Math-2
Lesson 2-1

Number Systems

Why do we need numbers?

Lebombo Plain (Africa)

Lebombo counting sticks appeared about 35,000 years ago!

How can you write the number “zero” using a counting stick?

How can you write a negative number using a counting stick?

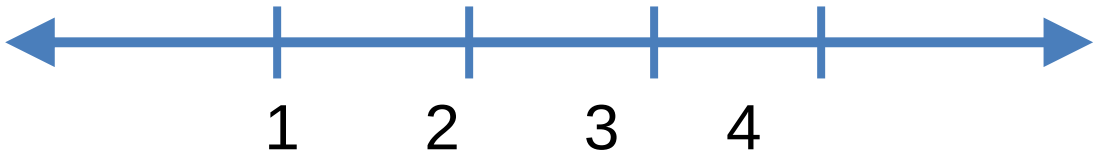
How do you count…?

1	|	6	𝍸 |
2	||	7	𝍸 ||
3	|||	8	𝍸 |||
4	||||	9	𝍸 ||||
5	𝍸	10	𝍸 𝍸

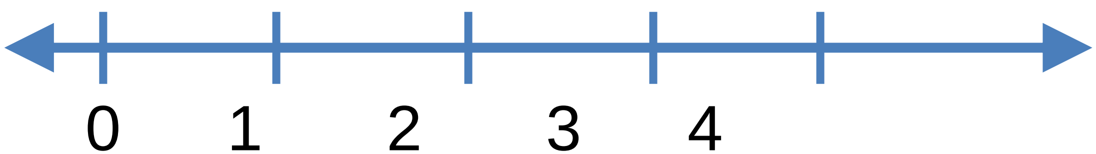
A few horses?

Vocabulary

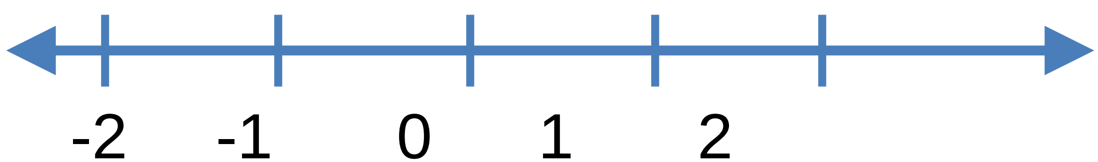
<u>Natural numbers</u>: the positive “counting” numbers that are usually shown on a number line.



<u>Whole numbers</u>: the natural numbers and the number <u>zero.</u>



<u>Integers</u>: the whole numbers and the negative “counting” numbers.



Can anyone interpret what the following means?

$$\text{Rational numbers} = \left\{ R : R = \frac{a}{b}; \text{a, b} \in \text{integers} \right\}$$

Vocabulary

Rational numbers: can be written as a ratio of integers: ½, -⅔, etc.

When converting a rational number into its decimal form (using division) the decimal with either “terminate” (1/2 = 0.5) or repeat (2/3 = 0.66666…).

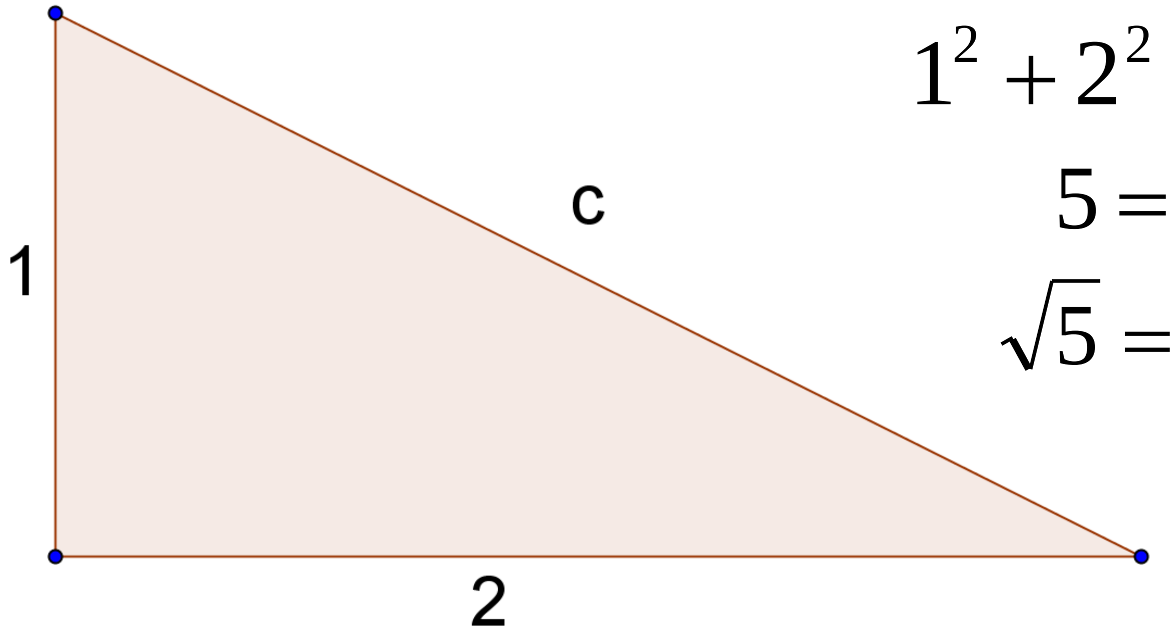
Write the integer -3 as a rational number.

