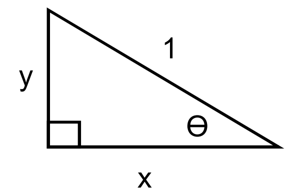
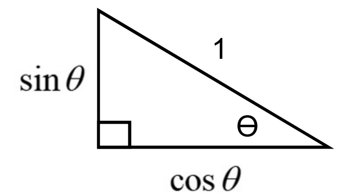
Math-2
8-6

Pythagorean Identity,
Tangent lines and Secant lines of circles,
Non-central/inscribed angles of Circles,
Dilations and Rotations on the XY Plane



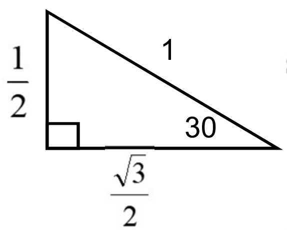


$\sin\theta = y$
$\cos\theta = x$

Back substitute to the triangle.

Write Pythagorean relationship for the triangle.

$$\sin^2\theta + \cos^2\theta = 1$$



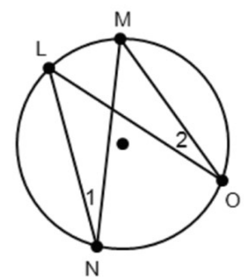
$$\sin 30 = \frac{1}{2} \qquad \cos 30 = \frac{\sqrt{3}}{2}$$

$$\sin^2\theta + \cos^2\theta = 1$$

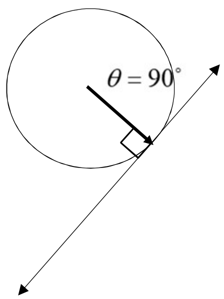
$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$$

$$\frac{1}{4} + \frac{3}{4} = 1$$

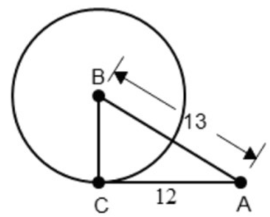
Find the measure of angles 1 and 2 if $m\angle 1 = 2x - 13$ and $m\angle 2 = x$.



<u>If</u> a tangent to a circle intersects at the endpoint of a radius, <u>then</u> the angle of intersection is 90°.



Segment AC is tangent to Circle B at point C. Find BC



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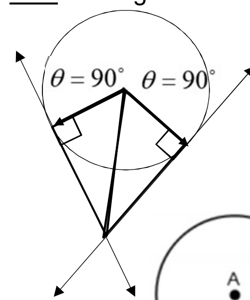
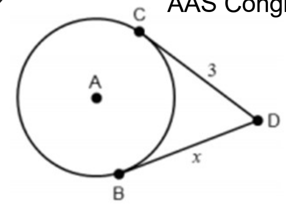


Are the two triangles congruent? If so, what congruence theorem can you use to prove congruence?

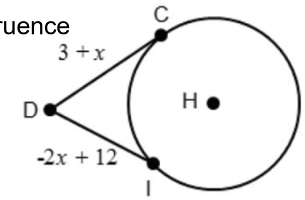
Shared side is congruent

Pair of legs are radii → congruent

AAS Congruence



$x = 3$

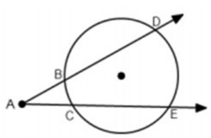


$-2x + 12 = 3 + x$

$x = 3$

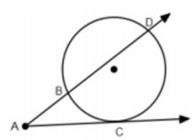
<u>If</u> two secant lines cut a circle <u>then</u> the angle of intersection is one half of the difference between the intercepted arcs.

Two Secants



$m\angle A = \frac{1}{2}(mDE - mBC)$

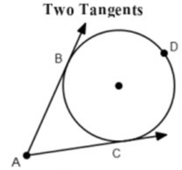
Secant and Tangent



$m\angle A = \frac{1}{2}(mDC - mBC)$

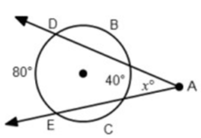
Same for a secant and a tangent line.

