = \sqrt{3}$ is an equivalent statement to $x^2 = 3$

$\sqrt{3}$ ≈ 1.732

≈ 1.7321

≈ 1.73205

≈ 1.732051

$\approx 1.7320508...$

There is <u>no equivalent number</u>

The decimal, is just an <u>approximation.</u>

<u>Radicals</u>

Index number

Radical symbol

$$\sqrt{3} \rightarrow \sqrt[2]{3}$$

Radicand

$$x = \sqrt[2]{3}$$
$$x^2 = 3$$

The "<u>square root of 3</u>" means:
"<u>what number squared equals 3</u>?"

$$x = \sqrt[3]{4}$$
$$x^3 = 4$$

The "<u>3rd root of 4</u>" means:
"<u>what number cubed equals 4</u>?"

$$x = \sqrt[5]{2}$$
$$x^5 = 2$$

The "5th <u>root of 2</u>" means:
"<u>what number used as a factor 5 times equals 2</u>?"

Adding and subtracting radicals

Can these two terms be combined using addition? $3x + 2x$

Write 3x as repeated addition $x + x + x$

Write 2x as repeated addition $x + x$

$$3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$$

When multiplication is written as repeated addition, "like terms" look exactly alike.

$$3\sqrt{x} + 2\sqrt{x} \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} \rightarrow 5\sqrt{x}$$

$$3\sqrt{6} + 2\sqrt{6} \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} \rightarrow 5\sqrt{6}$$

Define "like powers" "Same base, same exponent".

$$3x^4 + 2x^4 \rightarrow 5x^4$$

Define "like radicals" "Same radicand, same index number".

$$3\sqrt{6} + 2\sqrt{6} \rightarrow 5\sqrt{6}$$

Which of the following are "like radicals" that can be added?

$$\sqrt{2} + \sqrt{3}$$

$$\sqrt[4]{5} + \sqrt[4]{5}$$

$$2\sqrt{3} + 3\sqrt{2}$$

$$3\sqrt[5]{2} + 4\sqrt[5]{2}$$

$$\sqrt[4]{2} + \sqrt[3]{2}$$

$$6\sqrt[3]{4} + 6\sqrt[4]{4}$$

$$\sqrt{3}+\sqrt{2} \rightarrow \sqrt{3+2}=\sqrt{5}$$

Are they equivalent?

$$\sqrt{3} \approx 1.7321\ldots \qquad \sqrt{2} \approx 1.4142\ldots$$

$$\sqrt{3}+\sqrt{2} \approx 3.1462\ldots \qquad \sqrt{5} \approx 2.2630\ldots$$

$$\sqrt{3}+\sqrt{2} \neq \sqrt{5}$$

$$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$$

This is NOT a property of radicals.
NEVER DO THIS!!!!

$$\sqrt{4}+\sqrt{9} \rightarrow \sqrt{13}$$

$$\sqrt{4}+\sqrt{9} \rightarrow 2+3 \rightarrow 5 \neq \sqrt{13}$$

Simplify the following:

$$3\sqrt{2}+5\sqrt{2} \rightarrow 8\sqrt{2}$$

$$5\sqrt{3}-4\sqrt{3} \rightarrow \sqrt{3}$$

$$\sqrt{5}+3\sqrt{5} \rightarrow 4\sqrt{5}$$

$$7\sqrt{6x}+2\sqrt{6x} \rightarrow 9\sqrt{6x}$$

$$3\sqrt{x}+2\sqrt{x} \rightarrow 5\sqrt{x}$$

$$5\sqrt{2x}-\sqrt{5x}+3\sqrt{5x} \rightarrow 5\sqrt{2x}+2\sqrt{5x}$$

$7\sqrt{6}+2\sqrt{24}$ not “like terms” in their present form

$\sqrt{3} * \sqrt{2}$

$\sqrt{3*2} \to \sqrt{6}$

Will this work?

$\sqrt{3} \approx 1.7321...$ $\sqrt{2} \approx 1.4142...$

$\sqrt{3} * \sqrt{2} \approx 2.4495$

$\sqrt{6} \approx 2.4495...$

Product of Radicals Property

$\sqrt{a} * \sqrt{b} \to \sqrt{a*b}$

$$\sqrt{5} * \sqrt{2} = \sqrt{10}$$

$\sqrt{4} * \sqrt{9} \to \sqrt{4*9}$

$2*3 \to \sqrt{36}$

$2*3 \to 6$

$6 = 6$

Are these equivalent?

