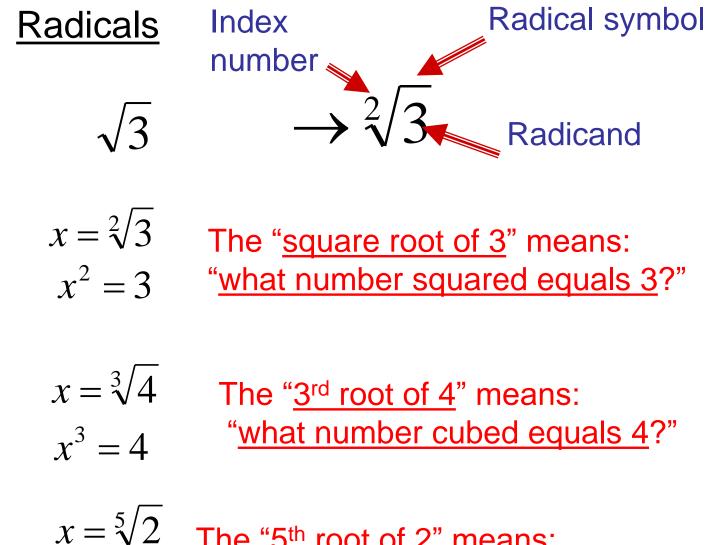
Math-2 Lesson 2-4 Radicals $\sqrt{3}$ What number is equivalent to the square root of 3? $x = \sqrt{3}$ Square both sides of the equation $(x)^2 = (\sqrt{3})^2$ $x^2 = 3$ $x = \sqrt{3}$ is an equivalent statement to $x^2 = 3$

 $\sqrt{3}$ ≈ 1.732 ≈ 1.7321 ≈ 1.73205 ≈ 1.732051 ≈ 1.7320508...



 $x = \sqrt[3]{2}$ The "5th root of 2" means: $x^5 = 2$ "what number used as a factor 5 times equals 2?"

Adding and subtracting radicals

Can these two terms be combined using addition? 3x + 2xWrite 3x as repeated addition x + x + xWrite 2x as repeated addition x + x $3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$

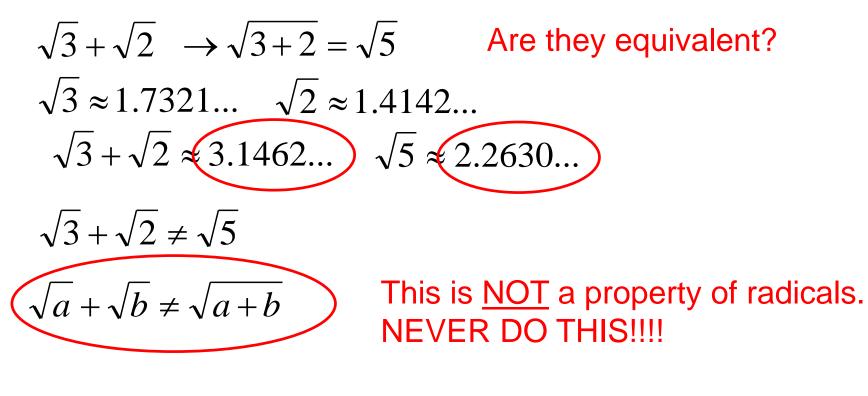
When <u>multiplication</u> is written as <u>repeated addition</u>, "like terms" look <u>exactly alike.</u>

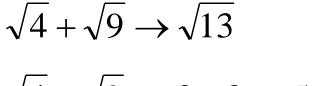
$$3\sqrt{x} + 2\sqrt{x} \quad \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} \rightarrow 5\sqrt{x}$$
$$3\sqrt{6} + 2\sqrt{6} \quad \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} \rightarrow 5\sqrt{6}$$

Define "like powers" "Same base, same exponent". $3x^4 + 2x^4 \rightarrow 5x^4$

Define "like radicals" "Same radicand, same index number". $3\sqrt{6} + 2\sqrt{6} \rightarrow 5\sqrt{6}$

Which of the following are "like radicals" that can be added? $\sqrt{2} + \sqrt{3}$ $4\sqrt{5} + 4\sqrt{5}$ $2\sqrt{3} + 3\sqrt{2}$ $3^{5}\sqrt{2} + 4^{5}\sqrt{2}$ $4\sqrt{2} + \sqrt[3]{2}$ $6^{3}\sqrt{4} + 6^{4}\sqrt{4}$