Are these all the same thing? $\frac{-3}{1}, \frac{3}{-1}, -\frac{3}{1}, \frac{-3}{-1}$

Why is -3 not equal to $\frac{-3}{-1}$?

If the triangle below is a right triangle, how can we find length ‘c’ (the hypotenuse)?

Pythagorean Theorem: If it’s a right triangle, then side lengths can be related by: $a^2 + b^2 = c^2$



$$1^2 + 2^2 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$

What numbers system does SQRT(5) number belong to?

Property of Equality: if the *same operation* is applied to both sides of an equal sign, then the resulting equation is an equivalent equation (has the same solution).

$$\boxed{x = \sqrt{4}}$$

$$(x)^2 = \left(\sqrt{4}\right)^2$$

Square both sides of the equation

$$x^2 = 4$$

$$\boxed{x = 2}$$

The same value of x makes both the 1st and last equation true (x = 2) .

Vocabulary

Irrational numbers: cannot be written as a ratio of integers: ½, -⅔, etc.

The decimal version of an irrational number never terminates and never repeats. (0 = 5.13257306…).

If we see the radical symbol, the number is usually irrational (unless it is a “perfect square).

$$\sqrt{3}$$

$$\sqrt{4} = 2 \text{ (rational \#)}$$

Identify the number system

(1) $\frac{2}{3}$ (2) $\sqrt{7}$ (3) 5.25 (4) 26 (5) π

Natural
Whole
Integer
Rational
Irrational

Exact vs. Approximate:

Exact: $\pm\sqrt{17}$

Approximate: $\approx \pm 4.1231056 \ldots.$

$\approx \pm 4.123106 \ldots.$

$\approx \pm 4.12311 \ldots.$

$\approx \pm 4.1231 \ldots.$

$\approx \pm 4.123 \ldots.$

$\approx \pm 4.12 \ldots.$

$\approx \pm 4.1 \ldots.$

Converting an irrational number into a *decimal* requires you to round off the decimal somewhere.

$$\sqrt{-1}$$

The square root of -1: $$x = \sqrt{-1}$$

really means, "what number squared equals -1".

$$x^2 = -1$$

What real number when squared becomes a negative number ?

It doesn't exist so it must be an "imaginary number"

