Math-2

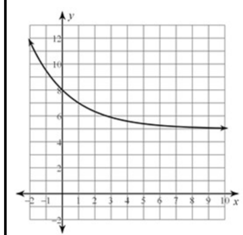
Lesson 9-5

More

Exponential Modeling

1. Money
2. Cooling

Find the equation of the graph.



1. Horizontal asymptote: y = 5

$y = AB^x + 5$

2. Passes through: (x, y) = (0, 8)

$8 = AB^0 + 5 \rightarrow 8 = A + 5 \rightarrow A = 3$

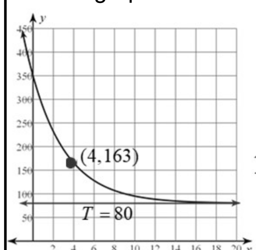
$y = 3B^x + 5$

3. Passes through: (x, y) = (1, 7)

$7 = 3B^1 + 5 \rightarrow 2 = 3B \rightarrow B = 2/3$

$\boxed{y = AB^x + k} \rightarrow \boxed{y = 3\left(2/3\right)^x + 5}$

Find the equation of the graph.



$\boxed{T(t) = AB^t + m}$

1. Horizontal asymptote: y = 80

$T(t) = AB^t + 80$

2. Passes through: (t, T) = (0, 350)

$350 = AB^0 + 80 \rightarrow 270 = A$

$T(t) = 270B^t + 80$

3. Passes through: (t, T) = (4, 163)

$163 = 270B^4 + 80 \rightarrow \frac{163-80}{270} = B^4 = 0.3074$

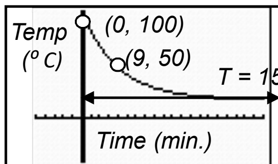
$\rightarrow B = \sqrt[4]{0.3074} = 0.7446$

$\rightarrow \boxed{T(t) = 270(0.7446)^t + 80}$

2. What will be the temperature in 10 minutes?

$T(10) = 270(0.7446)^{10} + 80$

$T(10) = 94.1\ F$



Boiling water (100º C) is taken off the stove to cool in a room at 15º C. After 9 minutes, the water's temperature is 50 C.

Write the modeling equation as a base 'b' exponential.

$T(t) = a(b)^t + k$

1) Horizontal Asymptote

$T(t) = a(b)^t + 15$

2) y-intercept

$100 = a(b)^0 + 15$

$a = 85$

3) "nice point"

$50 = 85(b)^9 + 15$

$\left(\frac{50-15}{85}\right) = (b)^9$

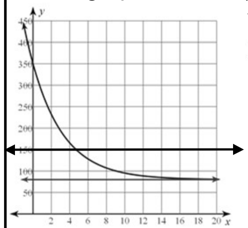
$\left(\frac{50-15}{85}\right)^{1/9} = b$

$b = 0.906$

4) Final equation

$\boxed{T(t) = 85(0.906)^t + 15}$

Find the equation of the graph.



1. Find 't' to reach 150 F

$T(t) = 270(0.7446)^t + 80$

$150 = 270(0.7446)^t + 80$

$\frac{150 - 80}{270} = (0.7446)^t$

→ $0.2593 = (0.7446)^t$

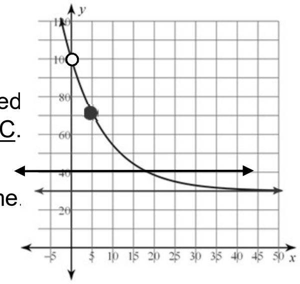
How do we solve for 't'?

1. "Guess and check" → build a table and try some values for 't'

t	4	4.2	4.4	4.5	4.6
T	0.307	0.290	0.273	0.265	0.258

2. Solve by graphing: $y_1 = 270(0.7446)^t + 80$
$y_2 = 150$

A cup of hot water is taken out of the microwave oven. Its initial temperature is 100 C. It is placed on the counter in a room whose temperature is 30 C. In 5 minutes it has cooled to 72 C. When will it reach 40 C.



1. Draw a graph that shows temperature as a function of time.

2. What is the equation of the graph? (use the following equation). $T(t) = AB^t + k$

3. Draw a horizontal line for T = 40

4. Solve by graphing

$y_1 = 70(0.903)^t + 30$
$y_2 = 40$

A cake taken out of the oven at temperature of 450° F. It is placed on in a room with an ambient temperature of 75°F to cool. 10 minutes later the temperature of the cake is 180°F. When will the cake be cool enough to put the frosting on (90°F)? (t=?, 90°F)

Start with either:

$T(t) = AB^t + k$

$T(t) = 375(0.8805)^t + 75$

$T(t) = 90$

Solve by graphing

A hard-boiled egg at temperature 212° F is placed in 60° F water to cool. 5 minutes later the temperature of the egg is 95° F. When will the egg be 75°C?