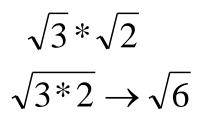


 $\sqrt{4} + \sqrt{9} \rightarrow 2 + 3 \rightarrow 5 \neq \sqrt{13}$

Simplify the following:

 $3\sqrt{2} + 5\sqrt{2} \rightarrow 8\sqrt{2}$ $5\sqrt{3} - 4\sqrt{3} \rightarrow \sqrt{3}$ $\sqrt{5} + 3\sqrt{5} \rightarrow 4\sqrt{5}$

 $7\sqrt{6x} + 2\sqrt{6x} \rightarrow 9\sqrt{6x}$ $3\sqrt{x} + 2\sqrt{x} \rightarrow 5\sqrt{x}$ $5\sqrt{2x} - \sqrt{5x} + 3\sqrt{5x} \rightarrow 5\sqrt{2x} + 2\sqrt{5x}$ $7\sqrt{6} + 2\sqrt{24}$ not "like terms" in their present form



Will this work?

 $\sqrt{3} \approx 1.7321...$ $\sqrt{2} \approx 1.4142...$ $3 * \sqrt{2} \approx 2.4495$ 6 ≈ 2.4495.

 $\sqrt{a} * \sqrt{b} \to \sqrt{a * b}$

$$\sqrt{5} * \sqrt{2} = \sqrt{10}$$

 $\sqrt{4} * \sqrt{9} \to \sqrt{4 * 9}$

 $2*3 \rightarrow \sqrt{36}$

Are these equivalent?

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$$2*3 \rightarrow 6$$

6 = 6

Although I only gave two examples, it actually DOES WORK for whole number radicand.

 $\sqrt{a} * \sqrt{b} = \sqrt{ab}$

Simplify the following:

$3\sqrt{8}*5\sqrt{2}$	$2\sqrt{3}*3\sqrt{5}$	$\rightarrow 6\sqrt{15}$
$3*\sqrt{8}*5*\sqrt{2}$	$7\sqrt{6} * 2\sqrt{5}$	$\rightarrow 14\sqrt{30}$
$3*5*\sqrt{8}*\sqrt{2}$ $15*\sqrt{8}*\sqrt{2}$	$\sqrt{5} + 3\sqrt{5}$	$\rightarrow 4\sqrt{5}$
$15 \times \sqrt{16}$	$7\sqrt{6} + 2\sqrt{6}$	$\rightarrow 9\sqrt{6}$
15*4 = 60		

<u>Simplify radicals</u>: use the Product of Radicals to "break apart" the radical into a "perfect square" times a number.

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$

$$\sqrt{18} \rightarrow \sqrt{9} * \sqrt{2} \rightarrow 3 * \sqrt{2} \rightarrow 3\sqrt{2}$$

Simplify

$$\sqrt{24} \rightarrow \sqrt{4} * \sqrt{6} \rightarrow 2\sqrt{6}$$

$$3\sqrt{32x^2} \rightarrow 3*\sqrt{16} * \sqrt{x^2} + \sqrt{2} \rightarrow 3*4*x*\sqrt{2} \rightarrow 12x\sqrt{2}$$

$$-2\sqrt{56x^3y} \rightarrow -2*\sqrt{x^2} * \sqrt{8*7xy}$$

$$\rightarrow -2*x*\sqrt{4} * \sqrt{2*7xy}$$

$$\rightarrow -4x\sqrt{14xy}$$

Stop Here

Can we add "unlike" radicals?

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$
Simplify $7\sqrt{6} + 2\sqrt{24} \rightarrow 7\sqrt{6} + (2*\sqrt{4}*\sqrt{6})$
 $\rightarrow 7\sqrt{6} + (2*2*\sqrt{6})$
 $\rightarrow 7\sqrt{6} + 4\sqrt{6}$
 $\rightarrow 11\sqrt{6}$
 $-3\sqrt{32} + 2\sqrt{8} \rightarrow (-3*\sqrt{16}*\sqrt{2}) + (2*\sqrt{4}*\sqrt{2})$
 $\rightarrow (-3*4*\sqrt{2}) + (2*2*\sqrt{2})$
 $\rightarrow -12\sqrt{2} + 4\sqrt{2}$
 $\rightarrow -8\sqrt{2}$

<u>Simplify radicals</u>: use the Product of Radicals to "break apart" the radical into a "powers of exponent 'm' " times a number.