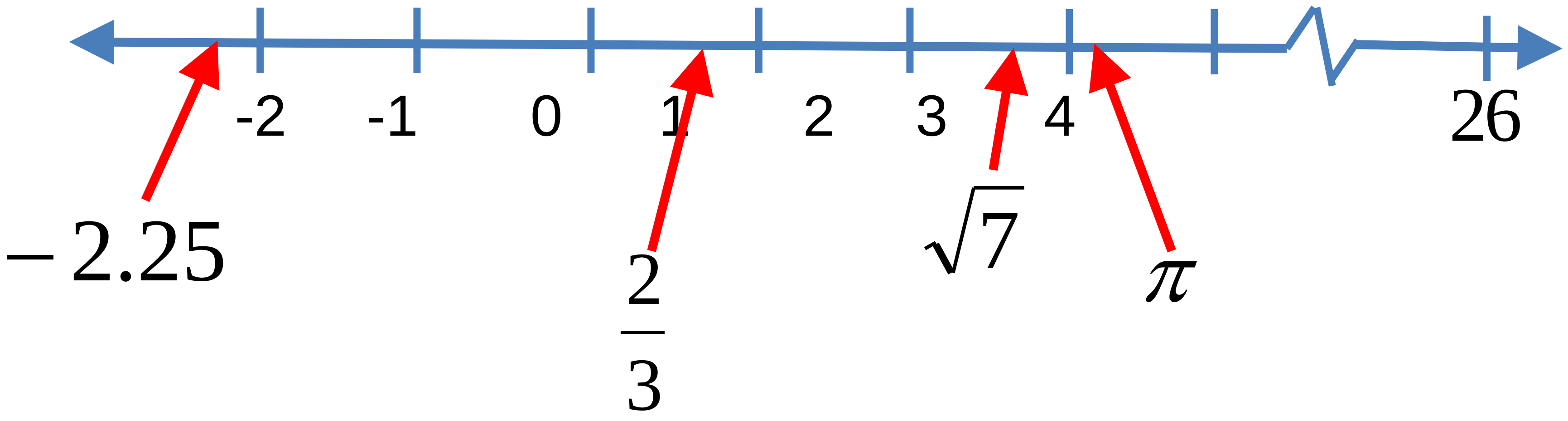
$$\sqrt{-3} = \sqrt{(-1)*3} = \sqrt{(-1)}*\sqrt{3} = i\sqrt{3}$$

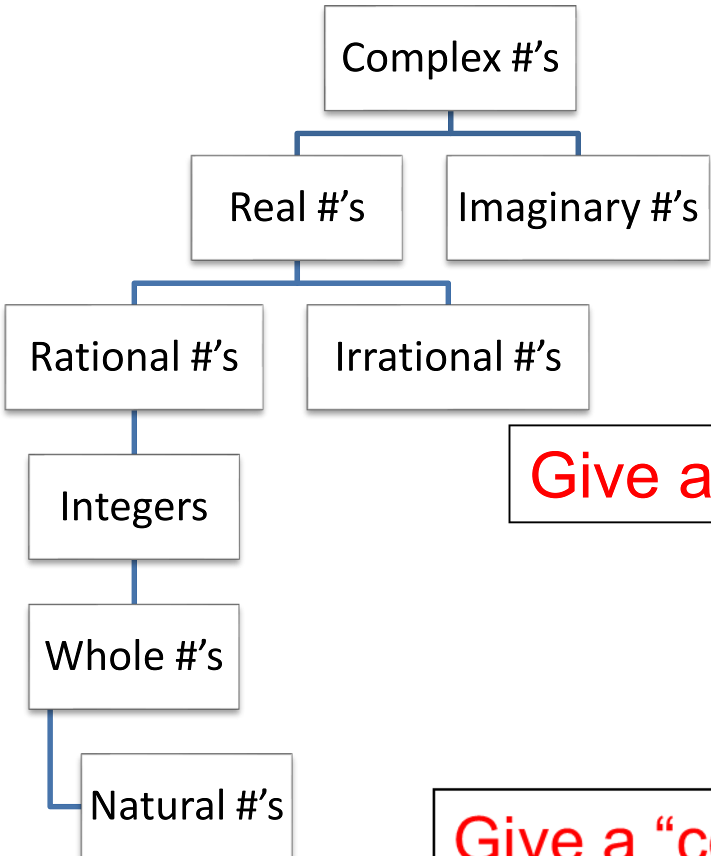
Vocabulary

imaginary numbers: a number that includes the square root of a negative number.

$$\sqrt{-1} \qquad i\sqrt{3} \qquad \sqrt{-3}$$

real numbers: a number that can be found on the number line.





$$1 + 2 = 3$$

$$\text{natural} + \text{natural} = \text{natural}$$

Is this always true?

$$4 \div 2 = 2$$

$$\text{natural} \div \text{natural} = \text{natural}$$

Is this always true?

Give a “counter example”: ________________

$$3 - 1 = 2$$

$$\text{natural} - \text{natural} = \text{natural}$$

Is this always true?

Give a “counter example”: ________________

$$2 * 3 = 6$$

$$\text{natural} * \text{natural} = \text{natural}$$

Is this always true?