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Although I only gave two examples, it actually DOES WORK for whole number radicand.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Simplify the following:

$$3\sqrt{8} * 5\sqrt{2}$$

$$3 * \sqrt{8} * 5 * \sqrt{2}$$

$$3 * 5 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{8} * \sqrt{2}$$

$$15 * \sqrt{16}$$

$$15 * 4 = 60$$

$$2\sqrt{3} * 3\sqrt{5} \rightarrow 6\sqrt{15}$$

$$7\sqrt{6} * 2\sqrt{5} \rightarrow 14\sqrt{30}$$

$$\sqrt{5} + 3\sqrt{5} \rightarrow 4\sqrt{5}$$

$$7\sqrt{6} + 2\sqrt{6} \rightarrow 9\sqrt{6}$$

Simplify radicals: use the Product of Radicals to “break apart” the radical into a “perfect square” times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$$\sqrt{18} \quad \rightarrow \sqrt{9} * \sqrt{2} \quad \rightarrow 3 * \sqrt{2} \quad \rightarrow 3\sqrt{2}$$

Simplify

$$\sqrt{24} \ \rightarrow \sqrt{4} * \sqrt{6} \ \rightarrow 2\sqrt{6}$$

$$3\sqrt{32x^2} \quad \rightarrow 3 * \sqrt{16} * \sqrt{x^2} + \sqrt{2} \quad \rightarrow 3 * 4 * x * \sqrt{2} \quad \rightarrow 12x\sqrt{2}$$

$$-2\sqrt{56x^3y} \quad \rightarrow -2 * \sqrt{x^2} * \sqrt{8 * 7xy}$$

$$\rightarrow -2 * x * \sqrt{4} * \sqrt{2 * 7xy}$$

$$\rightarrow -4x\sqrt{14xy}$$

Stop Here

Can we add "unlike" radicals?.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

Simplify

$$7\sqrt{6} + 2\sqrt{24} \rightarrow 7\sqrt{6} + (2 * \sqrt{4} * \sqrt{6})$$

$$\rightarrow 7\sqrt{6} + (2 * 2 * \sqrt{6})$$

$$\rightarrow 7\sqrt{6} + 4\sqrt{6}$$

$$\rightarrow 11\sqrt{6}$$

$$-3\sqrt{32} + 2\sqrt{8} \rightarrow (-3 * \sqrt{16} * \sqrt{2}) + (2 * \sqrt{4} * \sqrt{2})$$

$$\rightarrow (-3 * 4 * \sqrt{2}) + (2 * 2 * \sqrt{2})$$

$$\rightarrow -12\sqrt{2} + 4\sqrt{2}$$

$$\rightarrow -8\sqrt{2}$$

Simplify radicals: use the Product of Radicals to "break apart" the radical into a "powers of exponent 'm' " times a number.

$$\sqrt[m]{a} * \sqrt[m]{b} = \sqrt[m]{ab}$$

$$\sqrt[3]{x^4} \quad \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \quad \rightarrow x\sqrt[3]{x}$$

Simplify

$$\sqrt[4]{3x^5y} \rightarrow \sqrt[4]{x^4} * \sqrt[4]{3x} \quad \rightarrow x\sqrt[4]{3x}$$

$$3\sqrt[3]{16x^2y^5} \rightarrow 3*\sqrt[3]{8}*\sqrt[3]{y^3}*\sqrt[3]{2x^2y^2}$$

$$\rightarrow 3*\sqrt[3]{2^3}*\sqrt[3]{y^3}*\sqrt[3]{2x^2y^2}$$

$$\rightarrow 3*2*y*\sqrt[3]{2x^2y^2}$$

$$\rightarrow 6y\sqrt[3]{2x^2y^2}$$

Another way to Simplify Radicals **Factor, factor, factor!!!**

$$\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3*3*3}$$

What is the factor that is used ‘2’ times under the radical?

Bring that out factor (that is used 2 times).

$$\rightarrow 3\sqrt[2]{2*3} \quad \rightarrow 3\sqrt{6}$$

Using Properties of Exponents to reduce the writing:

$$\sqrt[4]{32x^6} \rightarrow \sqrt[4]{32*x^4*x^2}$$

$$\rightarrow x\sqrt[4]{32*x^2}$$

$$\rightarrow x\sqrt[4]{2^4*2^1*x^2}$$

$$\rightarrow 2x\sqrt[4]{2x^2}$$

Factor the numerator!

$$\frac{\sqrt{12}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}*\sqrt{6}}{\sqrt{2}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{6}$

$$\frac{\sqrt{48x^3}}{\sqrt{16x}} \rightarrow \frac{\sqrt{16x}*\sqrt{3x^2}}{\sqrt{16x}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{3x^2} \rightarrow x\sqrt{3}$

$$\frac{\sqrt{56x^3y}}{\sqrt{8xy}} \rightarrow \frac{\sqrt{8xy}*\sqrt{7x^2}}{\sqrt{8xy}}$$

Inverse Property of Multiplication $\rightarrow \sqrt{7x^2} \rightarrow x\sqrt{7}$

Simplify

$$\sqrt{\frac{32}{9x^2}}$$

$$\frac{\sqrt{50y^2}}{\sqrt{2y}}$$

$$\frac{\sqrt{49}}{\sqrt{7}}$$

$$\sqrt{\frac{48}{49}}$$

Rationalizing the denominator: using mathematical properties to change an irrational number (or imaginary) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2} * \sqrt{2} = \sqrt{2*2} = \sqrt{4} = 2$$

$$\sqrt{3} * \sqrt{3} = \sqrt{3*3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

