Math-2

Lesson 2 – 2
Properties of Exponents

Properties of Exponents

What is a power?

<u>Power</u>: An <u>expression</u> formed by repeated multiplication of the <u>base</u>.

$$3x^4$$

Coefficient

Base

Exponent

The exponent applies to the number or variable <u>immediately</u> to its left, not to the coefficient !!!

No Exponent? $3x = 3^1 x^1$

Usually, we don't write the exponent '1' (saves ink).

No Coefficient? $x^3 = 1 * x^3 = 1^1 * x^3$

Usually, we don't write the coefficient '1' (saves ink).

Negative? $-x^2 = (-1) * x^2 = (-1)^1 * x^2$

Usually, we don't write the coefficient '-1', we just put the "negative symbol" (saves ink).

Factor: a number that is being multiplied.

x^4 means "base x used as a factor 4 times"

Exponent

Base

$$x^4 = x * x * x * x$$

Power: is repeated multiplication $x^4 = x * x * x * x$

multiplication: is repeated addition $3x = x + x + x$

(<u>adding</u> two terms)

$$3x + 4x \quad = (x + x + x) + (x + x + x + x)$$

$$3x + 4x = 7x$$

$$2x^2 + 3x^2 = (x^2 + x^2) + (x^2 + x^2 + x^2)$$

$$2x^2 + 3x^2 = 5x^2$$

(<u>multiplying</u> two terms)

$$x^2 * x^3 = (x * x)(x * x * x)$$

$$x^2 * x^3 = x^5$$

$$3x^2 = ? \quad = 3x^2$$

There's no way to simplify this anymore.

$$(3x)^2 = ? \quad = (3x)(3x) \quad = 9x^2$$

$$4(3x)^2 = ?$$

In GEMA, exponents occur before multiplication.

$$= 4(9x^2) \quad = 4 * 9 * x^2 \quad = 36x^2$$

$$\left(\frac{x}{2}\right)^2 = ? \quad = \left(\frac{x}{2}\right)\left(\frac{x}{2}\right) \quad = \frac{x^2}{4}$$

$$\left(\frac{2}{3x}\right)^3 = ? = \left(\frac{2}{3x}\right)\left(\frac{2}{3x}\right)\left(\frac{2}{3x}\right) \quad = \frac{8}{27x^3}$$

Simplify

$$(4y)^2 = ? \quad = 16y^2$$

$$2(5x)^2 = ? \quad = 50x^2$$

$$\left(\frac{-2}{x}\right)^4 = ? \quad = \frac{16}{x^4}$$

$$\left(\frac{x}{2}\right)^3 = ? \quad = \frac{x^3}{8}$$

Multiply Powers Property

$$(x^2)(x^3) = (x * x)(x * x * x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)(x^3) = x^2 x^3 = x^{2+3} = x^5$$

‘x’ used as a factor five times

When you multiply powers having the same base, you add the exponents.

Exponent of a Power Property $(x^2)^3$

$$(x^2)^3 = (x*x)(x*x)(x*x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)^3 = \quad = x^6$$

‘x’ used as a factor six times

$$(x^2)^3 = x^{2*3} = x^6$$

you multiply the exponents.

Exponent of a Product Property

$$(xy)^2 = (xy)(xy) = x*y*x*y = x*x*y*y$$

$$= x^2y^2$$

$$(xy)^m = x^m y^m$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product!!

$$(x-y)^2 \neq x^2 - y^2$$

$$(x-y)^2 = (x-y)(x-y)$$

$$= x^2 - 2xy + y^2$$

Combination of

1. Power of a Product
2. Power of a Power

$$(3x^3y^4)^2 = (3^1x^3y^4)^2$$
$$= 3^2x^6y^8$$

Constants (integer, etc.) usually have an exponent of ‘1’.

‘x’ is a number, we just don’t know what it is. You treat all numbers the same (whether they are variables or constants).

$$3x^2(4x^3) = ? \quad = 3*4*(x^2)(x^3) \quad = 12x^5$$

You can re-arrange the order of multiplication.

<u>Coefficients</u> of the powers are handled separately from the base and the exponent.

$$(x^2)^5 = ? \quad = x^{10}$$

$$(5x^2)(2x^3) = ? \quad = 10x^5$$

$$(2x)\left(\frac{1}{2}x^3\right) = ? \quad = x^4$$

$$5(x)^3 x^4 = ? \quad = 5x^7$$

$$(2y^5)^3 = ? \quad = 8y^{15}$$

Be careful of exponents of negative numbers

$$(-x^3y^4)^2$$

$$=((-1)^1x^3y^4)^2$$

Turn negative signs into multiplication by -1.

$$=(-1)^2x^6y^8$$

This way you will be able to tell if the simplified version is positive or negative.