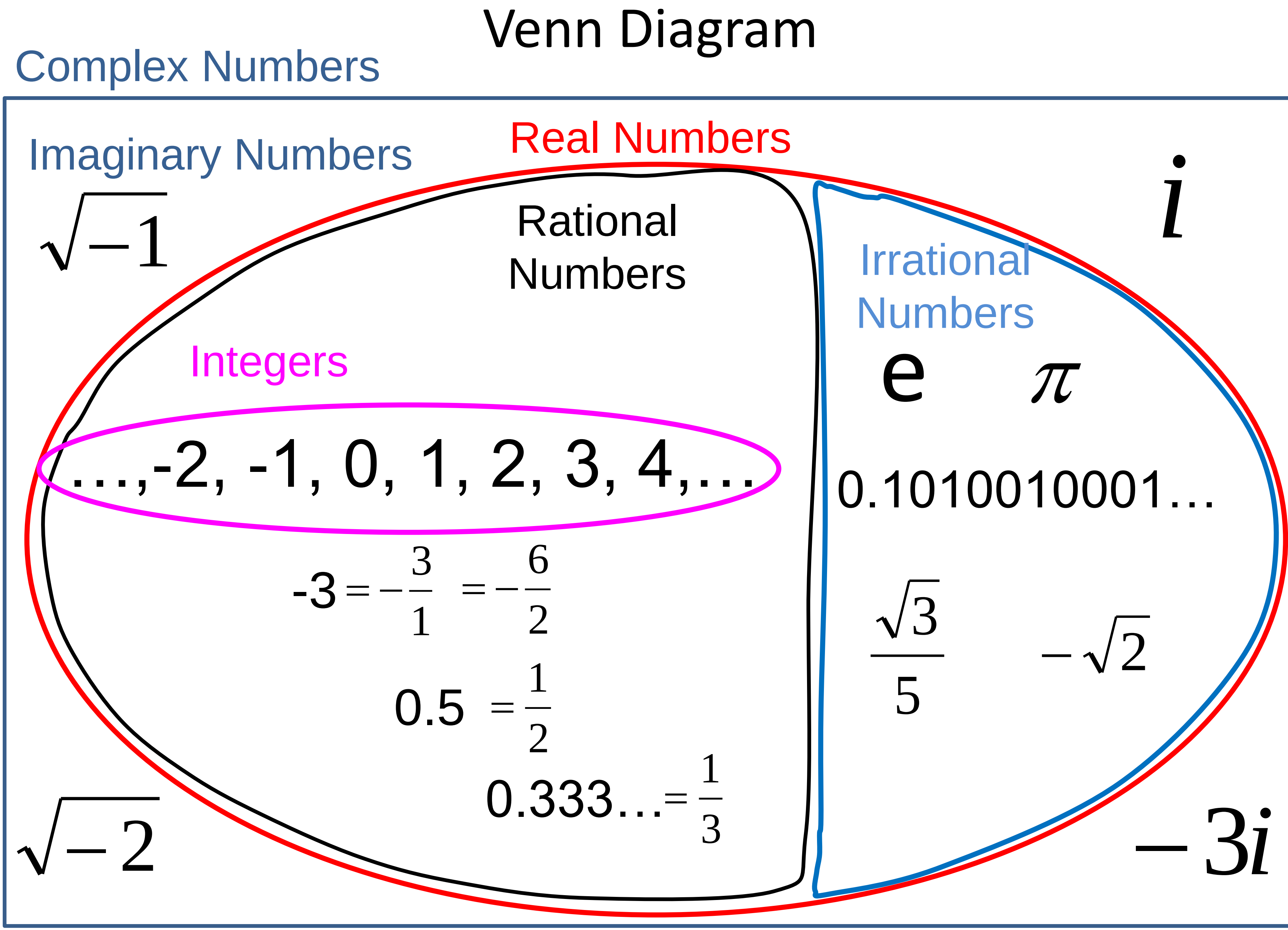
Vocabulary

<u>Closure</u>: a number system is "closed" for a particular operation (add, subtract, multiply, divide, etc.) when two numbers have an operation performed on them and the resulting number is <u>still in the number system</u>.

We say that <u>whole numbers</u> and <u>natural numbers</u> are <u>not closed</u> "under" subtraction.

Is there another operation for which the <u>whole numbers</u> or the <u>natural numbers</u> are <u>not closed</u>?

$$\frac{-7}{0} = ? \qquad \frac{1}{2} = ?$$


Venn Diagram
Complex Numbers
Imaginary Numbers
$\sqrt{-1}$
$\sqrt{-2}$
Real Numbers
i
$-3i$
Rational Numbers
Integers
…,-2, -1, 0, 1, 2, 3, 4,…
$-3 = -\frac{3}{1} = -\frac{6}{2}$
$0.5 = \frac{1}{2}$
$0.333\ldots = \frac{1}{3}$
Irrational Numbers
e
π
0.1010010001…
$\frac{\sqrt{3}}{5}$
$-\sqrt{2}$

Which number system came first?