$$\sqrt[m]{a} * \sqrt[m]{b} = \sqrt[m]{ab}$$

$$\sqrt[3]{x^4} \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \rightarrow x\sqrt[3]{x}$$

$$\underbrace{Simplify}_{4\sqrt{3x^5y}} \rightarrow \sqrt[4]{x^4} * \sqrt[4]{3x} \rightarrow x\sqrt[4]{3x}$$

$$3\sqrt[3]{16x^2y^5} \rightarrow 3 * \sqrt[3]{8} * \sqrt[3]{y^3} * \sqrt[3]{2x^2y^2}$$

$$\rightarrow 3 * \sqrt[3]{2^3} * \sqrt[3]{y^3} * \sqrt[3]{2x^2y^2}$$

$$\rightarrow 3 * 2 * y * \sqrt[3]{2x^2y^2}$$

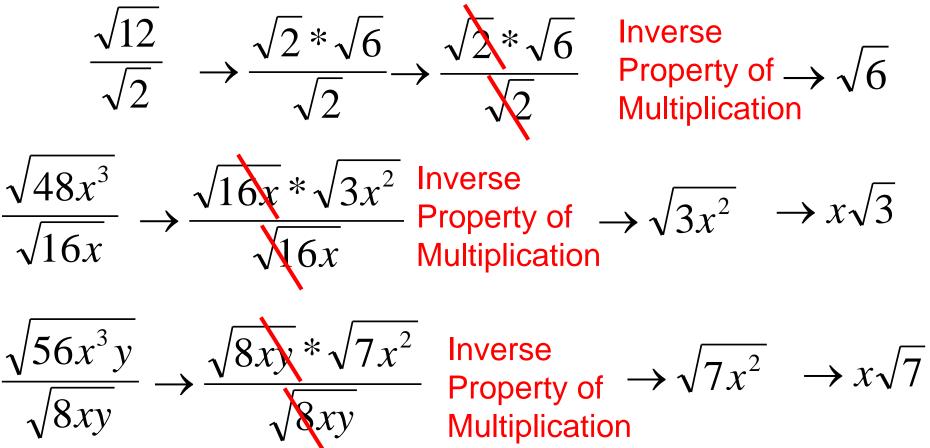
$$\rightarrow 6y\sqrt[3]{2x^2y^2}$$

Another way to Simplify Radicals Factor, factor, factor!!! $\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3} \times \sqrt[3]{3}$ What is the factor that is used '2' times under the radical? Bring that out factor (that is used 2 times). $\rightarrow 3\sqrt[2]{2*3} \rightarrow 3\sqrt{6}$

Using Properties of Exponents to reduce the writing:

 $\sqrt[4]{32x^6} \rightarrow \sqrt[4]{32*x^4*x^2}$ $\rightarrow x\sqrt[4]{32*x^2}$ $\rightarrow x\sqrt[4]{2^4*x^2}$ $\rightarrow 2x\sqrt[4]{2x^2}$

Factor the numerator!



Simplify

 $\frac{32}{9x^2}$ $\frac{\sqrt{50y^2}}{\sqrt{2y}}$ 49 48 49

<u>Rationalizing the denominator</u>: using mathematical properties to change an irrational number (or imaginary) in the denominator into a rational number.

We take advantage of the idea:

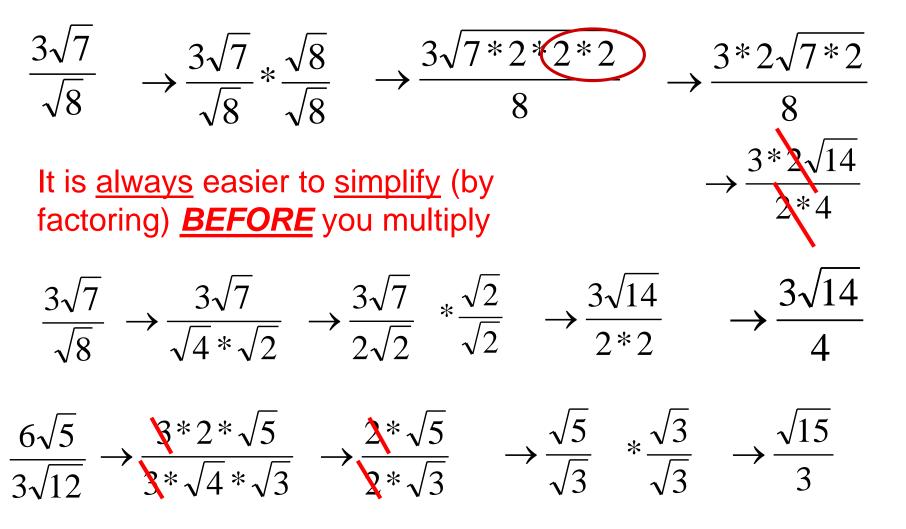
$$\sqrt{2} * \sqrt{2} = \sqrt{2 * 2} = \sqrt{4} = 2$$

