Math-2

Lesson 9-1
Combine Functions

Combining Functions Algebraically

Multiplication by a number

$f(x) = x^2 - 1$

$2f(x) = ?$ Replace "f(x)" with parentheses

2 (   )

Substitution step → $f(x) = x^2 - 1$

$2f(x) = 2(x^2 - 1)$ Simplify →

$f(x) = x^2 - 1$ $2f(x) = 2x^2 - 2$

Perform the indicated operation then simplify:

$f(x) = 4x + 2$ $g(x) = -5x + 3$

3f(x) = ? $= 3(4x + 2)$ $= 12x + 6$

-2g(x) = ? $= -2(-5x + 3)$ $= 10x - 6$

5g(-3) = ? $\Rightarrow g(-3)$ $= -5(-3) + 3$

$g(-3) = 18$

$\Rightarrow 5g(-3) = 5(18) = 90$

"Linear combinations of functions"

$f(x) = 4x + 2$ $g(x) = -5x + 3$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

f(x) - g(x) = ?   = (   ) − (   )

= ($4x + 2$) − ($-5x + 3$)

$= 4x + 2 + 5x - 3$

f(x) - g(x) $= 9x - 1$

Perform the indicated operation:

$$f(x) = 4x + 2 \qquad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$g(x) - 2\ f(x) \quad = (\quad\quad) - 2(\quad\quad)$$
$$= (-5x + 3) - 2(\ 4x + 2\ )$$
$$= -5x + 3 - 8x - 4$$
$$g(x) - 2\ f(x) = -13x - 1$$

Perform the indicated operation:

$$f(x) = 4x + 2 \qquad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$-3\ f(x) - 2\ g(x) \ = -3(\quad\quad) - 2(\quad\quad)$$
$$= -3(\ 4x + 2\ ) - 2(-5x + 3)$$
$$= -12x - 6 + 10x - 6$$
$$-3\ f(x) - 2\ g(x) = -2x - 12$$

Perform the indicated operation:

$$f(x) = x^2 + 4x - 21 \qquad g(x) = x - 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$f(x) \div g(x) \quad = \frac{(\quad\quad)}{(\quad\quad)} = \frac{(x^2 + 4x - 21)}{(x - 3)}$$
$$= \frac{(x + 7)(x - 3)}{(x - 3)}$$
$$f(x) - g(x) = x - 3$$

Perform the indicated operation:

$$f(x) = x^2 - 5x - 14 \qquad g(x) = x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$g(x) \div 2\ f(x) \quad = \frac{(\quad\quad)}{(\quad\quad)}$$
$$= \frac{(x + 3)}{2(x^2 - 5x - 14)} = \frac{(x + 3)}{2(x - 7)(x + 2)}$$

New Notation

$$f(x) - g(x) = (f - g)(x)$$

Rewrite the following in the new notation.

$$g(x) - 2\,f(x) = (g - 2f)(x)$$

$$-3\,f(x) - 2\,g(x) = (-3f - 2g)(x)$$

$$f(x) \div g(x) = \left(\dfrac{f}{g}\right)(x)$$

Perform the indicated operation:

$$j(x) = 15x^2 \qquad k(x) = -5x^3$$

$$(2j - 3k)(x) = 2(15x^2) - 3(-5x^3)$$
$$= 15x^3 + 30x^2$$

$$(2j - 3k)(-1) = 15(-1)^3 + 30(-1)^2$$

$$(2j - 3k)(-1) = 15$$

$$\left(\dfrac{2k}{j}\right)(x) = ? \quad = \dfrac{2(-5x^3)}{15x^2}$$

$$= \dfrac{\cancel{5}*(-2)*\cancel{x}*\cancel{x}*x}{\cancel{5}*3*\cancel{x}*\cancel{x}} = \dfrac{-2x}{3} = -\dfrac{2}{3}x$$

$$j(x) = 15x^2 \qquad k(x) = -5x^3$$

$(j + k)(-2)$ What does this mean?

$$(j + k)(x) = j(x) + k(x) = 15x^2 - 5x^3$$

$$j(x) + k(x) = 15x^2 - 5x^3$$

$$\qquad\qquad\text{OR, you can do it this way}$$

$$(j + k)(-2) = 15(\ )^2 - 5(\ )^3 \qquad (j + k)(-2) = j(-2) + k(-2)$$

$$= 15(-2\ )^2 - 5(-2\ )^3 \qquad j(-2) + k(-2) = 15(-2)^2 - 5(-2)^3$$

$$= 60 + 40 \qquad\qquad\qquad = 60 + 40$$

$$= 100 \qquad\qquad\qquad\qquad = 100$$

Perform the indicated operation:

$$f(x) = x^2 + 4x - 21 \qquad g(x) = x - 3$$

$(f - 2g)(-1)$

$f(-1) - 2g(-1)$

$$f(-1) = (-1)^2 + 4(-1) - 21 = -24$$

$$g(-1) = (-1) - 3 = -4$$

$$f(-1) - 2g(-1) = -24 - 2(-4)$$

$$= -24 + 8$$

$$= -16$$

Combining Functions Algebraically

Product: (fg)(x) = f(x) * g(x)

$$f(x) = x + 2 \qquad g(x) = x - 2$$

$$(fg)(x) = (x+2)(x-2)$$

$$(fg)(x) = x^2 - 4$$

$$f(x) = 3x \qquad g(x) = x - 4$$

$$(f*g)(2) = ? \qquad \textbf{OR} \qquad (f*g)(2) = f(2)*g(2)$$

$$(f*g)(x) = 3x(x-4)$$

$$f(2) = 3(2)$$

$$f(2) = 6$$

$$(f*g)(2) = 3(2)[(2)-4]$$

$$g(2) = (2) - 4$$

$$(f*g)(2) = 6(-2)$$

$$g(2) = -2$$

$$(f*g)(2) = -12$$

$$(f*g)(2) = (6)*(-2)$$

$$(f*g)(2) = -12$$

Which way is easier for you?

$$f(x) = 3x \qquad g(x) = x - 4$$

$$(2f + 3g)(-1) = ?$$

Method 1: Write as two separate functions then combine the result

$$(2f + 3g)(-1) = 2f(-1) + 3g(-1)$$

$$= 2[3(-1)] + 3[(-1) - 4]$$

$$= 2(-3) + 3(-5)$$

$$= -6 - 15$$

$$= -21$$

$$f(x) = 2x - 3 \qquad g(x) = 4x - 5$$

$$(3f + 2g)(-1) = ?$$

Method 2: Find linear combination of the functions THEN plug in.

$$(3f + 2g)(x) = 3(2x-3) + 2(4x-5)$$

$$= 6x - 9 + 8x - 10$$

$$= 14x - 19$$

$$(3f + 2g)(x) = 14x - 19$$

$$(3f + 2g)(-1) = 14(-1) - 19$$

$$= -14 - 19 \quad = -33$$