Man probably invented the natural number system first.

When the idea of zero was no longer "scary", then it was probably added to form the whole number system.

Then smart people started doing "math" with the numbers in the system.

$$1 + 2 = 3 \text{ duh!}$$

What's wrong with this?

$$1 - 2 = ?$$

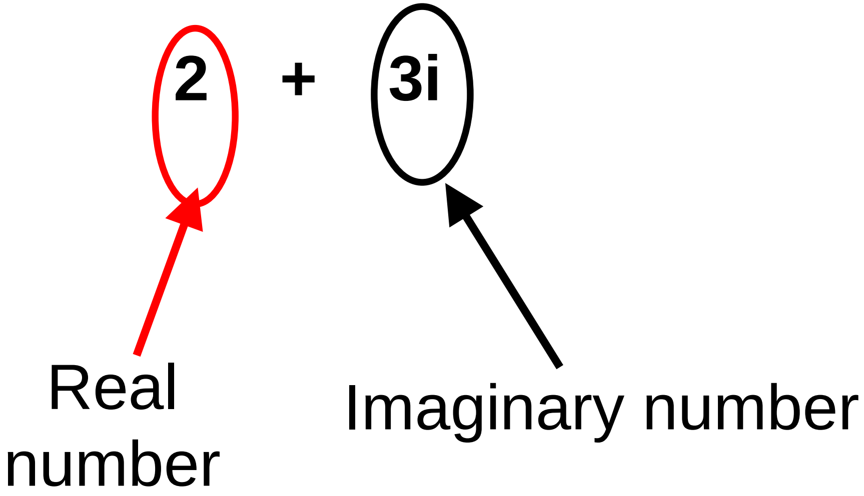
One subtract two is not in the whole number system!!!

They needed a new number system!!!

New number systems are needed when a number system is not "closed" for a particular operation (the square root of -1)

What number system is closed for all operations?

The <u>Complex Number System.</u> **a + bi**



Adding and Subtracting Complex #'s

$$(2 + 3i) + (4 + 7i) = ?$$

Real numbers are **NOT** "like terms" with imaginary numbers.

$$(2 + 4) \quad + \quad (3 + 7)i$$

$(2 - 3i) - (-4 - 5i) = ?$ $6 + 2i$

$7i - (2 - 3i) = ?$ $-2 + 10i$

$a - 3i = 4 + bi$ $a = 4, \; b = -3$

$a = ?, \; b = ?$

Multiplying Complex Numbers

3i * 4i

= 3 * i * 4 * i

= 3 * 4 * i * i

$$= 12i^2 \qquad i^2 = -1$$

$$= -12$$

Multiplying Complex Numbers

$2(4 + 3i) = 8 + 6i$

$(4 + 2i)(3 + 5i) = 4(3 + 5i) + 2i(3 + 5i)$

$= 12 + 20i + 6i + 10i^2$

$= 12 + 26i + 10(-1)$

$= 2 + 26i$

Additional material

1. The reason why we want to use "i" instead of $\sqrt{-1}$ is because mathematical operations are much easier for letters than with $\sqrt{-1}$

2. Multiplication is "repeated addition". x + x + x = 3x

   'x' used as an addend 3 times is the same as 3 times 'x'.

3. Exponents are "repeated multiplication". $x * x * x * x = x^4$

'x' used as a factor 4 times is the same as 'x' with an exponent of '4'.

4. If we combine items 1 and 3 we have:

$$i^3 = i^2 * i = (-1) * i = -i$$

5. “touching” means multiplication. $2x * 3x = 2 * x * 3 * x$

6. Commutative Property (of multiplication or addition): the order of the addends doesn’t matter. $2 + 3 = 3 + 2$

the order of the factors doesn’t matter $2 * 3 = 3 * 2$

→ You can rearrange the order if it makes it easier.

$$2x * 3x = 2 * x * 3 * x \quad = 2 * 3 * x * x \quad = 6x^2$$

7. We an only multiply (or add) a pair of numbers in one step.

$$2 * 3 * 4 \quad = (2 * 3) * 4 \quad = 6 * 4 \quad = 24$$