$$\sqrt{3} * \sqrt{3} = \sqrt{3 * 3} = \sqrt{9} = 3$$

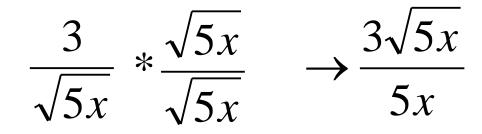
$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
Identity
Property of
Multiplication
multiplying by '1' doesn't change
the number.

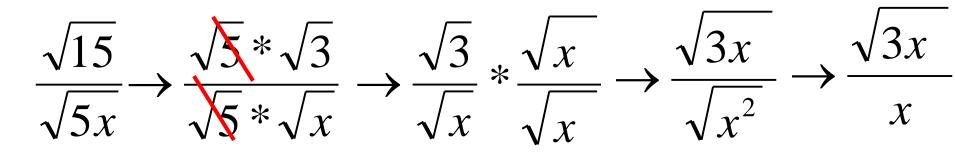
 $\frac{1}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \to \frac{\sqrt{6}}{6}$ $\frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \to \frac{2\sqrt{6}}{6} \to \frac{2*\sqrt{6}}{2*3} \to \frac{\sqrt{6}}{3}$ $\frac{25}{\sqrt{15}} * \frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{5 \times 5 \times \sqrt{15}}{5 \times 3} \rightarrow \frac{5\sqrt{15}}{3}$ $\frac{14}{3\sqrt{21}} * \frac{\sqrt{21}}{\sqrt{21}} \to \frac{14\sqrt{21}}{3*21} \to \frac{2*7*\sqrt{21}}{3*\sqrt{21}} \to \frac{2\sqrt{21}}{9}$

In all of the previous examples we just multiplied by "one in the form of" the denominator radical over the denominator radical.



What about variables?





What about higher index numbers?

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x * x}}{\sqrt[3]{x * x}} \to \frac{\sqrt[3]{x * x}}{\sqrt[3]{x * x * x}} \to \frac{\sqrt[3]{x^2}}{x}$$

How many more 'x's are needed in the denominator radicand? <u>Remember</u>: the cubed root of x-cubed equals x. $\sqrt[3]{\chi^3} = \chi$

We need two more x's under the denominator radical.

Using the <u>multiply powers property</u> we don't have to write out all the individual x's.

$$\frac{1}{\sqrt[3]{x}} * \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \to \frac{\sqrt[3]{x^2}}{x}$$

Rationalize the denominator

2x/3x1 3 4 $2\sqrt{3}$

1

What about higher index numbers?

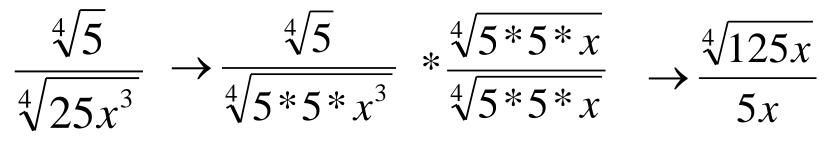
$$\frac{4}{12\sqrt[3]{4x}} \rightarrow \frac{1^{*4}}{4^{*3}\sqrt[3]{2^{*2}x}} \rightarrow \frac{1}{3\sqrt[3]{2^{*2}x}}$$

How many more <u>'2's' and 'x's</u> are needed in the denominator radicand?

We need <u>one more '2' and two more 'x's</u> under the denominator radical.

$$\rightarrow \frac{1}{3\sqrt[3]{2*2*x}} \quad *\frac{\sqrt[3]{2*x*x}}{\sqrt[3]{2*x*x}} \rightarrow \frac{\sqrt[3]{2x^2}}{3^{*3}\sqrt{2*2*2*x}} \times x^{*x}x$$
$$\rightarrow \frac{\sqrt[3]{2x^2}}{3^{*2}x} \quad \rightarrow \frac{\sqrt[3]{2x^2}}{6x}$$

What about higher index numbers?



How many more <u>'5's' and 'x's</u> are needed in the denominator radicand?

We need <u>one more '5's and one more 'x'</u>under the denominator radical.