Two Tangents

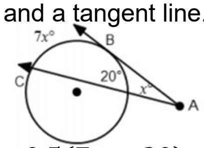


$m\angle A = \frac{1}{2}(mCDB - mBC)$

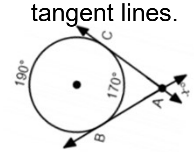
Same for two tangent lines.



$x = 0.5(80 - 40)$
$x = 20$

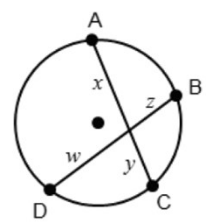


$x = 0.5(7x - 20)$
$2x = 7x - 20$
$x = 4$



$x = 0.5(190 - 170)$
$x = 10$

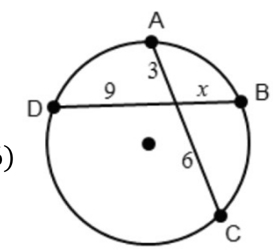
If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$xy = wz$

$9x = 3(6)$

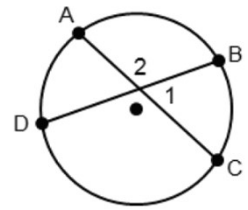
$x = 2$

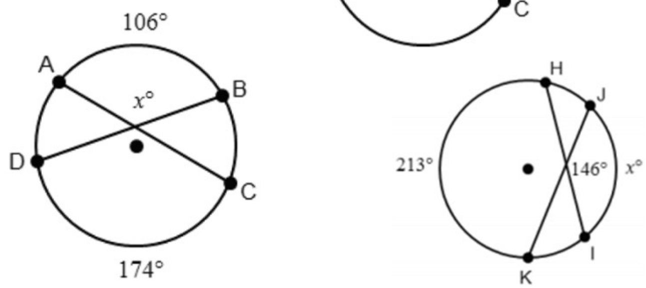


If 2 chords insect inside of a circle then the measure of the angle is the average of the two intercepted arcs.

$$m\angle 2 = \frac{1}{2}\left(mCD + mAB\right)$$

$$m\angle 1 = \frac{1}{2}\left(mBC + mAD\right)$$





Do the segments that connect the points form a parallelogram?

$F(2,1),\ G(1,4),\ H(5,4)$, and $J(6,1)$ → YES

The Hard Way

1. Find all the segment lengths (opposite sides of parallelograms are congruent).

$$GF = \sqrt{(2-1)^2 + (1-4)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$

$$HJ = \sqrt{(5-6)^2 + (4-1)^2} = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$$

$$FJ = \sqrt{(6-2)^2 + (1-1)^2} = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

$$GH = \sqrt{(5-1)^2 + (4-4)^2} = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

YES

2. Find all the slopes between the points (opposite sides are parallel).

$$m_{GF} = \frac{1-4}{2-1} = \frac{-3}{1} = -3 \qquad m_{HJ} = \frac{4-1}{5-6} = \frac{3}{-1} = -3$$

$$m_{FJ} = \frac{1-1}{6-2} = \frac{0}{4} = 0 \qquad m_{GH} = \frac{4-4}{5-1} = \frac{0}{4} = 0$$

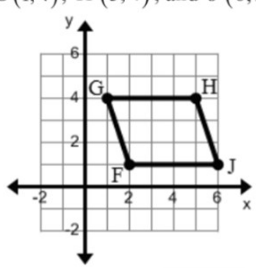
YES

Do the segments that connect the points form a parallelogram?