A cake taken out of the oven at temperature of 350° F. It is placed on in a room with an ambient temperature of 70°F to cool. Ten minutes later the temperature of the cake is 150°F. When will the cake be cool enough to put the frosting on (90°F) ?

You deposit $100 money into an account that pays 3.5% interest per year. The interest is "compounded" monthly. How much money will be in the account at the end of the 1st year?

$$A(1) = \$100 + \$100(0.035)$$

Original amount ($100) will still be in the account.

There will be a small amount of growth (3% of $100)

Factor out the common factor $100

$$A(1) = \$100(1+0.035) = \$100(1.035)$$

$$A(2) = \$100(1.035)^2$$

$$A(3) = \$100(1.035)^3$$

$$A(t) = \$100(1.035)^t$$

$$A(t) = A_0(1+r)^t$$

A bank pays 3% interest per year, and they pay you each month, what is the monthly interest rate?

$$\frac{0.03}{\cancel{year}} * \frac{\cancel{year}}{12\ months} \rightarrow \frac{0.03}{12\ month} \rightarrow \frac{0.03}{12}\ per\ month$$

$$\rightarrow 0.0025\ per\ month$$

A bank pays 5% interest per year, and they pay you each month, what is the monthly interest rate?

$$0.05\ per\ year \rightarrow \frac{0.05}{12}\ per\ month \rightarrow 0.0042\ per\ month$$

The exponential growth equation for money in a bank for account where the bank pays you more frequently than at the end of the year is:

$$A(t) = A_0(1 + r/k)^{k*t}$$

- Amount of $$ in the account as a function of time → $A(t)$
- Initial value → A_0
- Annual interest rate → r
- Years after the deposit → t
- # of times the bank pays you each year → k

"Compounding period" → the number of times the bank pays you each year.

"A bank pays 3% per year compounded monthly."

$$A(t) = A_0(1 + 0.03/12)^{12*t}$$

Values of "k"	
Words to look for	K
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365

You deposit $100 money into an account that pays 3.5% interest per year. The interest is "compounded" monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{k*t}$$

$$A(5) = 100\left(1 + \frac{0.035}{12}\right)^{12*5}$$

$$A(5) = \$119.09$$

Interest paid at the end of each month

$$A(t) = A_0(1+r)^t$$

$$A(5) = 100(1+0.035)^{(5)}$$

$$A(5) = \$118.77$$

Interest paid at the end of each year

You deposit $200 money into an account that pays 5.5% interest per year. How much money will be in the account at the end of the 20th year?

$$A(t) = A_0(1+r)^t$$

$$A(20) = \$200(1+0.055)^{(20)}$$

$$A(20) = \$583.55$$

You buy a car for $18,500. It depreciates at 15% per year. What is the value of the car (what you could sell it for) after 7 years?

$$V(t) = V_0(1-r)^t$$

$$V(t) = 18{,}500(1-0.15)^{(t)}$$

$$V(t) = 18{,}500(0.85)^{(t)}$$

What is the growth factor? Is it "growth" or "decay"?

$$V(7) = 18{,}500(0.85)^{(7)}$$

$$V(7) = \$5930.68$$

You deposit $100 money into an account that pays 3.5% interest per year. The interest is "compounded" monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1+{}^{r}\!/_{k})^{kt} \qquad A(5) = 100(1+{}^{0.035}\!/_{12})^{12(5)}$$

$$A(5) = \$119.09$$

What is the doubling time for this account?

$$200 = 100(1+{}^{0.035}\!/_{12})^{12t}$$

$$2 = (1.0029)^{12t}$$

$$y_1 = (1.0029)^t$$
$$y_2 = 2$$

Solve by graphing

You deposit $200 money into an account that pays 5.5% interest per year. The interest is "compounded" quarterly. How long will it take for your money to triple?

$$A(t) = A_0(1+{}^{r}\!/_{k})^{kt} \qquad 600 = 200(1+{}^{0.055}\!/_{4})^{4(t)}$$

$$3 = (1.0138)^{4t}$$

$$y_1 = (1.0138)^t$$
$$y_2 = 3$$

Solve by graphing