Math-2
Lesson 9-2

Function Composition

Function Notation

y = f(x) "y is a function of x"

'y' equals 'f' of 'x'

A function is a <u>rule</u> that matches input values to out put values.

f(x) = 2x + 1

(Input) x	(rule) 2x + 1	(output) y
2	2(2) + 1	5
3	2(3) + 1	7

f(2) = 5

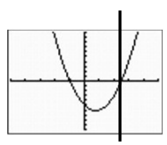
f(3) = 7

1. Is the following relation a function?

(-2, 5), (5, 6), (-2, 6), (7, 6)

No. Input value -2 has two output values.

2. Is the following relation a function?



Does the graph of the relation pass the "vertical line test" ?

Yes. Each input value has exactly one output value.

Compositions of Functions

f(x) = 2x → f(3) = ?

Means: wherever you see an 'x' in the function, replace it with a 3.

1. Replace the 'x' with a set of parentheses.

f(3) = 2( )

2. Put the input value '3' into the parentheses.

f(3) = 2(3)

3. Find the output value.

f(3) = 6

<u>Compositions</u> of Functions

$f(x) = x^2 - 3x + 2$ → f(2) = ?

Means: wherever you see an ‘x’ in the function, replace it with a ‘2’.

1. Replace the ‘x’ with a set of parentheses.

$$f(x) = (\ )^2 - 3(\ ) + 2$$

2. Put the input value ‘2’ into the parentheses.

$$f(x) = (2)^2 - 3(2) + 2$$

3. Find the output value.

f(2) = 0 **Cool, we found a zero of the function.**

Function Notation

	(Input) x	(rule) 2x + 1	(output) f(x)	
f(x) = 2x + 1	2	2(2) + 1	5	f(2) = 5
	3	2(3) + 1	7	f(3) = 7
f(x – 1) = 2x – 1	x – 1	2(x – 1) + 1	2x – 1	
f(3x) = 6x + 1	3x	2(3x) + 1	6x + 1	

If your input is an expression instead of a number: replace ‘x’ with parentheses and “plug in” the expression
→ parentheses, substitute, simplify

$f(x) = x^3 - 1$ $f(-2) = ?$

$f(-2) = (-2)^3 - 1$ $f(-2) = -9$

$f(x) = 2x^{\frac{1}{2}}$ $f(9) = ?$

$f(9) = 2(9)^{\frac{1}{2}}$ $f(9) = 6$

$f(x) = \dfrac{2(x-4)}{x^2 + x - 20}$ $f(-2) = ?$

$f(-2) = \dfrac{2(-2-4)}{((-2)^2 + (-2) - 20)}$ $f(-2) = \dfrac{2}{3}$

$$f(x) = 3x - 1$$

(Input) x	(rule) 3x - 1	(output) f(x)	
2	3(2) - 1	5	$f(2) = 5$
x^2	3( ) - 1	?	$f(x^2) = 3x^2 - 1$
$x + 2$	3( ) - 1	?	$f(x+2) = 3x + 5$
$3 - 2x$	3( ) - 1	?	$f(3-2x) = 8 - 6x$

Your turn:
input the expressions $f(x) = x^2 + 1$

$f(2) = ? \quad = 5$

$f(x^3) = ? \quad = x^6 + 1$

$f(x+2) = ? \quad = (x+2)^2 + 1 \quad = x^2 + 4x + 5$

$f(-2x+3) = ? \quad = (-2x+3)^2 + 1$

$= 4x^2 - 12x + 10$

Compositions of Functions

f(x) = 2x + 3 and $g(x) = x^2$

f(g(x)) = ?

1. The input value to f(x) is g(x).

$f(..) = 2(..) + 3$

2. Replace the 'x' in f(x) with a set of parentheses.

$f(x^2) = 2(x^2) + 3$

3. Put the input value (g(x)) into the parentheses.

$f(g(x)) = 2x^2 + 3$

4. Find the output value.

Compositions of Functions

f(x) = 2x and $g(x) = x^2$

Let's use f(x) as the input to g(x) $g(f(x)) = ?$

$g(..) = (..)^2$

1. Replace the 'x' with a set of parentheses.

$g(2x) = (2x)^2$

2. Put the input value "2x" into the parentheses.

$g(f(x)) = 4x^2$

3. Find the output value.

Function "composition"

$f(x) = x^2 + 1 \qquad g(x) = x^2$

$f(2) = ?$ **What does this mean?**

"Substitute '2' in for 'x' in the function f(x)."

$f(g(x)) = ?$ **What does this mean?**

"Substitute 'g(x)' in for 'x' in the function f(x)."

$f(g(x)) = (g(x))^2 + 1$

"Which means the same as…"

$f(x^2) = (x^2)^2 + 1 \quad = x^4 + 1$

Composition of Functions

f(x) = 2x + 1 g(x) = 3x + 2 h(x) = x + 5

$f(g(x)) = ? = 2(\quad) + 1 = 2(3x + 2) + 1$

$h(g(x)) = ?$ = ( ) + 5 = (3x + 2) + 5

$h(f(x)) = ?$ = ( ) + 5 = (2x + 1) + 5

$g(h(x)) = ?$ = 3( ) + 2 = 3(x + 5) + 2

$f(f(x)) = ? = 2(\quad) + 1 = 2(2x + 1) + 1$

One more layer! $f(x) = 3x$ $g(x) = x^2$

$f(g(4))$

4

$g(\ ) = (\ )^2$

$g(4) = (4)^2$

16

$f(\ ) = 3(\ )$

$f(16) = 3(16)$

48

$f(g(x)) = 3(g(x)) = 3x^2$

$f(g(4)) = 3(g(4)) = 3(4)^2$ **= 48**

New Notation for the Composition of Functions

$(f \circ g)(x) = f(g(x))$ "g" plugged into rule "f"

$f(x) = 4x - 1$ $g(x) = -5x + 3$

$(f \circ g)(x) = ?$ $= 4(\quad) - 1$ $= 4(-5x + 3) - 1$

"g" plugged into rule "f" $(f \circ g)(x) = -20x + 11$

$(g \circ f)(x) = ?$ $= -5(\quad) + 3$ $= -5(4x - 1) + 3$

"f" plugged into rule "g" $(g \circ f)(x) = -20x + 8$

$(f \circ f)(x) = ?$ $= 4(\quad) - 1$ $= 4(4x - 1) - 1$

"f" plugged into rule "f" $(f \circ f)(x) = 16x - 5$

$(g \circ g)(x) = ?$ $= -5(\quad) + 3 = -5(-5x + 3) + 3$

"g" plugged into rule "g" $(g \circ g)(x) = 25x - 12$

One more layer. $g(x) = x^2$ $f(x) = 3x$

$(g \circ f)(-1) = ?$ Rewrite in "old" notation

$g(f(-1)) = ?$ The input to f(x) is -1.

$f(-1) = 3(-1)$

$f(-1) = -3$ The output of f(-1) is -3.

The input to g(x) is -3.

$g(-3) = 9$

$g(f(-1)) = 9$