$F(2,1),\ G(1,4),\ H(5,4)$, and $J(6,1)$ → YES

The Easy Way

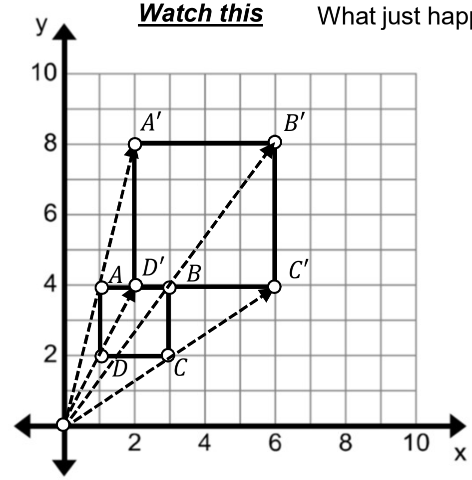
1. Graph the points



2. Are opposite sides congruent? YES

3. Are opposite sides parallel? YES

Watch this What just happened?

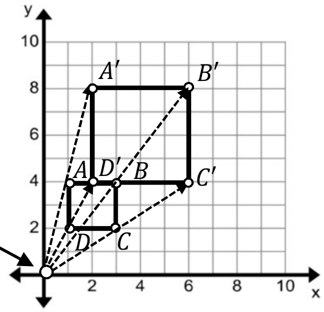


<u>Dilation</u>: a transformation that results in the same shape but a different size. It uses a *center of dilation* and a *scale factor* to create the proportional figure.

<u>Center of Dilation</u>: a fixed point in the x-y plane about which all points of the figure are expanded or contracted.

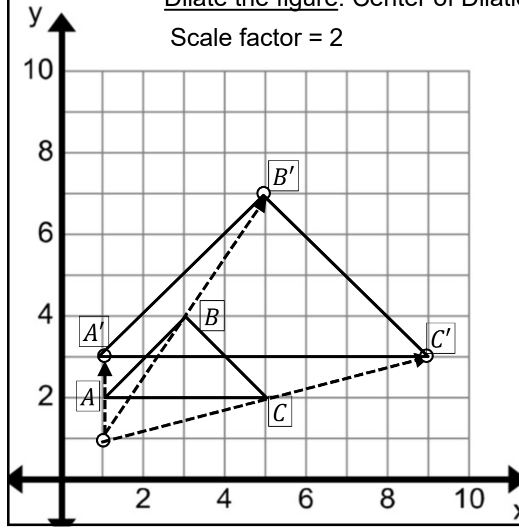
<u>Center of Dilation</u>: (0, 0)

<u>Scale factor</u> = $\frac{A'B'}{AB} = \frac{4}{2}$



<u>Dilate the figure</u>: Center of Dilation: (1, 1).

Scale factor = 2



Rotate the shape 90º in the clockwise direction.

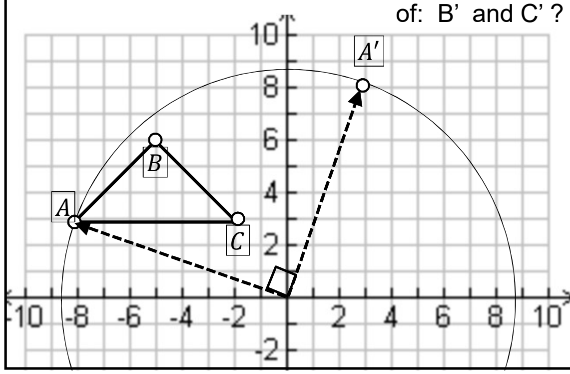
Compare the two x-y pairs. A: (-8, 3) A': (3, 8)

1) Moves the point to the adjacent quadrant of the x-y plane → +/- of points may (or may not change).

2) x-y values are exchanged.

What are the coordinates of: B' and C' ? B': (5, 5)

C': (3, 2)



Rotate the shape 180º in the clockwise direction.

A: (-8, 3) A': (8, -3) Moves the point to the opposite quadrant of the x-y plane → +/- of points change.

What are the coordinates of: B' and C' ?

B': (5, -6)

C': (2, -3)