$$=x^6y^8$$

$$(-2x^2y^6)^3$$

Negative coefficients have an exponent of ‘1’.

$$=((-2)^1x^2y^6)^3$$

$$=(-2)^3x^6y^{18}$$

A negative number raised to an odd exponent remains negative.

$$=-8x^6y^{18}$$

simplify

$$(-2x^2y^4z)^3 \quad = -8x^6y^{12}z^3$$

$$2(-m^4x^3)^5 \quad -2w^{20}x^{15}$$

$$-3(-2x^2yz^3)^4 \quad = -48x^{12}$$

What is the difference between?

$$(x)^4 \, and \; \mathrm{x}^4$$

$$(x^2)^3 \text{ and } (x^3)^2$$

$$x^4 x^3 \text{ and } x^3 x^4$$

$$(x+1)^2 \text{ and } (x+1)(x+1)$$

Negative Exponent Property "Grab and drag"

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^2}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

$$x^2 y^{-2} = \frac{x^2}{y^2}$$

$$\left(\frac{1}{x^3}\right)^{-2} = \left(\frac{x^3}{1}\right)^2 = x^6$$

<u>Negative</u> Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^2}$$

When you “Grab and drag” the <u>base and its exponent</u> across the “boundary line” between numerator and denominator, you just <u>change the sign</u> of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^2}$$

Quotient of Powers Property

$$\frac{x^5}{x^2} = \frac{x * x * x * x * x}{x * x} = x * x * x = x^3$$

$$\frac{x^5}{x^2} = x^5 x^{-2} = x^{5-2} = x^3$$

This is really a silly property. We don't even need to memorize this as a separate property. It's just the negative exponent property.

$$\frac{x^m}{x^n} = x^{m-n}$$

Power of a Quotient Property

$$\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right) \qquad = \frac{x^2}{y^2}$$

General form of
Power of a quotient:

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

This is another silly property. Isn’t it just exponent of a product?

Zero Exponent Property

Any base raised to the zero power simplifies to one.

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$2^0 = 1$$

$$(2x)^0 = 1$$

$$2x^0 = 2*1 = 2$$

<u>Combination</u>: (1) Negative Exponent, (2) Product of Powers, (3) Power of a Power, (4) Power of a Quotient

$$\left(\frac{3x^2}{2x^{-4}y}\right)^2 = \left(\frac{3x^2x^4}{2y}\right)^2 = \left(\frac{3x^6}{2y}\right)^2 = \left(\frac{3^1x^6}{2^1y^1}\right)^2$$

Use the negative exponent property to “grab and drag” same-based powers so that they are adjacent to each other.

Now you can combine the two same-based powers into one power using the multiply powers property.

$$= \frac{3^{1*2}x^{6*2}}{2^{1*2}y^{1*2}} = \frac{3^2x^{12}}{2^2y^2} = \frac{9x^{12}}{4y^2}$$

$$\frac{32x^{10}}{x^2y^{17}} = \frac{32x^{10}x^{-2}}{y^{17}} = \frac{32x^{10-2}}{y^{17}} = \boxed{\frac{32x^8}{y^{17}}}$$

"Grab and drag"

Product of powers: add the exponents of <u>same based powers</u>

$$\frac{3x^2}{2x^{-4}y} = \frac{3x^2x^4}{2y} = \frac{3x^{2+4}}{2y} = \boxed{\frac{3x^6}{2y}}$$

Do you "grab and drag (<u>up</u> or <u>down</u>)??

$$\frac{3x^2}{2x^{-4}y} = \frac{3}{2x^{-4}x^{-2}y} = \frac{3}{2x^{-4-2}y} = \frac{3}{2x^{-6}y} = \boxed{\frac{3x^6}{2y}}$$

<u>It doesn't matter!!!!</u>

Do you "grab and drag (<u>up</u> or <u>down</u>)??

It doesn't matter!!!!

$$\frac{3x^2}{2x^{-4}y} = \frac{3x^2x^4}{2y} = \frac{3x^{2+4}}{2y} = \boxed{\frac{3x^6}{2y}}$$

<u>Product of powers property</u>: add the exponents of like-based powers

$$\frac{3x^2}{2x^{-4}y} = \frac{3}{2x^{-4}x^{-2}y} = \frac{3}{2x^{-4-2}y} = \frac{3}{2x^{-6}y} = \boxed{\frac{3x^6}{2y}}$$

<u>Product of powers property</u>: add the exponents of like-based powers

<u>Make sure</u> when you're all done, there are <u>NO NEGATIVE EXPONENTS</u> remaining.

$$\left(\frac{x^2}{x^4}\right)^2$$

$$\left(\frac{yx^3}{xz}\right)^4$$

$$\left(\frac{3x^0}{2x^{-1}y}\right)^2$$

$$\left(\frac{2x^2yz^{-2}}{6x^4y^3z^3}\right)^2$$