Identity Property of Multiplication

multiplying by '1' doesn't change the number.

$$\frac{1}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{\sqrt{6}}{6}$$

$$\frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{2\sqrt{6}}{6} \rightarrow \frac{2 * \sqrt{6}}{2 * 3} \rightarrow \frac{\sqrt{6}}{3}$$

$$\frac{25}{\sqrt{15}} * \frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{5 * 5 ** \sqrt{15}}{5 * 3} \rightarrow \frac{5\sqrt{15}}{3}$$

$$\frac{14}{3\sqrt{21}} * \frac{\sqrt{21}}{\sqrt{21}} \rightarrow \frac{14\sqrt{21}}{3 * 21} \rightarrow \frac{2 * 7 ** \sqrt{21}}{3 * 7 * 3} \rightarrow \frac{2\sqrt{21}}{9}$$

In all of the previous examples we just multiplied by "one in the form of" the denominator radical over the denominator radical.

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{8}} * \frac{\sqrt{8}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7*2*2*2}}{8} \rightarrow \frac{3*2\sqrt{7*2}}{8}$$

$$\rightarrow \frac{3*2\sqrt{14}}{2*4}$$

It is always easier to simplify (by factoring) ***BEFORE*** you multiply

$$\frac{3\sqrt{7}}{\sqrt{8}} \rightarrow \frac{3\sqrt{7}}{\sqrt{4}*\sqrt{2}} \rightarrow \frac{3\sqrt{7}}{2\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{3\sqrt{14}}{2*2} \rightarrow \frac{3\sqrt{14}}{4}$$

$$\frac{6\sqrt{5}}{3\sqrt{12}} \rightarrow \frac{3*2*\sqrt{5}}{3*\sqrt{4}*\sqrt{3}} \rightarrow \frac{2*\sqrt{5}}{2*\sqrt{3}} \rightarrow \frac{\sqrt{5}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{15}}{3}$$

What about variables?

$$\frac{3}{\sqrt{5x}} * \frac{\sqrt{5x}}{\sqrt{5x}} \rightarrow \frac{3\sqrt{5x}}{5x}$$

$$\frac{\sqrt{15}}{\sqrt{5x}} \rightarrow \frac{\cancel{\sqrt{5}} * \sqrt{3}}{\cancel{\sqrt{5}} * \sqrt{x}} \rightarrow \frac{\sqrt{3}}{\sqrt{x}} * \frac{\sqrt{x}}{\sqrt{x}} \rightarrow \frac{\sqrt{3x}}{\sqrt{x^2}} \rightarrow \frac{\sqrt{3x}}{x}$$

What about higher index numbers?

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x}} \rightarrow \frac{\sqrt[3]{x*x}}{\sqrt[3]{x*x*x}} \rightarrow \frac{\sqrt[3]{x^2}}{x}$$

How many more ‘x’s are needed in the denominator radicand?

Remember: the cubed root of x-cubed equals x. $\sqrt[3]{x^3} = x$

We need two more x’s under the denominator radical.

Using the multiply powers property we don’t have to write out all the individual x’s.

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \rightarrow \frac{\sqrt[3]{x^2}}{x}$$

Rationalize the denominator

$$\frac{2x}{\sqrt{3x}}$$

$$\frac{3}{\sqrt[4]{x^2}}$$

$$\frac{2\sqrt{3y^3}}{\sqrt{5y}}$$

What about higher index numbers?

$$\frac{4}{12\sqrt[3]{4x}} \rightarrow \frac{1*4}{4*3*\sqrt[3]{2*2*x}} \rightarrow \frac{1}{3\sqrt[3]{2*2*x}}$$

How many more ‘2’s’ and ‘x’s are needed in the denominator radicand?

We need one more ‘2’ and two more ‘x’s under the denominator radical.

$$\rightarrow \frac{1}{3\sqrt[3]{2*2*x}} * \frac{\sqrt[3]{2*x*x}}{\sqrt[3]{2*x*x}} \rightarrow \frac{\sqrt[3]{2x^2}}{3*\sqrt[3]{2*2*2*x*x*x}}$$

$$\rightarrow \frac{\sqrt[3]{2x^2}}{3*2*x} \rightarrow \frac{\sqrt[3]{2x^2}}{6x}$$

What about higher index numbers?

$$\frac{\sqrt[4]{5}}{\sqrt[4]{25x^3}} \rightarrow \frac{\sqrt[4]{5}}{\sqrt[4]{5*5*x^3}} * \frac{\sqrt[4]{5*5*x}}{\sqrt[4]{5*5*x}} \rightarrow \frac{\sqrt[4]{125x}}{5x}$$

How many more <u>‘5’s’ and ‘x’s </u>are needed in the denominator radicand?

We need <u>one more ‘5’s and one more ‘x’ </u>under the denominator radical.